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## A theory of equity carve-outs and negative stub values under heterogeneous beliefs \*

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#### ABSTRACT

We develop a theory of new-project financing and equity carve-outs under heterogeneous beliefs. In our model, an employee of a firm generates an idea for a new project that can be financed either by issuing equity against the cash flows of the entire firm ("integration"), or by undertaking an equity carve-out of the new project alone ("non-integration"). While the patent underlying the new project is owned by the firm, the employee generating the idea needs to be motivated to exert optimal effort for the project to be successful. The firm's choice between integration and non-integration is driven primarily by heterogeneity in beliefs among outside investors (each of whom has limited wealth to invest in the equity market) and between firm insiders and outsiders: if the marginal outsider financing the new project is more optimistic about the prospects of the project than firm insiders, and this incremental optimism of the marginal outsider over firm insiders is greater regarding new-project cash flows than that about assets-in-place cash flows, then the firm will implement the project under non-integration rather than integration. Two other ingredients driving the firm's financing choice are the cost of motivating the employee to exert optimal effort, and the potential synergies between the new project and assets in place. We derive a number of testable predictions regarding a firm's equilibrium choice between integration and non-integration. We also provide a rationale for the "negative stub values" documented in the equity carve-outs of certain firms (e.g., the carve-out of Palm from 3Com) and develop predictions for the magnitude of these stub values.

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#### 1. Introduction

Starting with Miller (1977), a number of authors have theoretically examined the stock price implications of

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heterogeneous beliefs and short-sale constraints on stock valuations. Miller (1977) argues that when investors have heterogeneous beliefs about the future prospects of a firm, its stock price will reflect the valuation that optimists attach to it, because the pessimists will simply sit out the market (if they are constrained from short-selling). A number of subsequent authors have developed theoretical models that derive some of the most interesting cross-sectional implications of Miller's logic. In an important paper, Morris (1996) shows that the greater the divergence in the valuations of the optimists and the pessimists, the higher the current price of a stock in equilibrium, and hence lower the subsequent returns. In another important paper, Duffie, Gârleanu, and Pedersen (2002) show that, even when short-selling is allowed (but requires searching

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for security lenders and bargaining over the lending fee), the price of a security will be elevated and can be expected to decline subsequently in an environment of heterogeneous beliefs among investors if lendable securities are difficult to locate. Another important implication of heterogeneous beliefs among investors is that it can lead to a significant amount of trading among investors: see, e.g., Harris and Raviv (1993), who use differences in opinion among investors to explain empirical regularities about the relationship between stock price and volume. However, while the implications of heterogeneous beliefs among investors for capital markets have been examined at some length (see, e.g., Lintner, 1969 for one of the earliest contributions), the corporate finance implications of such beliefs have not been adequately studied (with some notable exceptions that we will discuss later). The objective of this paper is to theoretically analyze an important way of financing new projects, namely, equity carve-outs, in an environment of heterogeneous beliefs and short-sale constraints, and to analyze the puzzling phenomenon of negative stub values that have been known to arise in equity carve-outs.

There are several interesting questions that beg to be answered in the context of equity carve-outs. Should a new project be financed and implemented inside the existing firm or in a new external venture? What are the factors that determine whether a firm chooses to finance and develop a new project by integrating it with its assets in place or instead chooses to carve it out to outside investors and let it be developed externally? In particular, what is the extent to which the heterogeneity in beliefs among investors and between firm insiders and outsiders in the equity market plays a role in shaping the decisions made by firms when they finance new projects? How does the market value the prospects of new projects under heterogeneous beliefs? Does this valuation interact with the choice of organizational form under which a new project is funded and implemented?

Finally, what explains the phenomenon of negative stub values in some equity carve-outs? By stub value, we mean the difference between the market value of the parent firm as a whole after an equity carve-out and the market value of the parent firm's equity holdings in the carved-out firm. When this difference is negative, i.e., the equity market value of the parent firm is less than that of its equity holdings in the carved-out firm, we say that this equity carve-out is characterized by a "negative stub value" (at the current market values of the parent firm and the carved-out firm). A well known example of a negative stub value occurred in the case of the carve-out of Palm from 3Com, where, immediately after the carve-out, Palm was selling at several times the equity market value of 3Com, and the market value of 3Com was significantly less than that of its equity holdings in Palm. Such negative stub values have been considered to be puzzling in the financial economics literature, and have been cited by some as an example of investor irrationality: see, e.g., Lamont and Thaler (2003).

In this paper, we develop a theory of the financing of new projects under heterogeneous beliefs among investors in an equity market with short-sale constraints, and provide some novel answers to the above questions. We also provide an explanation for the phenomenon of negative stub values in a setting with fully rational investors who have differences in beliefs (heterogeneous priors). We consider a setting where an employee of a firm generates an idea for a new project. The firm needs to raise external financing to implement this project. It may choose to raise the investment capital needed either by issuing equity against the future cash flows of the entire firm, i.e., both assets in place and the new project ("integration"), or by undertaking an equity carve-out of the new project ("non-integration"). The equity market is characterized by heterogeneity in beliefs (heterogeneous priors), both between insiders and outsiders, and across outside investors. Further, while the patent underlying the new project is owned by the firm, the employee generating the idea needs to be motivated to exert optimal effort for the project to be successful.1

