Endogenous firm heterogeneity and the dynamics of trade liberalization☆

Josh Ederington a,⁎, Phillip McCalman b

a 335 Gatton Building, Department of Economics, University of Kentucky, Lexington, KY 40506, United States
b Department of Economics, University of California, 1156 High Street, Santa Cruz, CA 95064, United States

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Abstract

In this paper, we build a dynamic model with endogenous firm-level productivity that involves ex ante identical firms behaving differently in equilibrium. Heterogeneity arises in equilibrium as firms choose different dates to adopt a new technology. We investigate the effects of international trade on technological diffusion and show that trade has a generally positive impact on the equilibrium rate of adoption (and hence on firm-level productivity). In addition, the model can replicate the stylized fact that exporters are larger and more productive than non-exporters. Finally, we show how our model can be used to interpret the emerging empirical evidence on the firm-level productivity effects of CUSFTA.

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1. Introduction

The classic case for free trade is based on the static efficiency gains from lower trade barriers. However, a recent literature has emerged that stresses the dynamic productivity gains from liberalized trade. The most prominent theoretical papers on the link between trade and productivity are Melitz (2003) and Bernard et al. (2003). 1 Both of these models take plant-level productivity as given, and emphasize the potential productivity gains from trade-induced reallocation of market share across firms. 2 That is, they argue that trade liberalization will tend to reallocate production toward more efficient firms, generating an increase in aggregate industry productivity.

A key motivation for the development of these models is the well-documented heterogeneity that exists within industries, and empirical evidence that reallocation of production is an important source of productivity improvement in response to trade liberalization. However, the empirical evidence also points to a second prominent source of productivity gains in response to trade liberalization — increases in plant-level productivity (e.g., see Pavcnik, 2002; Syverson, 2004 for empirical evidence).

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⁎ Corresponding author.

E-mail addresses: ederington@uky.edu (J. Ederington), mccalman@ucsc.edu (P. McCalman).

1 For a review of this literature see Tybout (2002). The model of Melitz (2003) has now been extended to a multi-factor framework by Bernard et al. (2007).

2 Increased competition could also increase industry productivity by censoring the bottom tail of the productivity distribution (e.g., see Syverson, 2004 for empirical evidence).

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It is this second source of productivity gain that is the subject of this paper. In particular, we develop a dynamic model that treats firm-level productivity as endogenous, with one implication of the framework being that heterogeneous firms operate in the same industry in equilibrium. Such a model provides, as a starting point, an industry that is characterized not by a steady-state outcome, but by an evolution that depends on the choices of individual firms. In doing so, this framework offers an analysis that is complementary to the work of Melitz (2003) and Bernard et al. (2003). Specifically, we consider an environment where firms are ex ante identical, yet heterogeneity arises as an equilibrium outcome. Consequently, this framework admits not only reallocation effects but also firm-level productivity effects in response to trade liberalization.

To explain firm heterogeneity we focus on differences in adoption rates of new technologies across firms since this is one of the primary sources of heterogeneity for firms within the same industry. While one might expect profitable innovations in an industry to be adopted instantaneously (or with some small delay given costs of adjustment), the available evidence on technology adoption suggests a logistic diffusion curve with a lengthy diffusion process. Thus, in a study of 21 innovations, Jovanovic and Lach (1997) report an average of 15 years for a new technology to go from 10% usage to 90% usage. Likewise, as stated by Karshenas and Stoneman (1995):

The diffusion of new technology takes time, often a considerable period of time. Whether it be a new consumer technology spreading across households or a new producer (process) technology spreading across firms it would not be unusual for the time period between first use and say 90 percent usage of that technology to take several decades rather than several years.

This lengthy diffusion process leads naturally to persistent productivity differences across firms within an industry. In this paper, we emphasize differences in the rate of technology adoption as the endogenous source of plant-level heterogeneity. The specific question in which we are interested is how exposure to trade affects the speed of technology diffusion in an industry (and thus firm-level productivity). There is a small literature on the effects of trade on technology adoption (see Rodrik, 1992; Miyagiwa and Ohno, 1995), but only in the context of a single import-competing firm. Thus, this paper represents the first attempt to evaluate the link between trade and technology diffusion in a dynamic model of intra-industry trade with firm heterogeneity.

One reason that exogenous productivity models (such as those of Melitz, 2003 and Bernard et al., 2003) have achieved such influence in the trade literature is their ability to match some key aspects of the evidence on the link between exporting and productivity. In particular, numerous studies have found that exporting firms tend to be larger and more productive than non-exporting firms, yet there is no evidence that exporters have higher productivity growth. Thus, it is worth noting that our model of endogenous technology adoption is also capable of replicating these stylized facts. Specifically, at any point during the diffusion process, exporting firms will be, on average, larger and more productive than non-exporting firms; however, the subsequent productivity growth of these exporting firms will not necessarily be higher than that of the non-exporting firms. This is due to the fact that, in our model, firms that choose to export will also choose to adopt new cost-saving innovations earlier in the diffusion process while non-exporters adopt later in the process. Thus, we see our model of endogenous technology adoption as being complementary to the standard models of self-selection.

In addition, since the previous literature takes firm-level productivity as given, it cannot explain the impact of trade on firm productivity. Thus, in the final section of the paper, we present an asymmetric version of the model and show

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3 In the Bernard et al. (2003) model, exposure to trade can affect the measured productivity of firms by changing firm mark-ups. However, as in Melitz (2003), underlying firm productivity is taken as exogenous in the model.

4 Both Rodrik (1992) and Miyagiwa and Ohno (1995) consider the technology adoption decision of a single domestic firm engaging in Cournot competition with a foreign firm in the domestic market. In both papers, trade tends to slow down the speed of adoption by reducing the market share of the domestic firm. As we show in this paper, the implications of trade on technology diffusion are quite different in a model of intra-industry trade with free entry. As pointed out by Rodrik, a model of restricted entry is more appropriate for developing countries. We will also point out how the predictions of the model differ if entry is restricted. See also Crowley (2006).

5 Yeaple (2005) develops a static model of technology adoption and trade, where firm heterogeneity is derived from heterogeneity in the talent distribution of workers. He assumes a situation where adoption costs are such that high-tech firms export and low-tech firms do not, and shows that a reduction in trade costs will increase the share of exporting firms and, hence, the share of high-tech firms. As we discuss in Section 2, this result is nested in our model. However, the purpose of this paper is to investigate how exposure to trade affects the technology adoption decisions of the entire distribution of firms.
how it can be used to interpret the firm-level productivity effects of CUSFTA that have been documented by Trefler (2004) and Baggs et al. (2002). However, the main goal of the paper is not to provide a universal model that attempts to encompass the myriad empirical facts on the relationship between trade and productivity, but rather to provide a simple model of endogenous firm heterogeneity that is both capable of examining the positive effects of trade on the speed of technology diffusion, and can serve as a framework for future research. The basic conclusion of the paper is that trade tends to increase the rate of technology diffusion, with a reciprocal reduction in trade barriers causing the adoption process to both begin and end earlier. That is, both exporters and non-exporters adopt faster in a more liberal trading environment.

The structure of the paper is as follows. In Section 2, we solve for the open-economy equilibrium, and discuss the effects of trade liberalization on the endogenous speed of technology diffusion. In Section 3, we investigate the extent to which our model matches the empirical evidence on the Canadian/U.S. trade relationship.

2. Technology adoption in an open economy

In this section we present an open-economy model of technology adoption. To do so we extend a model of endogenous technology adoption (as in Reinganum, 1981; Fudenberg and Tirole, 1985; Götz, 1999) to allow for both international trade and endogenous entry/exit decisions. As in Götz (1999), we consider an industry characterized by monopolistic competition, which enables us to abstract from the question of whether or not firms commit to a date to adopt the superior technology, since in equilibrium all firms earn the same profits.6 In the model, trade affects firms by creating opportunities to export, but also by allowing foreign competition into the domestic market. Both of these forces alter firm profits and influence the entry and technology adoption decisions.

