Least Squares Estimation of a Panel Data Model with Multifactor Error Structure and Endogenous Covariates∗

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Abstract

We propose a method for estimating the slope parameter in an interactive effects panel data model with endogenous loadings and factors, and endogenous regressors.

JEL: C23, C33
Keywords: Panel data; Instrumental variables; Interactive fixed effects.

1. Introduction

Panel data models allow us to control for unobserved individual heterogeneity. Recently, attempts have been made to relax the traditional assumption of unique time invariant individual effects using multiple interactive effects (Bai, 2009; Pesaran, 2006). The natural extension to the standard panel data model with $N$ cross-sectional units and $T$ time periods, $y_{it} = \beta'x_{it} + \varepsilon_{it}$, imposes a multi-factor error structure on the error term $\varepsilon_{it} = \lambda_i'F_t + u_{it}$, where $\lambda_i$ is an $r \times 1$ vector of factor loadings and $F_t$ corresponds to the $r$ common factors, and where both $\lambda_i$ and $F_t$ are unobserved. The classical individual effects model can be obtained by setting $F_t$ and $r$ equal to one. Note however that both standard differencing and time trends methods are inconsistent in this setting since they will not remove the unobserved effects.

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Bai (2009) considers the estimation of the interactive effects model in large \( N \) and \( T \) panel data settings and treats both \( \lambda_i \) and \( F_t \) as fixed-effects parameters to be estimated, thus allowing for potential correlation between the unobservable interactive effects and the regressors \( x_{it} \). The finite sample performance of this method is relatively poor in micro-panels with small \( T \).

Pesaran (2006) estimates the model using cross-sectional averages of the data to proxy for the unobserved effects. The method relies on the assumption that only the latent factors \( F_t \) are allowed to be correlated with the regressors and the method is inconsistent if this assumption is violated. Moreover, Bai (2009) and Pesaran (2006) assume that the covariates \( x \) and the error term \( u \) are stochastically independent.

We show in this paper that the approach of Pesaran (2006) can be easily modified to accommodate instrumental variables estimation. Moreover, instrumental variables estimation is also shown to be an easy fix for the case where both factors and loadings are correlated with the regressors.

2. Model and Method

This paper considers the following model for \( i = 1 \ldots N \) and \( t = 1 \ldots T \),

\[
\begin{align*}
    y_{it} &= \beta' x_{it} + \gamma' d_t + \epsilon_{it}, \\
    \epsilon_{it} &= \lambda_i' F_t + u_{it}, \\
    x_{it} &= \Pi' w_{it} + G' d_t + \Lambda_i' F_t + A' \lambda_i + v_{it}.
\end{align*}
\]

We relax the assumptions of Bai and Pesaran by allowing that some of the components of \( v \) are stochastically dependent on \( u \). In this model \( x_{it} \) and \( d_t \) denote the individual and common observed regressors, while \( F_t \) denotes the unobserved common regressors with individual loadings \( \lambda_i \). It is assumed that we additionally observe a vector of instruments \( w_{it} \) which satisfies the appropriate identification conditions. The dimensions of the model are such that it is conformal to a design with \( k_1 \) regressors \( x_{it} \), \( k_2 \) common time trends \( d_t \), \( r \) factors \( F_t \) and \( m \geq k_1 \) instruments \( w_{it} \). In what follows we consider for simplicity the case of \( k_1 = m \).

Define \( z_{it} = (y_{it}, x_{it}')' \). Then,

\[
\begin{align*}
    z_{it} &= C_1' w_{it} + C_2' d_t + C_3' F_t + C_4' \lambda_i + \xi_{it},
\end{align*}
\]

where \( \xi_{it} = (\beta' v_{it} + u_{it}, v_{it}')' \) and \( C_1 = (\Pi \beta)', \Pi', C_2 = ((G \beta + \gamma)', G')', C_3 = ((\Lambda_i \beta + \lambda_i)', \Lambda_i')' \) and \( C_4 = ((A \beta)', A')' \).
Now consider cross-sectional averages,

\[ \tilde{z}_t = C'_1 \tilde{w}_t + C'_2 d_t + C'_3 F_t + C'_4 \tilde{\lambda} + \tilde{\xi}_t. \]