The most important ingredient driving a firm's choice between integration and non-integration in our model is heterogeneity in beliefs among outside investors (each of whom has limited wealth to invest in the equity market) and the difference in beliefs between firm insiders and outsiders. The difference between insiders' and outsiders' beliefs about the future cash flows of the new project creates a window of opportunity for the firm to raise capital at a lower cost by capturing the optimism of outside investors. While insiders would like to raise the entire amount of financing required by selling equity to the most optimistic outsiders, the fact that each outsider has only a limited amount of wealth to invest in the equity market forces them to go down the belief scale and sell equity to less optimistic outsiders until the entire amount of financing required is raised. The beliefs of the marginal investor financing the new project will be determined, to a large extent, by the average belief across outside investors, and the dispersion in beliefs across these investors. If outside investor beliefs are such that the marginal outsider financing the innovation is more optimistic about the prospects of the new project than firm insiders, and this incremental optimism of the marginal outsider over firm insiders is greater regarding the new project than about the firm's assets in place, then the firm will find it optimal (under some additional conditions) to finance and implement the project outside rather than integrate it within the firm.<sup>2</sup>

We also characterize the conditions under which firm insiders choose to finance the new project under integration (i.e., by selling equity in the combined firm) rather than non-integration. As in the case of non-integration, the firm will start with those outsiders who will yield them the highest equity value, and go down the belief scale until the entire amount of financing is raised. The correlation in outsider beliefs about the prospects of the new project

<sup>&</sup>lt;sup>1</sup> It is not crucial that the new idea is generated by a current employee of the firm. The crucial assumption here is that there are one or more employees of the firm whose effort is essential for the success of the project, and who need to be motivated to exert optimal effort.

<sup>&</sup>lt;sup>2</sup> As in the existing literature on heterogeneous beliefs (see, e.g., Miller, 1977; or Morris, 1996) we assume short-sale constraints throughout, so that the effects of differences in beliefs among investors are not arbitraged away. The above standard assumption is made only for analytical tractability: our results go through qualitatively as long as short selling is costly (see, e.g., Duffie, Gârleanu, and Pedersen, 2002).

and those about the firm's assets in place will substantially affect the identity of the marginal investor in the firm's equity in the case of integration. Outside investors in the combined firm will apply a discount to their valuation of the firm's assets in place if they are not as equally enthusiastic about these assets as they are about the new project. If, on the other hand, outsiders are equally or more enthusiastic about the prospects of the firm's assets in place as about its new project, then the beliefs of the marginal investor in the combined firm's equity will be sufficiently more optimistic than those of firm insiders, so that the firm will choose to implement the project under integration rather than non-integration.<sup>3</sup>

Two other ingredients driving a firm's choice between integration and non-integration are the cost of effort of the employee generating the new idea (project) and the potential synergy that the parent firm has in implementing the new project (arising, for example, from the new project sharing the parent firm's assets). The employeeentrepreneur's cost of effort affects the compensation to be provided to motivate him to exert optimal effort for project implementation. The cost of the optimal incentive scheme will be different for integration versus non-integration when the employee-entrepreneur is paid with the equity of the firm he is working for. It can be shown that it is cheaper for firm insiders to motivate the employee to exert optimal effort by compensating him with equity in the carved-out firm, so that this ingredient favors non-integration. The magnitude of the synergy in project implementation between the new project and the firm's assets in place affects the total cash flows generated by the firm. This synergy will be eliminated if the project is carved out as a separate firm, so that this ingredient favors integration.

Heterogeneity in beliefs across outside investors and differences in average investor beliefs across projects (technologies) provide a rationale for the presence of "negative stub values" in the equity carve-outs of certain firms (e.g., the carve-out of Palm from 3Com) in our setting. We show that, depending on the correlation in outsider beliefs about the prospects of the new project and those about the firm's assets in place, there may be a wedge between the market value of the parent firm's equity holdings in the carved-out firm (which is determined by the marginal investor financing the new project only) and the value attached to these equity holdings by the marginal investor in the parent firm. We demonstrate that negative stub values are possible, given that the market values of the parent firm and the carved-out firm are determined by different groups of investors in our setting.

It is important to note that, while outside investors and firm insiders have heterogeneous prior beliefs, all agents in our model are fully rational. As Morris (1995) has argued in an important paper, differences in beliefs are quite consistent with rationality. Thus, in our setting, rational agents with heterogeneous priors "agree to disagree" about the future prospects of the firm's assets in place as well as that of its new project. In other words, our model develops a theory of equity carve-outs and negative stub values in a fully rational setting with heterogeneous beliefs and short-sale constraints.

Our analysis generates a number of testable predictions for a firm's choice of organizational structure under which new projects will be funded and implemented. First, our model predicts that radically new technologies (characterized by greater uncertainty and therefore higher dispersion of investor beliefs) are more likely to be implemented outside the firm under a new venture. On the other hand. new projects that are increments of (closely related to) the firm's existing projects are more likely to be financed and implemented as part of the existing firm. Second, technologies about which outsiders are more optimistic, on average, are more likely to be "carved-out" and therefore implemented outside their parent firms. Third, new projects appealing to an investor base different from the current investor base of the parent firm are more likely to be implemented outside. Fourth, for projects where the effort of the employee generating the idea for the new project is more important for the implementation of the project, nonintegration will be the more probable organizational choice. Fifth, our model predicts that integration will be more likely if the synergy created between the new project and the firm's assets in place is greater, which would be the case, for example, when the new project is in the same industry as the parent firm. Finally, if the size of the new project is relatively small with respect to the size of the firm's assets in place, the parent firm is more likely to choose nonintegration as the preferred form of organization to better motivate the employee in charge of the project and to better capture the optimism of outside investors when raising new capital.<sup>5</sup> Many of the above implications are unique to our model and untested in the existing literature; we describe these in more detail in Section 5, and discuss how some of these can be tested.