2.1. Demand

We assume two identical countries, a home country and a foreign country. Each country has two sectors: one sector consists of a numeraire good, \( x_0 \), while the other sector is characterized by differentiated products. The preferences of a representative consumer are defined by the following intertemporal utility function:

\[
U = \int_0^\infty (x_0(t) + \log C(t))e^{-rt}dt
\]

where \( x_0(t) \) is consumption of the numeraire good in time \( t \) and \( C(t) \) represents an index of consumption of the differentiated product good. For \( C(t) \) we adopt the CES specification which reflects tastes for variety in consumption and also imposes a constant (and equal) elasticity of substitution between every pair of goods:

\[
C(t) = \left[ \int_0^{\tilde{n}} y(z, t)^{\rho}dz \right]^{1/\rho}
\]

where \( y(z, t) \) represents consumption of brand \( z \) at time \( t \) and \( \tilde{n} \) represents the number of available varieties in a representative country and is equal to the sum of domestic brands, \( n \), and foreign brands, \( n_F \). It is straightforward to show that, with these preferences, the elasticity of substitution between any two products is \( \sigma = 1/(1-\rho) > 1 \) and aggregate demand in each country for good \( i \) at any point in time is given by:

\[
y(i, t) = \frac{p(i, t)^{-\sigma}E}{\int_0^n p(z, t)^{1-\sigma}dz}
\]

where \( p(i, t) \) is the price of good \( i \) in time \( t \) and \( E \) represents the total number of consumers in the country.

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6 Reinganum (1981) assumes that firms commit to an adoption date at time \( t=0 \). This assumption can be interpreted to reflect an adoption process which has a long gestation period. However, in equilibrium firms earn different profits. Fudenberg and Tirole (1985) assume that firms do not commit to their adoption date, and hence profits are equalized in equilibrium. However, joint adoption can be the Pareto dominant equilibrium in their two-firm setting.
2.2. Production

All goods are produced using constant returns to scale technologies and a single factor of production, labor. Thus, production of any good (or brand) requires a certain amount of labor per unit of output. For simplicity, we assume that production of the numeraire good is defined by \( l = x_0 \) which ensures that the equilibrium wage is equal to unity.

We assume that varieties of the differentiated good can be produced using either of two types of technology. A low-productivity technology is always available to any firm and is purchased for \( F \) upon entering the industry. Production using the low-productivity technology is defined by \( l(t) = y(t) \). A high-productivity technology is also available at time \( t = 0 \), but requires an additional fee of \( x(t) \) where \( x(0) = \infty \), \( x(\infty) = \chi \), \( x' < 0 \) and \( x'' > 0 \). With this adoption cost function, earlier adoption is more expensive; however, the decreasing costs of technology adoption implies that eventually all firms will adopt the high-tech process. Production using the high-productivity technology is defined by \( l(t) = y(t)/\varphi \), where \( \varphi > 1 \).

2.3. Prices and profits

Firms in our model can produce for both the domestic market and the overseas market. Both markets are assumed to be symmetric in terms of preferences, technology, size and trade barriers (we relax these symmetry assumptions in Section 3). The Dixit–Stiglitz preference structure implies that profit-maximizing firms use a constant mark-up pricing rule for given marginal costs. The prices set by the low-tech firms and high-tech firms respectively in the domestic market are:

\[
\begin{align*}
  p_L &= \frac{1}{\rho} = \frac{\sigma}{\sigma - 1}, \quad p_H = \frac{1}{\rho \varphi} = \frac{\sigma}{\varphi (\sigma - 1)}.
\end{align*}
\]

(4)

When it comes to serving the foreign market, we assume that firms that choose to export face transport costs of the traditional iceberg form where \( b > 1 \) units of a good need to be shipped for one unit to arrive. Thus, firms that export will set higher prices in the foreign markets to reflect the higher marginal cost of serving those markets:

\[
\begin{align*}
  p^F_L &= \frac{\sigma b}{\sigma - 1}, \quad p^F_H = \frac{\sigma b}{\varphi (\sigma - 1)}.
\end{align*}
\]

(5)

Consistent with the empirical evidence on traded goods, we assume that the firms that choose to export face not only per-unit costs (i.e., transportation costs or tariffs) but also some fixed (sunk) costs to the export decision. The presence of such fixed costs to entering the export market represents the conventional wisdom as to why some firms export while other firms, in the same industry, do not. Indeed, given that profits decrease as more firms enter the export market, sunk export costs in our model will result in \textit{ex ante} identical firms endogenously sorting themselves into exporters and non-exporters.

The per-period profit of each firm is a function of both its own and rivals’ behavior with the profit differential for non-exporters (those firms that produce solely for the domestic market) given by:

\[
\pi^\text{nx}_H(t) - \pi^\text{nx}_L(t) = \frac{(\varphi^{\sigma - 1} - 1)\rho^{1-\sigma}E}{\sigma \int_{0}^{\infty} p(i, t)^{1-\sigma} dz}
\]

(6)

\footnote{These are standard assumptions in the technology diffusion literature; see, for example, Reinganum (1981) and Fudenberg and Tirole (1985). The only difference is that we assume \( x(t) \) is bounded from below to rule out the possibility of entry occurring after all the initial entrants have adopted. See the discussion about late entry in the Appendix.}

\footnote{While the constant mark-up implied by CES preferences is analytically convenient, it does rule out the possibility that firms lower their mark-ups in response to greater competition. A quadratic specification of preferences (as used in Ottaviano et al., 2002) is one way to incorporate such a mechanism into the model.}

\footnote{For evidence on the sunk costs of exporting see Roberts and Tybout (1997) and Bernard and Jensen (2004). Assuming fixed (sunk) costs to exporting is necessary to generate exporter sorting in models with CES preferences. However, using a quadratic specification, Melitz and Ottaviano (2005) show that such sorting can occur in the absence of fixed costs when mark-ups are endogenous.}
and the profit differential for exporters (those firms producing for both the domestic and overseas market) given by:

$$\pi_H^t(t) - \pi_L^t(t) = \frac{(q^{\sigma-1} - 1)\rho^{1-\sigma}(1 + b^{1-\sigma})E}{\sigma \int_0^\infty p(i, t)^{1-\sigma} dz}.$$  \hspace{1cm} (7)

It should be noted that, given our assumptions on preferences and the production function, firm profits (both foreign and domestic) are bounded to be positive. This will result in all entry occurring at time zero, and no firm ever has an incentive to exit (i.e., in equilibrium the number of firms, $n$, and the fraction of exporters, $s_x$, will not vary over time). Intuitively, this is due to the fact that we assume rational, forward-looking firms and no per-period fixed costs. The lack of per-period fixed costs ensures that profits are bounded to be positive in each period, so once the sunk costs of entry have been paid, no firm has an incentive to exit from either the domestic or foreign market. Likewise, the combination of positive per-period profits and rational, forward-looking firms implies that firms have little incentive to delay entry, and thus all firms enter at $t=0$. In the Appendix, we provide a proof that entry into both the domestic market and foreign market occurs at time zero (i.e., adjustments in $n$ and $s_x$ occur at time zero). In a separate paper, Ederington and McCalman (2006), we introduce per-period fixed costs and show that the stable case is an equilibrium for the set of industries where the fixed costs of production are small relative to the sunk costs of entry. Thus, the conclusions of this paper should be taken as applying to stable industries where the sunk costs of entry are sufficiently large. However, even though $n$ and $s_x$ are time-invariant in this set-up, they are endogenous and respond to changes in trade costs (although these adjustments will occur at time zero).

2.4. Adoption decision

The fraction of firms that have adopted the high-productivity technology at any point in time, $q(t)$, is determined by the firm’s selection of their optimal adoption dates. Thus, $q(t)$ represents the equilibrium distribution of firm productivity at any point in time, and it is the relationship between $q(t)$ and trade which is of interest in this paper. A firm chooses the adoption date, $T$, to maximize the discounted value of total profits:11

$$\Pi = \int_0^T e^{-rt} \pi_L(q(t))dt + \int_T^\infty e^{-rt} \pi_H(q(t))dt - X(T) - F$$

where $X(T) = e^{-rT} x(T)$. A firm’s profits depend both on its own adoption date, $T$, and the adoption decisions of rival firms (which are summarized by $q(t)$). Differentiating with respect to $T$ yields the first-order condition:

$$e^{-rT} [\pi_H(q(T)) - \pi_L(q(T))] = -X'(T).$$  \hspace{1cm} (8)

This first-order condition demonstrates the tradeoff faced by firms in the choice of when to adopt. The left-hand side is the gain in profits from adopting the high-productivity technology, while the right-hand side is the decrease in adoption costs from delaying adoption another period. From this first-order condition, we can establish our first main result:

**Proposition 1.** Exporting firms will adopt earlier in equilibrium than non-exporting firms.

**Proof.** Since operating profits are positive in both the domestic and foreign market, it is direct to show that the profit differential for an exporting firm is greater than the profit differential for a non-exporting firm for any $q \in [0, 1]$. Thus, the earlier adoption of exporting firms follows directly from the assumptions on $X(t)$.