Hence,

\[ F_t = (C_3 C'_3)^{-1} C_3 (\tilde{z}_t - C'_1 \tilde{w}_t - C'_2 d_t - C'_4 \tilde{\lambda} - \tilde{\xi}_t). \]

Under the regularity conditions of Pesaran (2006), as \( N \rightarrow \infty \), for all \( t \) we have \( \tilde{\xi}_t \rightarrow 0 \) and \( C_3 \rightarrow C_3 \) (constant). Hence, we can proxy for \( F_t \) by \( (\tilde{w}'_t, d'_t, \tilde{z}'_t, 1')' \). Notice, however that the required proxy \( \tilde{\lambda} \) is not observed and consistent estimation is not possible unless either \( A = 0 \) or \( \tilde{\lambda} = 0 \). Furthermore, a second source of endogeneity is present as long as \( E(u_{it} v_{it,k}) \neq 0 \). The analysis indicates that if valid instruments \( w_{it} \) are present performing an instrumental variable regression on equation 2.1 augmented by \( (\tilde{w}'_t, d'_t, \tilde{z}'_t)' \) will lead to consistent estimates of \( \beta \). The instruments address both the endogeneity of the regressors \( x_{it} \) due to the correlation between \( u_{it} \) and \( v_{it} \) and the correlation of the factor loadings with the regressors.

In practice, the model can be estimated consistently using instrumental variables as follows. First, augment equation 2.1 using the cross-sectional means \( \bar{y}_t \) and \( \bar{x}_t \). Notice, that this introduces the additional endogenous variable \( \bar{x}_t \). Second, estimate the augmented equation using instrumental variables \([d_t; \tilde{w}_t; 1]\) as the first stage regressors. Inferential procedures could be implemented by accommodating the variance formulas introduced in Pesaran (2006) or by considering the bootstrap, as in Section 4. Once the coefficients on the observables have been estimated consistently, each model can be transformed into a traditional factor model and the factors and factor loadings consistently estimated using maximum likelihood or PCA. If the number of factors is unknown, it can also be estimated consistently using the eigenvalue method of Harding and Nair (2009) even in the presence of factor dynamics.

3. Monte Carlo

We generate the dependent variable considering a design similar to Bai (2009) and Pesaran (2006):

\[
\begin{align*}
y_{it} &= \beta_0 + \beta_1 x_{it} + \gamma d_t + \lambda'_i F_t + u_{it} \\
x_{it} &= \pi_0 + \pi_1 w_{it} + gd_t + l_i' F_t + a \lambda'_i F_t + v_{it} \\
F_{jt} &= \rho_f F_{jt-1} + \eta_{jt} \\
\eta_{jt} &= \rho_\eta \eta_{jt-1} + e_{jt}
\end{align*}
\]
for \( j = \{1, 2\}, \ldots, t = -49, \ldots, 0, \ldots, T \) in the last two equations. The random variables are \( d_t \sim \mathcal{N}(0, 1) \), \( \lambda_{t1}, \lambda_{t2} \sim \mathcal{N}(1, 0.2) \), \((u_{it}, v_{it})' \sim \mathcal{N}(0, \Omega)\), and \( e \) and \( w \) are Gaussian random variables. The parameters are assumed to be: \( \beta_0 = \pi_0 = l = 2 \), \( \beta_1 = \gamma = \pi_1 = g = 1 \), \( \rho_f = 0.90 \), \( \rho_\eta = 0.25 \), and \( \Omega_{11} = \Omega_{22} = 1 \). We consider three designs: (I) \( a = 0 \) and \( \Omega_{12} = \Omega_{21} = 0 \); (II) \( a = 2 \) and \( \Omega_{12} = \Omega_{21} = 0 \); (III) \( a = 2 \) and \( \Omega_{12} = \Omega_{21} = 0.5 \).

<table>
<thead>
<tr>
<th>Statistics</th>
<th>N</th>
<th>T</th>
<th>OLS</th>
<th>FE</th>
<th>2SLS</th>
<th>IVFE</th>
<th>ICCE</th>
<th>CCE</th>
<th>IVCCE</th>
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<tbody>
<tr>
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<tr>
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<td>5</td>
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<td>0.3147</td>
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<td>-0.0210</td>
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<td>0.2829</td>
<td>0.3355</td>
<td>0.1569</td>
<td>0.1762</td>
<td>0.0283</td>
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<tr>
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<tr>
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<tr>
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<td>0.7272</td>
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<td>0.4443</td>
<td>0.2091</td>
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</table>

Table 1. Bias and root MSE of panel data regression estimators. Results are based on 1000 replications.