<sup>&</sup>lt;sup>3</sup> We rule out structures such as tracking stock, which involve implementing the new project within the firm, but financing it using a separate class of equity issued against the new project's cash flows alone. From the point of view of the effect of heterogeneity in investor beliefs, arrangements such as tracking stock are quite close to equity carve-outs, though they may allow the firm to partially preserve the synergies between the firm's assets in place and its new project. To keep our analysis simple, we focus on the two extreme organizational structures of integration and non-integration (equity carve-outs), and do not analyze hybrid arrangements such as tracking stock here.

<sup>&</sup>lt;sup>4</sup> Morris (1995) provides a detailed discussion of the role of the common prior assumption in economic theory. Kurz (1994) provides the foundations for heterogeneous but rational priors.

<sup>&</sup>lt;sup>5</sup> One real-world example of spinoffs or carve-outs driven by differences in shareholder optimism between assets in place and a new project is the spinoff of Sunpower, which makes solar panels, from Cypress Semiconductor, an established Silicon Valley firm. To quote Daniel Gross' article in Slate.com ("The Prius bubble," *Slate*, July 22, 2006): "Investors have thronged to the stock the way college students flock to Cancun on spring break." Another example is the spinoff of CoGenesys Inc., which focuses on the early stages of drug development, from the bio-tech firm Human Genome Sciences Inc. Here, the motivation was not only differences in investor optimism, but also the need to motivate Craig Rosen and Steven C. Mayer (former employees of Human Genome Sciences) to make CoGenesys a success: they were given a 13% equity stake in the new firm and the rights to develop some of the CoGenesys drugs as part of the carve-out ("A biotech firm's new formula," *Washington Post*, July 31, 2006).

Our model also has two predictions regarding negative stub values. First, it predicts that negative stub values in the equity carve-outs of certain firms are more likely to arise if (a) the dispersion in investor beliefs about the new project is higher, (b) investors are more optimistic about the new project, (c) the correlation between investor beliefs regarding the new project and those regarding the firm's assets in place is lower (i.e., when the investor bases for the new and existing projects of the firm are quite different), and (d) the relative size of the new project is not too small. Second, our model predicts that, whenever negative stub values are present, the heterogeneity in investor beliefs about the value of the subsidiary firm will be much higher than the heterogeneity in beliefs about the value of the parent firm. Therefore, the model predicts that, when the stub value is negative, the turnover in the shares of the subsidiary firm will be much higher than that in the shares of the parent firm given that trade is generated by differences in beliefs: see, e.g., Harris and Raviv (1993). Evidence consistent with this is presented by Lamont and Thaler (2003), who study mispricing in tech-stock carve-outs and find that, when the law of one price is violated, the higher priced security has turnover that is many times higher than the turnover of the lower priced security (indicating that the heterogeneity in investor beliefs about the carved-out firm is much greater than that about the parent firm). They find that, in the case of the well known Palm-3Com carve-out, the turnover in the shares of Palm (the carved-out firm's security) was vastly higher than the turnover in the shares of 3Com (the parent firm's security).

Our paper is related to three broad strands in the theoretical finance and economics literature. The first is the emerging literature on firm and investor behavior under heterogeneous prior beliefs. As discussed earlier, several authors have examined the asset pricing and trading implications of heterogeneous beliefs (see, e.g., Harrison and Kreps, 1978; Morris, 1996; Duffie, Gârleanu, and Pedersen, 2002; and Chen, Hong, and Stein, 2002 for contributions to this literature; and Scheinkman and Xiong, 2004 for a review). Several authors have argued that prior beliefs should be viewed as primitives in the economic environment (Kreps, 1990) and that it may be appropriate for economists to allow for differences in prior beliefs to understand economic phenomena (Morris, 1995). Allen and Gale (1999) examine how heterogeneous priors among investors affect the source of financing (banks versus equity) of new projects.<sup>6</sup> Bayar, Chemmanur, and Liu (2010) develop a theory of capital structure, price impact, and long-run stock returns under heterogenous beliefs. Dittmar and Thakor (2007) study the choice of capital structure in a firm when insiders and outsiders disagree about the firm's prospects; Boot, Gopalan, and Thakor (2006) study an entrepreneur's choice between private and public financing in a similar setting of disagreement between insiders and outsiders. Harris and Raviv (1993) use differences of opinion to explain empirical regularities about the relationship between stock price and volume. Finally, Garmaise (2001) examines the implications of heterogeneous beliefs for security design.