Proposition 1 is a reflection of the fact that exporting firms, due primarily to their larger scale from operating across multiple markets, gain more by adopting the innovation and thus choose to adopt earlier in the diffusion process. A direct implication of Proposition 1 is that at any point during the diffusion process, exporting firms will, on average, be more productive than non-exporting firms in equilibrium. Taken together these features of the model replicate the findings in the empirical literature that exporters are larger and more productive than non-exporters (see Bernard and

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10 Specifically, the only reason to delay entry into either the domestic or foreign market is to save on interest on paying off the sunk costs of entry.

11 Since the foreign market entry decision is endogenous, exporters and non-exporters make equivalent profits in equilibrium. Thus, the decision of when to adopt will be made to maximize profits conditional on that decision, and will not be made to strategically influence that export decision.
Jensen, 1995, 1999; Clerides et al., 1998; Baldwin and Gu, 2003). It is also consistent with the firm-level empirical evidence that exporting is associated with higher levels of investment (e.g., see Alvarez and Lopez, 2005; Hallward-Driemeier et al., 2002).\footnote{For a survey of the exporting and productivity literature using firm-level data, see Wagner (2005).} It should be noted that this result (the existence of a positive relationship between exporting and productivity) is not due to any learning or positive feedback from the action of exporting to production functions, but rather due to the fact that, in the model, forward-looking firms that choose to enter the export market will, simultaneously, choose to adopt new technologies at a faster rate.

We are now in a position to calculate the equilibrium diffusion path of technology in an industry that operates in an open economy. Note that the model admits heterogeneity in two dimensions: firms differ in terms of their technology choices (high-tech/low-tech) and their market participation (exporter/non-exporter). Both of these divisions will be derived as equilibrium outcomes. For now we will take both as given and denote the fraction of firms that choose to export as \(q\) and the fraction of firms that are high-tech by \(s\). Exporting firms receive profits from both the home and foreign market, and, as is implied by Proposition 1, adopt in the range where \(q(t) < s\), i.e., when the number of high-productivity firms is less than the number of exporters).

Given these divisions and the pricing equations, the price index in a country (when \(n^E\)) is:

\[
\int_0^T p(i, t)_{1-s} dt = (q(t)_{1-s} + (1 - q)_{1-s})n + (q(t)_{h1-s} + (s - q)_{h1-s})n^E.
\]

Note that, for now, we impose symmetry across countries (\(q\) and \(s\) are the same for both countries and \(n^E = n\)). Substituting in for the price index during this time period, one derives that the profit differential between low-technology (\(\pi^L\)) and high-technology (\(\pi^H\)) exporting firms is given by:

\[
\pi^H - \pi^L = \frac{(\varphi^{a-1} - 1)(1 + b^{1-a})E}{(\varphi^{a-1} - 1)(1 + b^{1-a}) + (1 + s_b^{1-a})n\sigma}.
\]

Note that the profit differential (\(\pi^H - \pi^L\)) is decreasing as the number of firms producing with the high-tech production process (\(q\)) increases. This is because adoption by rival firms lowers the market share of other firms and, thus, the gain to adopting a cost-saving innovation. It is this property of the model that leads to the gradual diffusion of the new technology through the industry, as firms must tradeoff the increased operating profits from early adoption against the lower adoption costs of later adoption. This tradeoff defines a range of adoption dates that all return the same present value of profits for exporters.

Given this indifference, (8) and (9) can be solved for the fraction of firms that have optimally adopted at any time over the range where \(q \leq s\):

\[
q^*(t) = \frac{-e^{-rt}E}{X^*(t)n\sigma} - \frac{1 + s_b^{1-a}}{(\varphi^{a-1} - 1)(1 + b^{1-a})} \quad \text{for } t \in [T_L, T]
\]

where \(T_L\) is date at which the first (exporting) firm will adopt the new technology and \(T\) is the adoption date of the final exporting firm.

Alternatively, the adoption decision for the non-exporters occurs in the range where \(q(t) > s\). The profit differential for a non-exporting firm, which receives profits only from sales in the domestic market, is given by:

\[
\pi^N = \pi^N = \frac{(\varphi^{a-1} - 1)E}{(\varphi^{a-1} - 1) + (1 + s_b^{a-1}b^{1-a})n\sigma}
\]

Combining (8) and (11) yields the distribution function for the non-exporters:

\[
q^{**}(t) = \frac{-e^{-rt}E}{X^*(t)n\sigma} - \frac{1 + s_b^{a-1}b^{1-a}}{(\varphi^{a-1} - 1)} \quad \text{for } t \in [\bar{T}, T_F]
\]

where \(\bar{T}\) is the date at which the first non-exporting firm adopts the new technology, and \(T_F\) is the adoption date at which the final (non-exporting) firm adopts.
Taken together, (10) and (12) describe the diffusion of the new production process through the industry in an open-economy setting:

\[
q(t) = \begin{cases} 
0 & \text{for } t \in [0, T_L] \\
\frac{-e^{-rt}E}{X'(t)n\sigma} - \frac{1 + s_x b^{1-\sigma}}{(\varphi^{\sigma-1} - 1)(1 + b^{1-\sigma})} & \text{for } t \in [T_L, \bar{T}] \\
\frac{-e^{-rt}E}{X'(t)n\sigma} - \frac{1 + s_x \varphi^{\sigma-1} b^{\sigma-1}}{(\varphi^{\sigma-1} - 1)} & \text{for } t \in [\bar{T}, T_H] \\
1 & \text{for } t \in [T_H, \infty]
\end{cases}
\]

The diffusion of technology in the open economy is shown in Fig. 1.\(^\text{13}\) Since adoption costs are initially very high, no firm will adopt earlier than \(T_L\) (resulting in per-period firm operating profits of \(E/\sigma n\) during this period). However, as adoption costs fall, exporting firms begin to adopt the cost-saving innovation (i.e., the new technology diffuses through exporting firms during the time period \(t \in [T_L, T]\)). Since the rate of diffusion is defined by the first-order condition, exporting firms are indifferent between adopting at any date within this range (although the fraction of firms that will profitably adopt is pinned down by the model). Then there is a lull in the adoption process, followed by the non-exporting firms beginning to adopt the innovation during the time period \(t \in [\bar{T}, T_H]\). Once again, the first-order condition ensures that non-exporting firms are indifferent between adopting at dates within this range. Finally, all firms will have adopted the innovation by time \(T_H\).

Of obvious interest is how changes in trade barriers (i.e., \(b\)) affect this diffusion process and thus firm productivity. Differentiating (10) and (12) with respect to the variable trade costs, one can derive the direct effect of an increase in \(b\) on the adoption decision of both exporters and non-exporters:

Proposition 2. Holding \(n\) and \(s_x\) constant, lower trade costs (i.e., a lower \(b\)) will increase the rate of adoption of exporters and decrease the rate of adoption of non-exporters.

\(^{13}\) Fig. 1 and Proposition 1 also suggest that timing issues in the empirical trade/productivity literature are potentially important. For example, if one had data on firm productivity for some point before \(T_L\) and another point after \(T_H\), then firm productivity growth would be the same for exporters and non-exporters. In contrast, if the data started before \(T_L\) and ended before \(\bar{T}\), then exporting firms would have higher productivity growth than non-exporting firms. Finally, if one had data for a point after \(\bar{T}\) and another after \(T_H\), then non-exporters would have higher productivity growth than exporters. Thus, models of gradual technology diffusion suggest that the observation that exporting firms have higher productivity levels at a point in time does not necessarily imply that their subsequent rates of productivity growth should be higher.
Proof. Differentiating (10) and (12) with respect to \( b \) yields:

\[
\frac{\partial q^x(t)}{\partial b} = \frac{(1 - s_x)(1 - \sigma)}{(\sigma^{\alpha-1} - 1)(1 + b^{1-\alpha})^2 b^\alpha} < 0 \quad \text{for } t \in [T_L, T]
\]

(14)

\[
\frac{\partial q^{nx}(t)}{\partial b} = -\frac{\sigma^{\alpha-1} s_x(1 - \sigma)}{(\sigma^{\alpha-1} - 1)b^\alpha} > 0 \quad \text{for } t \in [\bar{T}, T_H].
\]

(15)

The immediate implication of Proposition 2 is that trade can potentially have differential effects on the productivity of different types of firms. Specifically, the expanded scale of the exporting firms increases their incentive to adopt cost-saving innovations. Meanwhile, non-exporters face increased foreign competition (thus reducing market share) without any compensating increase in foreign markets. This decreased market size for the non-exporters in turn reduces the incentives to adopt new technologies. However, the above calculations are incomplete as they ignore the fact that trade liberalization will also have indirect effects since changes in \( b \) affect the equilibrium values of \( n \) and \( s_x \). Thus, in the next section we endogenize the entry and exit decisions of firms to determine the overall impact of trade on firm productivity.