In Table 1 we compare the performance of several estimators for different samples sizes. We consider the ordinary least squares estimator (OLS), the within estimator (FE), the two-stage least
squares estimator (2SLS), the instrumental variables estimator applied to the within transformation (IVFE), the infeasible common correlated effect estimator (ICCE) which observes $\lambda'F_t$, the common correlated effects estimator (CCE) which proxies for the unobserved factors using cross-sectional averages and the instrumental variables common correlated effects estimator (IVCCE) proposed in this paper, which applies instrumental variables estimation to the regression equation augmented by the cross-sectional averages of the observables.

The first MC design corresponds to a situation with interactive effects, where the regressors are exogenous and the factor loadings are not correlated with the regressors. As we would expect IV, IVFE, ICCE, CCE and IVCCE are all unbiased.

The second MC design corresponds to a model where the regressors are exogenous but the factor loadings are correlated with the regressors. In this case we notice that CCE is biased since it does not fully proxy for the latent factors. IVCCE however automatically corrects this problem. This model could have been estimated using the concentrated least squares approach of Bai (2009), however due to the small $T$ dimension common in micro-econometric applications, the IVCCE estimator has better mean squared error properties than an approach involving PCA which produces large mean-squared errors (Online Appendix to Bai, 2009).

The final MC design allows for both endogenous regressors and factor loadings correlated with the regressors. In this case the CCE estimator is severely biased while the IVCCE estimator continues to perform very well. Notice that the ICCE estimator is also severely biased due to the endogenous regressors and shows that knowing the true factors is not as important as correcting for endogeneity when it is present.

4. Empirical Application

We consider data from the Milwaukee Parental Choice program (MPCP), the same data as Rouse (1998). Consider a model with interactive fixed effects:

\[
T_{it} = \delta c_{it} + \beta' x_{it} + \lambda' F_t + u_{it}
\]

where $T$ measures educational attainment, $c$ is actual attendance to choice school, $x$ is a vector of exogenous variables that includes grade level of the test, gender, income, application lotteries, indicators for years from application to the program, and a dummy variable for whether the test was imputed. The variable $w$ is an indicator variable for whether the students was randomly selected to
attend choice schools. We allow for possible correlation between $c$ and $(\lambda', F', u)$, and by definition, $w \perp u$.

<table>
<thead>
<tr>
<th>Treatment Variable</th>
<th>OLS</th>
<th>FE</th>
<th>CCE</th>
<th>2SLS</th>
<th>IVFE</th>
<th>IVCCE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Enrolled in choice school</td>
<td>0.699</td>
<td>-0.772</td>
<td>0.720</td>
<td>3.530</td>
<td>4.276</td>
<td>3.585</td>
</tr>
<tr>
<td></td>
<td>(1.175)</td>
<td>(1.264)</td>
<td>(1.176)</td>
<td>(2.071)</td>
<td>(2.265)</td>
<td>(2.067)</td>
</tr>
</tbody>
</table>

Table 2. Estimates of the causal effect of choice schools on math test scores.

Table 2 presents results. The OLS, FE, and CCE estimates are likely to be biased because the treatment variable is suspected to be endogenous. Additionally the standard within transformation does not get rid of the individual specific effect, and thus $E((c_{it} - \bar{c}_i)\lambda_i(F_t - \bar{F})) \neq 0$ if we consider for simplicity the case of one factor. 2SLS and IVFE might be biased if there are interactive fixed effects also. The instrument could be correlated with the multifactor error term: $E((w_{it} - \bar{w}_i)\lambda_i(F_t - \bar{F})) \neq 0$ in the IVFE case and $E(w_{it}\lambda_iF_t) \neq 0$ in the IV case. These conditions might be interpreted as suggesting that the selection to the program could be related to parents’ motivation and time factors like the availability of seats in the schools. The IVCCE suggests that the program benefited students on mathematics, yet the gain is less than that implied by standard IVFE methods.

References