The second strand of literature that our paper is related to is the theoretical literature on equity carve-outs and corporate spinoffs. A prominent example is Nanda (1991), who analyzes equity carve-outs by extending the Myers and Majluf (1984) asymmetric information framework to a setting where firms can raise financing by issuing equity against the new project as well as against the combined firm. Unlike our paper, his focus is on explaining the positive announcement effect in equity carve-outs that has been documented by a number of empirical papers starting with Schipper and Smith (1986); neither does his model explain negative stub values, which we are able to do using our heterogeneous beliefs framework. A second example is Aron (1991), who studies the relationship between corporate spin-offs and managerial incentives, and demonstrates that corporate spinoffs improve such incentives when the stock market value of a product line that is spun off provides a much clearer signal of managerial productivity than accounting measures generated when that division belongs to the parent firm. Unlike the above papers and others in the literature which rely on either asymmetric information or moral hazard to explain spinoffs and carve-outs, ours is the first paper in the literature to develop a model of equity carve-outs that incorporates the role of heterogeneous beliefs among investors. Ours is also the first paper to develop a theoretical analysis of negative stub values in equity carve-outs.7

The third strand of literature our paper is related to is the theoretical and empirical literature on the development of new firms and the choice between internal versus external development of innovations. The pioneering paper by Aghion and Tirole (1994) analyzes research and development (R&D) activity in an incomplete contracting framework, and studies how the allocation of property rights on innovations between two firms may affect both the frequency and magnitude of these innovations. Gromb and Scharfstein (2002) analyze the choice between the financing of new ventures in start-ups (entrepreneurship) versus in established firms (intrapreneurship). In their model, the above choice is driven by adverse selection in the external labor market: while an employee who fails at implementing a project within an established firm can be redeployed to another job in the firm, employees who fail at being entrepreneurs must seek new jobs in an imperfectly informed external labor market. Cassiman and Ueda (2006) analyze why an established firm chooses not to commercialize a seemingly good innovation while a start-up firm may do so. In a setting where the established firm can commercialize only a limited number of innovations and innovations have varying degrees of fit with the firm's existing capabilities, they show that an established firm may optimally reject a seemingly good innovation and wait for

<sup>&</sup>lt;sup>6</sup> See also Abel and Mailath(1994), who demonstrate that in certain special settings with heterogeneous beliefs, even projects that all investors believe have negative expected value if undertaken may be financed by these investors.

<sup>&</sup>lt;sup>7</sup> While they do not explicitly model equity carve-outs, Duffie, Gârleanu, and Pedersen (2002) use a stylized example to suggest that negative stub values may be generated due to heterogeneous beliefs in an environment of costly short-selling.

a future innovation with a better fit with its capabilities. Finally, Amador and Landier (2003) also study the choice between internal and external development of new ideas in a firm financed by a venture capitalist. In their setting, this choice is driven by the trade-off between the cost reduction generated due to the sharing of assets when implementing projects internally versus the flexibility generated by contingent contracting with the employee-turned-entrepreneur when implementing projects externally. It is worth noting that, while the above papers study the choice between internal and external development of innovations, none of these papers analyze the relationship between this choice and conditions in the external equity market (and in particular, the heterogeneity in beliefs across investors in the equity market), which is the focus of our paper.<sup>8</sup>

The rest of the paper is organized as follows. Section 2 presents the basic features of our model. Section 3 presents the analysis of a firm's equilibrium choice between integration and non-integration. Section 4 analyzes situations under which negative stub values arise in equity carve-outs. Section 5 summarizes the empirical implications of the model, and Section 6 concludes. The proofs of all lemmas and propositions are in the Appendix.

#### 2. The model

There are three dates in the model. At time 0, insiders of a firm hold a fraction  $\gamma$  of the firm's equity, with the remaining shares held by a group of current shareholders. We assume that there is a continuum of outside investors in the market, with an aggregate wealth of W, which is uniformly distributed across all investors. We further assume that current shareholders of the firm have exhausted their wealth and therefore cannot participate in a new equity issue. Finally, we assume that short-selling of equity is not allowed. All agents are risk neutral and the risk-free rate of return is normalized to zero.

The firm has an ongoing project A (assets in place). Its cash flows will be realized at time 2. Firm insiders and employees believe that with probability  $\theta_a^f$ , the cash flow from project A will be  $X_a^H$  and with probability  $(1-\theta_a^f)$  it will be  $X_a^L$ , where  $X_a^H > X_a^L$ . In contrast, outside investors in the market have heterogeneous beliefs about the cash flows from this project. Their beliefs about the success probability of project A are uniformly distributed in the interval  $[\theta_a^m - d_a, \theta_a^m + d_a]$ .

At time 0, an employee of the firm ("employee-entrepreneur" from now on) comes up with the idea for an innovative project B, which requires an investment capital of *I* at time 1. The firm owns the property rights on

this new project (i.e., the patent). Market participants also have heterogeneous beliefs about the cash flows from project B, which will be realized at time 2. Firm insiders and the employee-entrepreneur believe that with probability  $\theta_b^f$ , the cash flow from project B will be  $X_b^H$  and with probability  $(1-\theta_b^f)$  it will be  $X_b^L$ , where  $X_b^L < I < X_b^H$ . In contrast, outside investors' beliefs about the success probability of project B are uniformly distributed in the interval  $[\theta_b^m - d_b, \theta_b^m + d_b]$ .

The parameters  $\theta_a^m$  and  $\theta_b^m$  represent the average beliefs of investors about the existing project and the new project, respectively, and  $d_a$  ( $d_b$ ) is the dispersion in outside investors' beliefs about project A (project B). We use  $\theta_a$  to index an agent whose belief about the success probability of project A is  $\theta_a$ . For example, agent  $\theta_a$  believes that with probability  $\theta_a$  project A's time 2 cash flow will be  $X_a^H$ , and with probability  $1-\theta_a$  it will be  $X_a^L$ . Investors' beliefs about the cash flows of projects A and B are illustrated in Fig. 1.9 If a project q has success probability  $\theta_q$ , where  $q \in \{a,b\}$ , its expected value of time 2 cash flows, denoted by  $V(\theta_a)$ , is given by

$$V(\theta_a) = \theta_a X_a^H + (1 - \theta_a) X_a^L, \tag{1}$$

where  $X_q^H > X_q^L$ . We call a project q successful if the high cash flow outcome  $X_q^H$  is realized at time 2. We assume that the project B has a positive net present value based on firm insiders' beliefs. Further, there are enough outsiders who believe that the new project has positive net present value so that, regardless of whether the project B is financed under non-integration or integration, the marginal outside investor providing funding for implementing the project believes it to have net present value large enough that the firm insiders' participation constraint is satisfied (i.e., they are better off implementing the new project by selling equity to outsiders than not implementing it).