2.5. Zero-profit entry conditions

The model can be closed by solving for the equilibrium number of firms, \( n \), in the industry and the fraction of exporters, \( s_x \). First, given that operating profits are positive in every period, firms (once they have entered) will remain in the market. Entry into both the domestic and foreign market will occur until the present value of profits for both types of firms is zero. This decreased market size for the non-exporters in turn reduces the incentives to adopt new technologies. However, the above calculations are incomplete as they ignore the fact that trade liberalization will also have indirect effects since changes in \( b \) affect the equilibrium values of \( n \) and \( s_x \). Thus, in the next section we endogenize the entry and exit decisions of firms to determine the overall impact of trade on firm productivity.

Substituting in the respective profit functions, one derives the zero-profit condition for exporters as:

\[
\Pi^x = \left(1 - e^{-rT_L} + e^{-rT_H}\right) + \frac{\sigma^{\alpha-1}(e^{-rT} - e^{-r\bar{T}})}{1 + ((1 + b^{1-\alpha})\sigma^{\alpha-1} - 1)s_x} \left(\frac{E(1 + b^{1-\alpha})}{n\sigma r} + \frac{X(T_L) - \sigma^{\alpha-1}X(T)}{(\sigma^{\alpha-1} - 1)}\right)
\]

(16)

Substituting in the respective profit functions, one derives the zero-profit condition for non-exporters as:

\[
\Pi^{nx} = \left(1 - e^{-rT_L} + e^{-rT_H}\right) + \frac{e^{-r\bar{T}} - e^{-rT}}{1 + ((1 + b^{1-\alpha})\sigma^{\alpha-1} - 1)s_x} \left(\frac{E(1 + b^{1-\alpha})}{n\sigma r} + \frac{X(T_L) - X(T)}{(\sigma^{\alpha-1} - 1)(1 + b^{1-\alpha})}\right)
\]

(17)

\[\text{Substituting in the respective profit functions, one derives the zero-profit condition for non-exporters as:}\]

\[\Pi^{nx} = \left(1 - e^{-rT_L} + e^{-rT_H}\right) + \frac{e^{-r\bar{T}} - e^{-rT}}{1 + ((1 + b^{1-\alpha})\sigma^{\alpha-1} - 1)s_x} \left(\frac{E(1 + b^{1-\alpha})}{n\sigma r} + \frac{X(T_L) - X(T)}{(\sigma^{\alpha-1} - 1)(1 + b^{1-\alpha})}\right)
\]

(17)

\[\text{Similar calculations reveal the zero-profit condition for non-exporters as:}\]

\[\Pi^{nx*} = \left(1 - e^{-rT_L} + e^{-rT_H}\right) + \frac{e^{-r\bar{T}} - e^{-rT}}{1 + ((1 + b^{1-\alpha})\sigma^{\alpha-1} - 1)s_x} \left(\frac{E(1 + b^{1-\alpha})}{n\sigma r} + \frac{X(T_L) - X(T)}{(\sigma^{\alpha-1} - 1)(1 + b^{1-\alpha})}\right)
\]

(17)

\[\text{Similar calculations reveal the zero-profit condition for non-exporters as:}\]

\[\Pi^{nx*} = \left(1 - e^{-rT_L} + e^{-rT_H}\right) + \frac{e^{-r\bar{T}} - e^{-rT}}{1 + ((1 + b^{1-\alpha})\sigma^{\alpha-1} - 1)s_x} \left(\frac{E(1 + b^{1-\alpha})}{n\sigma r} + \frac{X(T_L) - X(T)}{(\sigma^{\alpha-1} - 1)(1 + b^{1-\alpha})}\right)
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\]

(17)
These respective zero-profit conditions are plotted in Fig. 2. The exporter (non-exporter) line represents combinations of \( n \) and \( s_x \) such that the zero-profit condition holds for exporters (non-exporters). Total differentiation of the present value condition reveals the relationship between \( n \) and \( s_x \) as:

\[
\frac{dn}{ds_x} = -\frac{\partial \Pi^I / \partial s_x}{\partial \Pi^I / \partial n}.
\]  

(18)

Since \( \Pi^I \) is declining in both \( n \) and \( s_x \), this implies that the zero-profit lines are downward sloping. Furthermore, straightforward calculations reveal that the zero-profit line for exporters is steeper than the zero-profit line for non-exporters. This implies that, for a given set of parameter values, a unique equilibrium exists. Specifically, if the sunk cost of exporting is sufficiently low the equilibrium will involve everyone exporting \((s_x=1)\), while if the sunk cost of exporting is sufficiently high no one exports \((s_x=0)\). However, for a moderate range of export costs there will be an interior equilibrium where some positive fraction of firms export \((0<s_x<1)\). In Appendix A.5 we provide a discussion of these conditions and in the analysis that follows, we will concentrate on interior equilibria where \(0<s_x<1\), as this is the most empirically relevant case.\(^{15}\)

In this case, as is evident from Fig. 2, the open-economy equilibrium values of \( n \) and \( s_x \) are given by the intersection of the two zero-profit lines. To determine the impact of trade on \( n \) and \( s_x \), note that when \( s_x=0 \), \( \Pi^x \) is independent of \( b \), while at \( s_x=1 \), \( \Pi^x \) is increasing in \( b \). However, when \( s_x=0 \), \( \Pi^x \) decreases in \( b \), while at \( s_x=1 \), \( \Pi^x \) is independent of \( b \). This implies that \( n \) is increasing in \( b \), while \( s_x \) is decreasing in \( b \). This mechanism is captured in Fig. 3.

Intuitively, the increased exporter profits resulting from lower trade barriers induce the entry of other firms into the export market (i.e., an increase in \( s_x \)).\(^{16}\) Likewise, the reduction in profits resulting from increased foreign competition results in the exit of firms from the market (i.e., a decrease in \( n \)).

It should be apparent that these indirect effects, at least partially, counter the direct effects. That is, the increase in exporter profits caused by lower trade barriers is countered by an increase in the number of exporters which reduces overseas profits. Likewise, the decrease in non-exporter profits is matched by a reduction in the total number of firms in the market, thus ensuring that the zero-profit condition for non-exporters is satisfied. Given all these different effects,

\(^{15}\) Note also that the model does not predict which firms become exporters, only the number of firms that export. The model can easily be extended to allow for exogenous heterogeneity that could serve as a mechanism to identify which firms export. However, this still leaves unanswered the question of how firms acquired these exogenous advantages.

\(^{16}\) This finding replicates the results of Yeaple (2005), which assumes a situation where fixed adoption costs are such that only high-tech firms export and low-tech firms do not (in the context of our model this is equivalent to analyzing the steady state between \( \bar{T} \) and \( T \)). Under this assumption, he shows that a reduction in trade costs will increase the share of exporting firms and, hence, the share of high-tech firms. As can be seen, the same outcome occurs in our framework: lower trade barriers increase the number of exporters and hence the number of high-tech firms during the middle steady state.
the question arises whether we can derive any general statements about the effects of trade on the overall rate of technology adoption. It turns out that we can. As we show in the following proposition, the overall effect of trade will be to increase the rate of technology diffusion as lower trade barriers result in adoption both starting earlier and ending earlier.

**Proposition 3.** Lower variable trade costs (i.e., a lower $b$) will result in the process of adoption starting earlier (i.e., $T_L$ will occur earlier) and ending earlier (i.e., $T_H$ will occur earlier).

**Proof.** See Appendix.

Proposition 3 is of interest because it implies that trade has a generally positive impact on firm productivity as it will speed up the adoption of a new technology through an industry. Specifically, assume two industries which are identical in all respects other than the size of barriers to trade. The direct implication of Proposition 3 is that technology adoption will occur more rapidly in the industry with lower trade barriers.