We also assume that, at time 0, there exists a further source of uncertainty among outside investors about the correlation of their beliefs about the future prospects of the firm's assets in place (project A) and those of the new project B. At time 0, each outside investor knows his belief  $\theta_a$  about project A's success probability with certainty, but he has only an ex ante expectation as to what his time-1 belief about the new project B's success will be, conditional on his belief  $\theta_a$  about project A. In other words, at time 0 each investor only has a prior probability about his beliefs about project B, and therefore the correlation between his beliefs about the future prospects of projects A and B. To model and measure the ex ante (time 0) degree of statistical dependence (i.e., ex ante correlation)

<sup>&</sup>lt;sup>8</sup> The empirical literature that studies the creation of new firms, innovations by start-ups vs. large firms, and entrepreneurial spawning by public corporations (e.g., Henderson and Clark, 1990; Audretsch, 1991; Gans, Hsu, and Stern, 2002; and Gompers, Lerner, and Scharfstein, 2005) is also related to our paper. Our paper is also indirectly related to the literature on the generation and implementation of new ideas: see, e.g., Biais and Perotti (2008). Our paper is also broadly related to the literature on strategic alliances between firms (e.g., Mathews, 2006; Palia, Ravid, and Reisel, 2008; Robinson, 2008) and alternative ways of financing the firm in the context of an R&D race (see, e.g., Fulghieri and Sevilir, 2009).

<sup>&</sup>lt;sup>9</sup> Thus, we allow insiders to be more or less optimistic relative to the average outsider about the cash flows from the firm's projects A and B, depending on stock market conditions. This is a reasonable assumption, given that, during certain time periods, outsiders may be very enthusiastic about investing in projects in certain industries (but not in others), while insider beliefs about the prospects of the firm's projects can be expected to remain steady over time. See footnote 5 for a real-world example of an equity carve-out driven by differences in shareholder optimism between the assets in place and the new project of a firm.

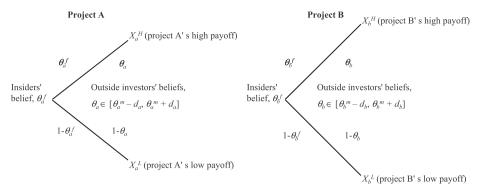


Fig. 1. Beliefs of insiders and outsiders about the cash flows from project A and project B.

between investor beliefs about the success probabilities of the existing project A and the new project B,  $\theta_a$  and  $\theta_b$ , respectively, we use the parameter  $\rho$ , which can take any value in the closed interval [-1,+1]. Note that  $\rho = +1$  is the case of perfectly positive ex ante correlation and  $\rho = -1$  is the case of perfectly negative ex ante correlation in outsiders' beliefs about project A and project B. This implies that, at time 0, the prior assessment of each outside investor is that his time-1 belief about project B will be either perfectly positively correlated with his belief about project A (with probability  $(1+\rho)/2$ ) or perfectly negatively correlated (with the remaining probability  $(1-\rho)/2$ ). The actual realization of each investor's belief about project B occurs at time 1: consistent with his prior belief at time 0, the realized value of the correlation between the investor's beliefs about project A and project B will be either +1 or -1.

The assumption that outside investors realize their own beliefs about the future prospects of project B only at time 1 (and have only a prior probability assessment of these beliefs at time 0) is made for analytical simplicity. In particular, this allows us to derive closed-form solutions for the equity value of the combined firm (in the case of integration), even for situations where the ex ante (time 0) correlation  $\rho$  in investor beliefs about projects A and B is between -1and +1. If we make the alternative assumption that each investor realizes his belief about project B at time 0 itself, we will be able to develop closed-form solutions for the combined firm's equity value only for the cases where the correlation between investor beliefs about projects A and B is either +1 or -1.<sup>10</sup> However, even under this alternative assumption, numerical simulations show that our results remain qualitatively similar to those presented in the paper for values of the correlation in investor beliefs about projects

A and B lying between -1 and +1 as well. These simulations are available to interested readers upon request.

If outside investors' beliefs about the existing project A and the new project B are perfectly positively correlated at time 1, the following one-to-one mapping holds for an investor from his belief  $\theta_a$  about project A to his belief  $\theta_b$  about project B:

$$\theta_b = \theta_b^m + \frac{d_b}{d_a} (\theta_a - \theta_a^m). \tag{2}$$

Thus, if investor beliefs about the two projects are perfectly correlated at time 1, agent  $\theta_a$  will have the same preference ranking for project A and project B among all outside investors in the economy. In other words, the most optimistic investor about project A will be also the most optimistic investor about project B. Similarly, the second most optimistic investor about project A will also be the second most optimistic investor about project A will also be the second most optimistic investor about project B, and so on. We can invert the mapping given in (2) to find the belief of an investor about project A, whose time-1 belief about project B is equal to  $\theta_b$ , where  $\theta_b \in [\theta_b^m - d_b, \theta_b^m + d_b]$ .

An exactly opposite relationship will hold between investors' belief rankings about project A and project B at time 1, if these beliefs are perfectly negatively correlated at time 1. In this case, the most optimistic investor about project A will be the most pessimistic investor about project B. Similarly, the second most optimistic investor about project A will be the second most pessimistic investor about project B, and so on. For any investor  $\theta_a$ , the following one-to-one mapping holds in the case of perfectly negative correlation between his belief  $\theta_a$  about project A and his belief  $\theta_b$  about project B at time 1:

$$\theta_b = \theta_b^m - \frac{d_b}{d_a} (\theta_a - \theta_a^m). \tag{3}$$

Thus, agent  $\theta_a$ 's time-0 expectation of his time-1 belief about project B conditional on his belief  $\theta_a$  about project A is given by

$$\begin{split} \mathbf{E}[\theta_b|\theta_a] &= \frac{(1+\rho)}{2} \left[ \theta_b^m + \frac{d_b}{d_a} (\theta_a - \theta_a^m) \right] + \frac{(1-\rho)}{2} \left[ \theta_b^m - \frac{d_b}{d_a} (\theta_a - \theta_a^m) \right] \\ &= \theta_b^m + \rho \frac{d_b}{d_a} (\theta_a - \theta_a^m). \end{split} \tag{4}$$

<sup>10</sup> While we adopt this modeling approach for the correlation between the cash flows of the firm's assets in place and its new project mainly for analytical simplicity, there may be many real-world situations where outsiders may have only a prior assessment of the probability distribution of project B's cash flows, and therefore this correlation when they first become aware of the firm's new project (at time 0 in our model) and revise this correlation upwards or downwards as additional information becomes available to them in the prospectus for an equity issue undertaken to fund this new project (at time 1 in our model).

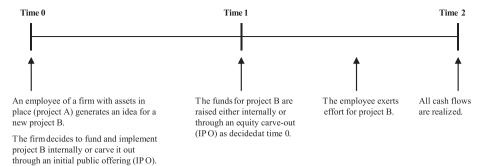


Fig. 2. Sequence of events.

It is easy to verify that the unconditional expectation of  $\theta_b$  at time 0 is equal to  $\theta_b^m$  and its unconditional dispersion at time 0 is equal to  $d_b$ . At time 0, we use  $\theta_a$  to index an outside investor in the economy. When the uncertainty about the relationship between  $\theta_a$  and  $\theta_b$  is resolved at time 1, however, we can use outside investors' beliefs about either project to index an outside investor in the economy.

Since project B is generated by the employee-entrepreneur, we assume that he is indispensable for its successful implementation. We assume that the employee-entrepreneur has only two possible effort levels: high effort (e=1) or no effort (e=0). If he does not exert effort for project B, i.e., e=0, the probability of success will be zero, i.e.,  $\theta_b=0$ . If he exerts high effort, i.e., e=1, he incurs a private effort cost of  $\Psi>0$ . The employee-entrepreneur's effort is unobservable to both firm insiders and outsiders. His reservation utility is normalized to 0.11

The objective of firm insiders is to maximize the expected time-2 payoff to current shareholders based on firm insiders' beliefs  $\theta_a^f$  and  $\theta_b^f$  by choosing the optimal organizational form under which the new project (project B) will be implemented and funded. The sequence of events in our model is summarized in Fig. 2. The choice of organization will be made at time 0, and it will be implemented and financed at time 1. The firm has no slack, and the external funding I for the new project will be raised from outside investors at time 1 under either form of organization. The aggregate wealth of outside investors is large enough so that W > 2I.

#### 2.1. Non-integration

The first choice available to firm insiders to manage the new project is to implement and fund it outside the firm (non-integration). If they choose to do so at time 0, they will conduct an equity carve-out to raise the amount *I* for the new investment in an IPO at time 1.<sup>13</sup> The new firm will then solely consist of project B. Therefore, it will issue equity claims against cash flows from project B only. The remaining fraction of outstanding shares of the carved-out firm will be held by the parent firm. Thus, the insiders and current shareholders of the existing parent firm will continue to hold equity in the new firm indirectly through their equity holdings in the parent firm. The employee-entrepreneur will be offered an incentive compatible compensation contract using the new firm's equity to guarantee his effort provision for project B. The total number of shares held by the parent firm's current shareholders in the new firm before the carve-out is normalized to one.

In the case of non-integration, since the carved-out firm runs project B only, its market value is determined purely by the marginal outside investor's belief about project B at time 1. If the equity of the new firm is issued at the offer price of  $P_b$  per share to raise an amount of I for project B at time 1, all outside investors whose share valuation is higher than the offer price  $P_b$  will participate in the IPO. The total market value  $V_b$  of the new firm at time 1 will then be equal to the expected valuation of the firm's time-2 cash flows by the marginal IPO investor based on his belief  $\overline{\theta}_b$  about project B:

$$V_b = V(\overline{\theta}_b), \tag{5}$$

where the marginal IPO investor's belief  $\overline{\theta}_b$  is implicitly given by

$$\int_{\overline{\theta}_b}^{\theta_b^m + d_b} \frac{W}{2d_b} d\theta = I.$$
 (6)

(footnote continued)

of outsiders with respect to the firm's new project. In this case, the amount raised by the firm may exceed I, and will be the amount that maximizes the firm insiders' surplus conditional on their own beliefs. The optimal amount raised will then depend on the following trade-off: as the firm sells more shares, insiders are able to capture value from a larger number of outsiders by selling them a larger number of shares at an overvalued price, but the price per share falls, since the belief of the marginal outside investor, which determines the price at which these shares are sold, will be less optimistic. However, given that the focus of this paper is not on the determination of the optimal amount of equity raised by the firm, but on the optimal choice between integration and non-integration, we assume here that the firm raises only the minimum amount required, I, to fund the firm's project due to considerations of corporate control or other reasons we do not model here. Modeling the optimal amount of external financing raised complicates our model considerably without changing the qualitative nature of our results.