The fact that trade is predicted to have a positive impact on firm productivity may seem somewhat surprising since it implies that lower trade barriers will increase the rate of adoption, not only of early-adopting exporters, but also of late-adopting non-exporters. The explanation for this seemingly paradoxical result lies in the relative impact of the direct and indirect effects of trade on endogenous technology adoption decisions. The key insight is that the impact of trade barriers is not constant over time. At the beginning and the end of the diffusion process all firms (both exporters and non-exporters) have similar technology (either all high-tech or all low-tech), and this symmetry implies that trade barriers represent a similar impediment to foreign competition at both points in time. However, in the middle of the diffusion process (when $s_x = q$) the asymmetry between exporters and non-exporters is most pronounced as exporters enjoy a relative cost advantage. It is exactly these conditions that also make tariffs relatively important for the profits of domestic firms, especially low-tech non-exporters who are now more dependent on tariffs for their survival.\(^{17}\)

The fact that the impact of tariffs varies through time has important implications for the evolution of profits over time when tariffs change, especially since the present value of net profits are constrained to equal zero in equilibrium. If tariffs are lowered, it is clearly the case that this depresses profits proportionately more in the middle of the diffusion phase (when technological asymmetries are most pronounced between exporters and non-exporters) than either at the beginning or the end of the diffusion process (when there is greater technological symmetry). Since the zero-profit condition must hold, the losses from the middle of the diffusion process must be compensated by gains during periods of symmetry.

\(^{17}\) One way to see this is to calculate the elasticity of the price index with respect to changes in the trade barrier. It is direct to derive that a change in trade costs has a proportionately greater impact on the price index in the middle of the diffusion phase (i.e., when $q = s_x$) than at either the beginning or the end. Note that, due to the zero-profit condition, a simple change in market size has no effect on technology diffusion since it is netted out by changes in the number of firms, $n$. However, changes in trade costs do affect the speed of adoption since the impact of a trade barrier is not constant over time.
This implies that \( \pi_L (q=0) \) and \( \pi_L (q=1) \) must both be higher in equilibrium when tariffs are lowered. Since these profits determine respectively \( T^L \) and \( T^H \), it follows immediately that adoption must start earlier and end sooner when tariffs are liberalized.

Thus, the extension of endogenous technology adoption models to an open-economy setting provides two main results. First, that a natural result of endogenous technology adoption is the observed fact that exporters will be larger and more productive than non-exporters. Second, that trade tends to shift forward the process of technology diffusion, thus providing support for the conventional wisdom that trade can create productivity gains by encouraging productivity-enhancing investments on the part of firms. In the next section of the paper, we use this framework to shed light on some of the documented effects of the U.S.–Canada Free Trade Agreement.

3. The productivity effects of the Canada–U.S. Free Trade Agreement

In previous sections, we focused exclusively on a symmetric model; the countries were symmetric, as was trade policy. Given the widespread use of reciprocal trade agreements, the analysis of symmetric changes in trade policy is certainly of interest. However, even in the context of a reciprocal agreement much debate concerns the costs of lowering domestic trade barriers rather than the benefits from increased foreign market access. This bias is evident in empirical studies of trade agreements which almost universally attempt to quantify the impact of a reduction in domestic trade barriers at the national, industry and firm level. An exception to this rule has been the research focused on the Canadian experience in the context of the free trade agreement with the U.S., as the implications of variation in both Canadian and U.S. tariffs are quantified. A particularly attractive aspect of this setting is that it provides a relatively clean trade policy experiment: notably, the agreement was not part of a broader policy regime change in either country, and the changes in trade policy are reflected in detailed tariff rates that vary both by commodity and through time. Moreover, the Canada–U.S. Free Trade Agreement (CUSFTA) covers the majority of Canadian trade (82% of Canadian exports are destined for U.S., and 68% of Canadian imports originate in the U.S.). These conditions are rarely satisfied in other empirical studies of trade liberalization. Thus, in the following section we summarize the empirical evidence on CUSFTA. As will been seen, the results of these studies establish an important link between openness and firm-level productivity, a link that is missing in the previous theoretical literature.

3.1. Empirical evidence

A series of papers (Head and Ries, 1999; Trefler, 2004; Baggs et al., 2002) have exploited the detail of the data provided by the Canadian experience to identify the impact of the Canadian manufacturing sector from both Canadian and U.S. tariff concessions. These studies provide a rich perspective on the Canadian experience of free trade agreements. Here we focus on two key results that present a challenge to the previous theoretical literature which has concentrated on the connection between trade and industry-level productivity. These results relate to the responses of plant-level productivity to both Canadian and U.S. tariff concessions.

With respect to changes in U.S. trade policy, Trefler (2004) estimates that U.S. concessions resulted in large within-plant productivity gains for Canadian firms. In particular, plants in industries that experienced the greatest reductions in U.S. trade barriers are estimated to have raised their productivity by an average of 14%. This is an enormous increase in plant-level productivity. Baggs et al. (2002) complement the analysis of Trefler by examining the impact of U.S. tariff concessions on the distribution of firm-level productivities. They conclude that significant productivity increases are achieved only by the top quartile of the productivity distribution. Consequently, the benefits of improved access to the U.S. market are concentrated among the high-productivity firms.

With respect to Canadian trade policy, Trefler (2004) estimates that Canadian concessions had little impact on mean plant productivity. However, Baggs et al. (2002) offer further insight into this result by examining the impact of Canadian concessions on the distribution of plant-level productivity. They find that the impact was not uniform, with firms at the lower end of the productivity distribution improving their productivity performance in the face of Canadian tariff reductions. Moreover, they estimate that Canadian tariff concessions had a disproportionately negative impact on the most productive firms. This differential response to Canadian reductions provides a possible explanation for Trefler’s result that, on average, plant-level productivity was not significantly affected by Canadian tariff reductions.

These findings present a challenge to the predictions of the existing theoretical literature, since this literature has taken firm-level productivity as exogenously determined. Consequently, the previous literature cannot explain either
the mean response in productivity to reductions in trade barriers (as estimated by Trefler, 2004) or the changes in the productivity distribution estimated by Baggs et al. (2002) at the firm level. In the following section we present an asymmetric version of the model in an effort to provide insight into these intriguing and important results.

3.2. Asymmetric model

In order to represent the above features of the Canadian/U.S. trade relationship, the model needs to incorporate independent domestic and foreign tariffs. In addition, the pronounced difference in size and technological capabilities between the U.S. and Canada needs to be incorporated into the model. To replicate these characteristics, yet keep the framework tractable, it will be assumed that both the number of U.S. (foreign) firms \((n_f)\), share of exporters \((s_{bf}=1)\) and the state of their technology is exogenous \((q_f=1)\). This is equivalent to a small country assumption (i.e., Canadian trade does not affect the U.S. equilibrium) and provides a tractable way to analyze the implications of domestic tariffs \((b_h)\) and foreign tariffs \((b_f)\) on the home country (Canada). As a result of these assumptions, it follows that the profit differential for an exporter is (for \(q<s_s\)):

\[
\pi^H_q - \pi^L_q = \frac{(q^\sigma - 1)E b_f^{1-\sigma}}{\sigma [(q^\sigma - 1) + s_s] b_f^{1-\sigma} n_f + n_f q^\sigma} + \frac{(q^\sigma - 1)E}{\sigma [(q^\sigma - 1) + n_f q^\sigma b_f^{1-\sigma}].}
\]

Likewise, the profit differential for a non-exporter is given by (for \(q > s_s\)):

\[
\pi^{nx}_q - \pi^{nx}_L = \frac{(q^\sigma - 1)E}{\sigma [(q^\sigma - 1) + n_f q^\sigma b_f^{1-\sigma}].}
\]

From these profit differentials, we can derive the impact of changes in domestic and foreign tariffs on the adoption rates of exporters and non-exporters respectively.