<sup>&</sup>lt;sup>11</sup> We assume that the only compensation provided to the employee is equity and that he has limited liability. However, relaxing this assumption and adding a fixed wage component as well will not change our results qualitatively.

 $<sup>^{12}</sup>$  Recall that firm insiders hold a fraction  $\gamma$  of shares outstanding.

<sup>&</sup>lt;sup>13</sup> When outsiders' valuation of the new project is greater than that of firm insiders, it may be beneficial for the latter to sell equity that raises an amount larger than *I* to take advantage of the optimistic beliefs

Thus, Eq. (6) shows that the marginal outside investor is determined by starting with the most optimistic outside investor willing to invest in the firm (whose belief is given by  $(\theta_b^m + d)$ ) and working down the ladder of outside investors' beliefs until the entire amount I is raised by selling equity. Solving (6) for the marginal outside investor's belief  $\overline{\theta}_b$ , we obtain

$$\overline{\theta}_b = \theta_b^m + d_b \left( 1 - \frac{2I}{W} \right). \tag{7}$$

If the firm issues  $E_e$  new shares at the offer price  $P_b$  per share to raise an amount I in the equity carve-out of project B, the size of the equity issue, I, must be equal to the market value of the new shares offered to outside investors in the equity carve-out: i.e.,  $I = P_b \times E_e$ . The market share price  $P_b$  is equal to the total market value of the firm  $V_b$  divided by the number of shares outstanding  $(1 + E_e)$  after the equity carve-out:  $P_b = V_b/(1 + E_e) = V(\overline{\theta}_b)/(1 + E_e)$ .

The market value  $V_{parent}$  of the existing parent firm is determined by the parent firm's marginal investor's valuation of project A and his valuation of the parent firm's equity holdings in the newly carved-out firm. <sup>14</sup> We assume that the parent firm has already raised funding for project A in its history from its current shareholders who are most optimistic about project A, and it has exhausted the wealth of all such current shareholders (for whom the belief  $\theta_a$  about the success probability of project A is greater than  $(\theta_a^m + d_a)$ ). Thus, in the case of non-integration, the marginal investor of the parent firm has the belief  $(\theta_a^m + d_a)$  about project A. Moreover, we assume that the parent firm issues no new equity claims against the cash flows from project A to fund project B.

Since the firm insiders want to induce the employeeentrepreneur's effort, *e*=1, for project B, the employeeentrepreneur's optimal equity compensation scheme is given by the solution to the following problem<sup>15</sup>:

$$\begin{aligned} & \min_{\alpha} & \alpha V(\theta_b^f) \\ & \text{s.t.} & \alpha V(\theta_b^f) \geq \Psi, \end{aligned} \tag{IR1}$$

$$\alpha V(\theta_b^f) - \Psi \ge \alpha X_b^L,$$
 (IC1)

where  $\alpha$  is the fraction of the new firm's equity held by the employee-entrepreneur. The employee-entrepreneur's individual rationality constraint is given in (IR1), and his incentive compatibility constraint is given in (IC1). We denote the number of shares of equity offered to the entrepreneur as  $E_e$ '. Since the total number of shares outstanding after the IPO is  $(1+E_e)$ , the fraction of equity held by the employee-entrepreneur is given by<sup>16</sup>:  $\alpha = E_e'/(1+E_e)$ . Thus, the fraction of

equity held by the parent firm in the carved-out firm is equal to  $\beta = (1-E_e)/(1+E_e)$ .

**Proposition 1** (Equity issue under non-integration). If the firm chooses to implement the new project B outside the firm and raise an amount I for investment in the new project through an IPO. it has to issue a total of

$$E_e = \frac{I}{V(\overline{\theta}_h) - I} \tag{8}$$

new shares to outside investors at time1, where  $\overline{\theta}_b = \theta_b^m + d_b(1-2I/W)$  represents the belief of the marginal investor financing the investment I required for the new project. The market value  $V_b$  of the carved-out firm will be equal to  $V(\overline{\theta}_b)$ .

The number of shares of the IPO firm that is offered to the employee-entrepreneur in exchange for his effort provision for project B is

$$E_{e'} = \frac{\Psi}{\theta_b^f(X_b^H - X_b^I)} \frac{V(\overline{\theta}_b)}{V(\overline{\theta}_b) - I}.$$
 (9)

The employee-entrepreneur's fraction of equity in the new firm is  $\alpha = \Psi/(\theta_b^f(X_b^H - X_b^L))$ , and he extracts a surplus of  $\Psi X_b^L/(\theta_b^f(X_b^H - X_b^L))$ . The fraction of equity held by the parent firm in the carved-out firm is given by

$$\beta = 1 - \frac{I}{V(\overline{\theta}_b)} - \frac{\Psi}{\theta_b^f(X_b^H - X_b^L)}.$$
 (10)

In the case of non-integration, Proposition 1 shows that the number of shares,  $E_e$ , and the fraction of equity issued to outside investors,  $I/V(\overline{\theta}_b)$ , decrease with outside investors' average belief  $\theta_b^m$  about project B, and the dispersion in their beliefs  $d_b$  about project B. This is due to the fact that the marginal investor's belief  $\overline{\theta}_b$  about project B is increasing in these two parameters for a given level of required investment I. In other words, the cost of raising capital to finance the innovation decreases with the marginal investor's optimism about project B.