Finally, in the empirical work we are analyzing, identification of the productivity impact of U.S. and Canadian trade liberalization is based on panel data and is derived primarily from cross-industry variation (i.e., a comparison of the productivity gains of firms in industries that received large tariff reductions with the gains of firms in industries where tariff reductions were smaller). That is, the empirical research is implicitly comparing firm productivity gains in an industry that experiences a reduction in tariffs to an industry that does not receive such a reduction and so, in our theoretical analysis, we want to consider the impact of tariff liberalization on subsequent productivity growth. Thus, we consider the case where trade liberalization occurs at some period \(T_d\). In this case, the discounted value of total profits for a firm are:

\[
\Pi = \int_0^{T} e^{-rt} \pi_L(q(t), \bar{b}_h, \bar{b}_f)dt + \int_0^{T} e^{-rt} \pi_L(q(t), b_h, b_f)dt \int_T^{\infty} e^{-rT} \pi_H(q(t), b_h, b_f)dt - X(T) - F - S.
\]

Thus, we want to consider the comparative statics of a change in \(b_h\) and \(b_f\) (holding initial tariffs, \(\bar{b}_h\) and \(\bar{b}_f\), constant). Note that the timing of the trade agreement, \(T_d\), will have no impact on the direct effects of trade liberalization, but will affect the strength of the indirect effects. Specifically, the later the trade agreement occurs, the less impact trade liberalization will have on the discounted value of total profits and thus the smaller the effect on \(n\) and \(s_s\). For expositional purposes, the proofs in the following sections will consider the case where \(T_d=0\). As we discuss

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18 Moreover, in Melitz (2003) a reduction in domestic trade barriers have no impact on either firm or industry-level productivity. However, Melitz and Ottaviano (2005) does provide a model which is capable of explaining how reductions in foreign barriers can impact the measured productivity of firms by influencing firm mark-ups.

19 Since differences in size between the two countries and the share of foreign firms that export both enter the profit expressions by merely scaling \(b_h\) and \(b_f\) to avoid clutter they will be suppressed in the following analysis.

20 While it was generally the case that Canada had higher tariffs than the U.S., liberalization usually involved tariff reductions of equal steps over similar time horizons. Consequently, the identification of separate effects for Canadian and U.S. tariffs comes from variation across sectors rather than from within sectors.

21 It should be noted that, given our assumption that firms are rational and forward-looking, adjustments in the number of firms will occur at time zero, even when the trade liberalization episode occurs later.
in the following sections, having the trade agreement occur later in time will not affect our predictions with respect to early-adopting firms (since the direct and indirect effects have the same sign), but can have an impact on our theoretical predictions for the late-adopting firms.

3.2.1. U.S. tariff concessions

First, take the case of a decrease in U.S. (foreign) tariffs (i.e., $b_f$ decreases). It should be apparent that a decrease in $b_f$ will directly increase exporter profits and consequently the profit differential for exporting firms, while having no direct impact on non-exporters. Thus, the direct impact of a U.S. tariff concession on Canadian firms will be to increase the speed of technology adoption, albeit only for the more productive, exporting firms.\(^{22}\)

However, to determine the total effect of foreign trade policy on firm-level productivity, the impact on $n$ and $s_x$ needs to be considered. This impact is demonstrated by Fig. 4. The mechanism in this case operates entirely through the increase in exporter profits. The obvious impact of such a policy is that the share of exporters increases. However, less obviously, the number of domestic firms decreases. How does this happen? For firms in the export market, their incentive to adopt the high-tech equipment increases as U.S. tariffs are reduced. Accordingly, the degree of high-tech competition in the domestic market increases, lowering the profits of non-exporting firms, and encouraging exit among non-exporters.\(^{23}\)

To determine the total effect on firm productivity of U.S. tariff concessions, we must account for both the direct and indirect effects of a decrease in $b_f$. As we argue in the following proposition, the total effect of a U.S. tariff concession on Canadian firm productivity is positive as it speeds up the rate of technology adoption.

**Proposition 4.** Lower foreign trade costs (i.e., a lower $b_f$) will result in the process of adoption for domestic firms starting earlier (i.e., $T_L$ will occur earlier) and ending earlier (i.e., $T_H$ will occur earlier).

**Proof.** See Appendix.

Thus, the above proposition provides a rationale for the findings of both Trefler (2004) and Baggs et al. (2002) that U.S. trade concessions increased the productivity of Canadian firms. The intuition behind this result is similar to that of Proposition 3. Specifically, adoption begins earlier because the reduction in foreign tariffs will directly increase the rate of adoption of domestic exporters. In addition, adoption will end earlier because the reduction in foreign tariffs

\(^{22}\) This provides a possible rationale for the finding by Baggs et al. (2002) that the productivity gains from a U.S. tariff concession are concentrated among the more productive Canadian firms.

\(^{23}\) Thus the number of Canadian firms declines as U.S. trade barriers fall. This result is consistent with the findings of Head and Ries (1999), and provides a theoretical foundation for an outcome that they label “perverse” since it lacked any theoretical underpinning at the time. Note that the models in Melitz (2003) and Bernard et al. (2003) can also explain this result, although the mechanism differs from the one emphasized here. In their frameworks, an increase in demand raises real wages, driving the least productive firms out of the market.
contracts the domestic industry (i.e., reduces the number of domestic firms), thus increasing the adoption rate of the non-exporting late-adopters.

As noted previously, the above result is derived for the case where the trade agreement occurs at time zero. If one were to consider a tariff liberalization episode that occurred sufficiently far from the time of entry, the prediction for the timing of $T_H$ could change. Specifically, while one would still find that initial adoption, $T_L$, occurs earlier, it is possible that the time of adoption for the final adopters, $T_H$, could occur later. This is due to the fact that a weakening of the indirect effect means that it might fail to outweigh the direct effect at the end of the adoption phase. This is of potential interest, because it provides a possible explanation for the finding by Baggs et al. (2002) that the productivity gains of U.S. trade liberalization were concentrated among the most productive Canadian firms, with productivity growth actually decreasing among the less productive firms.

3.2.2. Canadian tariff concessions

Next, take the case of a decrease in Canadian (domestic) tariffs (i.e., $b_h$ decreases). It should be apparent that a decrease in $b_h$ will directly decrease both exporter and non-exporter profits by reducing domestic profits of both types of firm. Thus, the direct impact of a Canadian tariff concession will be to decrease the speed of technology adoption. However, this profit decrease will induce the exit of firms from the market, which in turn will encourage faster adoption. To determine the total effect of domestic trade policy on firm productivity, the impact on $n$ and $s_x$ needs to be considered. Not surprisingly, the reduction in domestic profits will decrease the equilibrium number of Canadian firms. However, in this case the impact on the share of exporters $s_x$ is ambiguous (although it can be shown that the total number of exporters $n \cdot s_x$ will decrease).

As mentioned previously, the direct impact of the Canadian tariff concession (which delays adoption) is countered by the indirect impact (a reduction in the number of firms which speeds adoption). As we argue in the following proposition, the net impact of the Canadian tariff concessions is mixed. Specifically we can show that, for a sufficiently small technological innovation (i.e., $\phi \sigma^{-1} < 2$), adoption will start later (i.e., the first adopters will begin adopting later), but end earlier (i.e., the last adopters will begin adopting earlier). \(^{24}\)

**Proposition 5.** Lower domestic trade costs (i.e., a lower $b_h$) will result in the process of adoption for domestic firms starting later if $\phi \sigma^{-1} < 2$ (i.e., $T_L$ will occur later) and it will also end earlier (i.e., $T_H$ will occur earlier).

**Proof.** See Appendix.

The explanation for this differential effect lies in the offsetting effects that a reduction in the domestic tariff has on the rate of adoption. As discussed before, reducing domestic tariffs directly slows down adoption rates (by reducing the market share of domestic firms), but also indirectly increases them through the reduction in the number of domestic firms. However, this reduction in $n$ (the number of domestic firms) has a relatively small effect on profits and the profit differential at the beginning of the adoption process (when the majority of domestic firms are smaller, low-tech types) and a larger effect at the end of the adoption process (when the majority of domestic firms are larger, high-tech types). As we verify in the Appendix, the negative direct effect outweighs the positive indirect effect at the beginning of the adoption process (delaying adoption of the early adopters), but the positive indirect effect dominates at the end of the adoption process (speeding up the adoption of the late adopters). \(^{25}\) Thus, the above proposition provides a potential rationale for the finding by Baggs et al. (2002) that the impact of a domestic tariff concession is mixed, with productivity of the most efficient firms decreasing (which is consistent with $T_L$ occurring later) and productivity of the least efficient increasing (which is consistent with $T_H$ occurring earlier).

4. Conclusion

The starting point of this paper is the recognition that industries and firms are constantly evolving, and that understanding this evolution is necessary in order to understand the full implications of trade policy. We take a key

\(^{24}\) The requirement that $\phi \sigma^{-1} < 2$ is a sufficient condition for $T_L$ to occur later, not a necessary condition.

\(^{25}\) Once again, this result is derived for the case where the trade agreement occurs at time zero. If one were to sufficiently delay the timing of the trade agreement, then the resulting weaker indirect effect could result in $T_H$ occurring later as well.
determinant of this evolution to be the diffusion and adoption of new technology. By its nature, new technology tends to be employed only gradually by firms, generating an important source of heterogeneity among firms in the same industry. By developing a model that has firm heterogeneity as an equilibrium outcome, we are able to explain a number of empirical findings regarding firm-level behavior as well as investigate the impact of trade on technology adoption and firm-level productivity. Thus, we see our model as being complementary to the influential and emerging line of research that assumes productivity varies exogenously across firms and focuses on the impact of trade on industry-level productivity. As in that literature, we find that trade has a generally positive impact on productivity (and, perhaps surprisingly, this positive impact applies to both exporting and non-exporting firms). Specifically, we find that liberalization of trade barriers tends to increase the rate of technology adoption, in that the process of adopting a new technological innovation both starts and ends earlier.

Appendix A

A.1. Proof of Proposition 3

First, we establish the following lemma

**Lemma 1.** A decrease in a given trade barrier, \( b \), implies that \( T_L \) occurs earlier iff \( \frac{d\Pi}{db} \bigg|_{q=0} < 0 \). Likewise, \( T_H \) occurs earlier iff \( \frac{d\Pi}{db} \bigg|_{q=1} < 0 \).

**Proof.** First, note that for a given adoption cost schedule, \( T_L \) will occur earlier the higher is the profit differential for exporting firms (\( \pi_H^x - \pi_L^x \)) evaluated at \( q=0 \). Likewise, \( T_H \) will occur earlier the higher is the profit differential for non-exporting firms (\( \pi_H^x - \pi_L^x \)) evaluated at \( q=1 \). Thus, Lemma (1) follows directly from the realization that \( \pi_H^x - \pi_L^x = \phi^x \pi_L^x \) for \( i \in (x, nx) \).

Note, from (17) that the zero-profit condition for non-exporters is defined by:

\[
\Pi^x = \Pi_1^x + \Pi_2^x + \Pi_3^x + \Pi_4^x - F - X(T_H) = 0
\]  

(19)

where \( \Pi_1^x = \int_0^{T_L} e^{-rt}\pi_L^x(q=0)dt \); \( \Pi_2^x = \int_0^{T_L} e^{-rt}\pi_L^x(q(t))dt + \int_0^{T_H} e^{-rt}\pi_L^x(q(t))dt \); \( \Pi_3^x = \int_0^{T_L} e^{-rt}\pi_L^x(q=1)dt \), and \( \Pi_4^x = \int_0^{T_H} e^{-rt}\pi_L^x(q=1)dt \).

Totally differentiating (19) and applying the envelope theorem, one derives that:

\[
\frac{d\Pi^x}{db} = 0 = \frac{d\Pi_1^x}{db} + \frac{d\Pi_2^x}{db} + \frac{d\Pi_3^x}{db} + \frac{d\Pi_4^x}{db}.
\]  

(20)

By direct calculation, one derives that \( \frac{d\Pi_2^x}{db} > 0 \), and thus, from (20):

\[
\frac{d\Pi_1^x}{db} + \frac{d\Pi_3^x}{db} + \frac{d\Pi_4^x}{db} < 0.
\]  

(21)

Likewise, the zero-profit condition for exporters is defined by:

\[
\Pi^e = \Pi_1^e + \Pi_2^e + \Pi_3^e + \Pi_4^e - F - S - X(-T) = 0
\]  

(22)

where \( \Pi_1^e = \int_0^{T_L} e^{-rt}\pi_L^e(q=0)dt \); \( \Pi_2^e = \int_0^{T_L} e^{-rt}\pi_L^e(q(t))dt + \int_0^{T_H} e^{-rt}\pi_L^e(q(t))dt \); \( \Pi_3^e = \int_0^{T_L} e^{-rt}\pi_L^e(q=1)dt \), and \( \Pi_4^e = \int_0^{T_H} e^{-rt}\pi_L^e(q=1)dt \).

Similar calculations reveal that \( \frac{d\Pi_1^e}{db} < 0 \) and thus:

\[
\frac{d\Pi_1^e}{db} + \frac{d\Pi_3^e}{db} + \frac{d\Pi_4^e}{db} > 0.
\]  

(23)

From the respective profit functions, one derives that \( (\Pi_1^e + \Pi_1^e) = (1 + b^{L-\sigma})(\Pi_1^x + \Pi_1^x) \) and \( \Pi_3^e = \phi^e \Pi_3^x \). Using these derivations, it is straightforward to show that both (20) and (23) can only be satisfied if:

\[
\frac{d(\Pi_1^x + \Pi_4^x)}{db} < 0, \quad \frac{d\Pi_3^x}{db} > 0
\]  

(24)
Finally, \( \frac{d(I_T^T + I_L^T)}{dh} < 0 \) implies that \( \frac{d\pi_T}{dh} < 0 \) as \( I_T^T + I_L^T = \pi_T^T(q = 1) \) where \( A = [1 - e^{-Tr} + e^{-rL}] / r \). Thus, by Lemma 1 a reciprocal reduction in trade costs (a decrease in \( b \)) will increase the profit differential at \( q = 1 \) and will result in \( T_H \) occurring earlier.

Similarly, \( \frac{d(I_T^T + I_L^T)}{ds} < 0 \) implies that \( \frac{d\pi_T}{ds} < 0 \). Thus, since \( I_T^T + I_L^T \) is a similar monotonic function of the profit differential for exporters at \( q = 0 \), a reciprocal reduction in trade costs (a decrease in \( b \)) will increase this profit function and result in \( T_L \) occurring earlier.

**A.2. Proof of Proposition 4**

The zero-profit condition for non-exporters is given by (19). The impact of \( b_f \) on \( T_H \) follows directly from the impact of \( b_f \) on \( I_T^T \). It is direct to derive that \( I_T^T \) is decreasing in \( b_f \) (this calculation simply reflects the fact that an increase in \( b_f \) will lead to an increase in \( n \) which implies that \( \pi_T^N(q = 1) \) is also decreasing in \( b_f \). Consequently, by Lemma 1, \( \frac{d\pi_T^N}{db_f} > 0 \).

The argument to determine \( \frac{d\pi_L^N}{db_f} > 0 \) is more involved. Note that an exporter derives operating profits from both the domestic market and the foreign market. Denote operating profits from the domestic market as \( \pi_T^N \) and operating profits from the foreign market as \( \pi_T^F \). Then one can express the zero-profit condition for exporters as:

\[
\Pi^T = \Pi_T^N + (q^{σ-1} - 1)\Pi_T^N + \Pi_T^F + \Pi_T^F - S - X(I) = 0
\]  

where:

\[
\Pi_T^N = \int_0^{T_1} e^{-r_t}\pi_T^N(q = 0)dt; \quad \Pi_T^F = \int_0^{T_1} e^{-r_t}\pi_T^F(q(t))dt + (q^{σ-1} - 1)\int_0^{T_H} e^{-r_t}\pi_T^N(q(t))dt, \quad \Pi_T^F = \int_0^{∞} e^{-r_t}\pi_T^F(q = 1)dt.
\]

Totally differentiating (25) and noting that \( \frac{d\Pi_T^N}{dh} = 0 \) one derives that:

\[
\frac{d\Pi_T^T}{db_f} = \frac{d\Pi_T^N}{db_f} + \frac{d\Pi_T^F}{db_f} = (q^{σ-1} - 1)\frac{d\Pi_T^N}{db_f}.
\]  

Given the respective profit functions, direct calculations reveal that \( \frac{d\pi_T^N}{db_f} < 0 \) and \( \frac{d\pi_T^F}{db_f} < 0 \). Thus, by (20), it must be the case that \( \frac{d\pi_T^T}{db_f} > 0 \). Likewise, direct calculations reveal that \( \frac{d\pi_T^F}{db_f} = 0 \). Therefore, \( \frac{d\pi_T^T + d\pi_T^F}{db_f} < 0 \). However, using the respective profit functions, one can derive that this sum can only be negative if that \( \frac{d\pi_T^F}{db_f} < 0 \). Thus, since \( \frac{d\pi_T^F}{db_f} < 0 \) and \( \frac{d\pi_T^T}{db_f} > 0 \), this implies that \( \pi_T^F(q = 0) \) is decreasing in \( b_f \). Consequently, \( \frac{d\pi_T^T}{db_f} > 0 \).

**A.3. Proof of Proposition 5**

Totally differentiating (25) and noting that \( \frac{d\Pi_T^N}{ds} = 0 \) and \( \frac{d\Pi_T^F}{ds} = 0 \) one derives that

\[
(q^{σ-1} - 1)\frac{d\Pi_T^N}{db_f} + \frac{d\Pi_T^F}{db_f} = 0.
\]

Direct calculations reveals that \( \frac{d\Pi_T^T}{db_f} > 0 \) (this calculation simply reflects the fact that an increase in \( b_h \) will lead to an increase in the number of exporters, \( s,n \)). Thus, \( \frac{d\Pi_T^T}{db_f} > 0 \). A comparison of \( \Pi_T^T \) and \( \Pi_T^N \) confirms that if \( \Pi_T^N \) is increasing in \( b_h \) then so is \( \Pi_T^T \). Thus, by (27), must be decreasing in \( b_h \) which implies that the profit differential for non-exporters at \( q = 1 \) is decreasing in \( b_h \), and consequently, \( \frac{d\pi_T^N}{db_h} > 0 \).

To characterize \( \frac{d\Pi_T^T}{db_h} > 0 \), note from (27) that

\[
\frac{d\Pi_T^T + d\Pi_T^F}{db_h} = (2 - q^{σ-1})\frac{d\Pi_T^N}{db_h} - \frac{d\Pi_T^F}{db_h} \quad \text{provided that } q^{σ-1} < 2 \text{ it follows that } \frac{d\Pi_T^F}{db_h} > 0.
\]

Thus implies that \( \frac{d\Pi_T^F}{db_h} > 0 \). It can be shown that if \( \frac{d\Pi_T^F}{db_h} > 0 \) then \( \frac{d\Pi_T^F}{db_h} \mid q=α > 0 \). This implies, by Lemma 1, that \( \frac{d\Pi_T^T}{db_h} > 0 \).

**A.4. Proof: No late entry into the domestic or foreign market (adjustments in \( n \) and \( s_x \) occur at \( t = 0 \))**

First, note that, in the absence of exit, \( \pi_L(t) \mid q = s_x \) is non-increasing. Second, note that \( \pi_L(t=0) > rF \) (otherwise, the present discounted value of profits for firms that enter at time \( t = 0 \) and adopt at \( T_H \) will be negative) and \( \pi_L(t=∞) < rF \)
(otherwise, the present discounted value of profits for firms that enter at time \( t=0 \) and never adopt will be positive). Low-tech firms that enter after \( t=0 \) will choose their entry date, \( T_e \), to maximize:

\[
\Pi = \int_{T_e}^{\infty} e^{-rt}\pi_L(t)\,dt + \int_{T_e}^{\infty} e^{-rt}\pi_H(t)\,dt - X(T_e) - e^{-rT_e}F.
\]

Thus, no firm will enter after \( t=0 \) as a low-tech firm, since if they enter when \( \pi_L(t=0) > rF \), they would have made greater discounted profits by entering as a low-tech firm earlier, and if they enter when \( \pi_L(t=\infty) < rF \), they would have made greater discounted profits by entering as a high-tech firm later. High-tech firms that enter after \( t=0 \) will choose their entry date to maximize:

\[
\Pi = \int_{T_e}^{\infty} e^{-rt}\pi_H(t)\,dt - X(T_e) - e^{-rT_e}F.
\]

Differentiating with respect to \( T_e \) yields the first-order condition:

\[
\pi_H(t = T_e) - rF = -X'(T_e)e^{T_e}.
\]  

(28)

The above first-order condition demonstrates the trade-off faced by firms in the choice of when to enter. The left-hand side is the lost profits from waiting one more period to enter the industry while the right-hand side is the gain from the decrease in adoption costs from delaying entry another period. Note that the first-order condition for optimal adoption requires that, between \( T_L \) and \( T_H \), the profit differential is fixed at:

\[
\pi_H(t) - \pi_L(t) = -X'(t)e^{T_e}.
\]  

(29)

Thus, there are three points in time when the entry first-order condition is satisfied: first, before \( T_L \) when \( \pi_L > rF \); second, between \( T_L \) and \( T_H \) when \( \pi_L = rF \); and finally, after \( T_H \) when \( \pi_L < rF \). However, note that the first two potential entry dates are when \( \pi_L(t=0) \geq rF \) and, since \( \pi_L \) is non-increasing over time, this implies that these firms would have made greater discounted profits from entering earlier as a low-tech firm. Thus, the only possible entry date is the one after \( T_H \). However, provided that lower bound for adoption costs, \( \lambda \), is sufficiently high the present value of profits for such potential late entrants, given by (28), will be negative in equilibrium (i.e., when the present value of profits for \( t=0 \) entrants is zero). Thus, no late entry into the domestic market will occur.

To show that no late entry into the foreign market occurs, define \( \pi^F \) as the per-period profits from the foreign (export) market. For exporters to be present at \( t=0 \) it must be the case that \( \pi^F_L(t=0) > rS \). In addition, note that, in the absence of exit, \( \pi^F(t) \) and \( \pi^F'(t) \) are non-increasing. Thus, potential late entry into the foreign (export) market will only occur at a firms adoption date. Therefore, non-exporting firms that choose to export at a later date will choose an export date, \( T_e \), to maximize:

\[
\Pi = \int_{T_e}^{T_e} e^{-rt}\pi^*_L(t)\,dt + \int_{T_e}^{\infty} e^{-rt}\pi^*_H(t)\,dt - X(T_e) - e^{-rT_e}S - F.
\]

Differentiating with respect to \( T_e \) yields the first-order condition:

\[
\pi^*_H(t = T_e) - \pi^*_L(t = T_e) - rS = -X'(T_e)e^{T_e}.
\]  

(30)

Since \( \pi^*_L(t=0) > rS \) then late entrants will adopt before incumbent exporters (i.e., before \( T_L \)). However, since \( \pi^*_L(t < T_H) > rS \), these firms would have made greater discounted profits from entering the export market at \( t=0 \) as a low-tech firm. Thus, no late entry into the foreign (export) market will occur.
A.5. Conditions for an interior solution involving $0 < s_x < 1$

Start by assuming that export costs are sufficiently low that everyone exports. In this case firms operate in both markets and to simplify notation we can identify the profits of a typical firm (i.e. the profits from a set-up where everyone is symmetric and technology is constant over time).

$$\pi(\tilde{\phi}_x) = \frac{\tilde{\phi}_x^{\sigma-1}E}{n\sigma}$$

where $\tilde{\phi}_x$ is the productivity level that converts our non-stationary per-period profits into an annuity. That is $\tilde{\phi}_x$ is implicitly defined by the following condition:

$$\frac{\tilde{\phi}_x^{\sigma-1}E}{n\sigma} = \left(1 - e^{-rT_L} + e^{-rT_H}\right)E \frac{X(T_L) - X(T_H)}{\tilde{\phi}_x^{\sigma-1} - 1}.$$ 

The zero-profit condition can then be written compactly as:

$$\frac{\tilde{\phi}_x^{\sigma-1}E}{n\sigma} - X(T_H) - S - F = 0. \quad (31)$$

For a single non-exporter to consider operating we need the following to be true:

$$\frac{\tilde{\phi}_x^{\sigma-1}E}{2n\sigma} - X(T) - F \geq 0. \quad (32)$$

Note that $\tilde{\phi}_x$ is defined analogously to $\tilde{\phi}_x$ and $\tilde{T}$ is the optimal adoption date for the non-exporter. Note that if condition (32) is not met then we can raise $F$ and lower $S$, while keeping $S+F$ constant, until this condition is met. Such a trade-off ensures that the equilibrium values of $n$, $T_L$, and $T_H$ remain unchanged and therefore (31) holds.

To relate these conditions to those that exist in the literature with exogenous productivity define $f = F + X(\tilde{T})$ and $f_x = S + X(T_H) - X(\tilde{T})$. Using these definitions along with (31) and (32) one derives:

$$f_x + f \frac{2\tilde{\phi}_x^{\sigma-1}}{\tilde{\phi}_x^{\sigma-1}} \geq f(1 + \frac{2\tilde{\phi}_x^{\sigma-1}}{\tilde{\phi}_x^{\sigma-1}}).$$

Note that unlike the exogenous productivity literature, these conditions are composed of endogenous variables. Nevertheless, they do imply that for a only subset of firms to export requires that $f_x > f$.

References


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