The fraction of equity given to the employee-entrepreneur,  $\alpha = \Psi/(\theta_b^f(X_b^H - X_b^I))$ , compensates his effort cost  $\Psi$  and ensures that he does not shirk. It is increasing in the employee-entrepreneur's effort  $\cot \Psi$  and decreasing in his marginal productivity of effort  $\theta_b^f(X_b^H - X_b^I)$ . The employee-entrepreneur's equity compensation does not depend on the marginal investor's beliefs about project B, since the employee-entrepreneur and the firm insiders have the same belief  $\theta_b^f$  about project B. Due to the limited liability of equity and the fact that the employee-entrepreneur will benefit from the low state cash flow  $X_b^L$  even if he does not exert effort, he earns an expected surplus of  $\Psi X_b^L/(\theta_b^f(X_b^H - X_b^I))$  in excess of his effort cost.

Finally, the fraction of equity  $\beta$  retained by the parent firm in the new carved-out firm is given by Eq. (10). Notice that the higher the fraction of equity  $I/V(\overline{\theta}_b)$  issued to outsiders to raise funding I for the new project and the

<sup>&</sup>lt;sup>14</sup> See Section 4 for a detailed analysis of the parent firm's market value in the context of negative stub values in equity carve-outs.

 $<sup>^{15}</sup>$  If the employee-entrepreneur does not exert effort for project B, the cash flow from project B will be  $X_b^L$  with probability 1, which is less than the required investment amount *I*. Clearly, in this case, the project would not be worth implementing.

<sup>&</sup>lt;sup>16</sup> We assume for simplicity that the firm's current shareholders can compensate the employee-entrepreneur from their own outstanding equity holdings, and therefore, that the firm does not need to issue new equity for the employee-entrepreneur. This allows us to separate valuation effects of heterogeneous beliefs from incentive effects. Thus,

<sup>(</sup>footnote continued)

the parameters are such that  $E_e^{\prime} < 1$ . If we allow for the issuance of new equity for the employee-entrepreneur, all results remain qualitatively similar. The proof of this claim is available to interested readers upon request.

fraction of equity  $\alpha = \Psi/(\theta_b^f(X_b^H - X_b^I))$  used to motivate the employee-entrepreneur, the lower the fraction of equity  $\beta$ , i.e., the higher the dilution in the ownership of the parent firm in the carved-out firm.

#### 2.2. Integration

Firm insiders can choose to implement project B within their existing organization, that is, project B can be integrated into the same organizational structure as project A (integration). If the firm decides to develop project B internally at time 0, it will have to issue new equity to outside investors at time 1 to raise the amount I. Even though the money is raised to finance project B, new shareholders will have equity claims against cash flows from both the existing project A and the new project B. The employee-entrepreneur is also offered a fraction  $\alpha_c$  of the combined firm's equity. The fraction of equity given to the employee-entrepreneur will compensate him for his effort costs  $\Psi$  for project B. and induce him to exert a high level of effort (e = 1). The total number of shares held by current shareholders before equity issuance is normalized to one. The total market value of equity of the combined firm at time 1 will be equal to the valuation of the marginal outside investor financing the combined firm. If insiders choose to develop the new project B internally, total cash flows at time 2 will increase by s, since there are synergies to be realized by integrating project B with project A.<sup>17</sup> These synergies are realized if and only if the employee-entrepreneur exerts effort.

The valuation of the integrated firm and the identity of the marginal investor, by whose beliefs the market value of the combined firm is determined in the case of integration, crucially depend on the correlation in outsiders' beliefs,  $\theta_a$  and  $\theta_b$ , about the success probabilities of project A and project B, respectively. In this section, we first obtain (Section 2.2.1) closed-form solutions for the beliefs of the marginal investor of the integrated firm for any value of the ex ante correlation  $\rho$  in outsiders' beliefs about projects A and B, where  $\rho \in [-1,+1]$ . We then characterize the value of equity to be issued under integration to finance the new project (Section 2.2.2).

#### 2.2.1. The marginal investor in the case of integration

As assumed above, outside investors already know their beliefs about the existing project A at time 0, but they learn about their beliefs about the new project B only at the time of equity issuance (time 1). Further, at time 0, they know that their beliefs about project B at time 1 will be either perfectly positively correlated with their beliefs about project A, with probability  $(1+\rho)/2$ , or perfectly negatively correlated with them, with probability  $(1-\rho)/2$ , where the parameter  $\rho$  can take any value in the closed interval [-1,+1]. The case where  $\rho=-1$  corresponds to the extreme case of perfectly negative ex ante correlation, and

the case where  $\rho=+1$  corresponds to the other extreme case of perfectly positive ex ante correlation. As  $\rho$  continuously increases from -1 to +1, the ex ante correlation in investors' beliefs about the two projects increases, and at time 0, insiders can calculate the expected beliefs of the marginal investor about project A and project B as a function of the correlation parameter  $\rho$ , and therefore the expected market value of their firm in the case of integration.

The following lemma shows that the determination of the marginal investor of the integrated firm at the time of the equity issue (time 1) will be based on the firm insiders' objective to sell the combined firm's equity to outside investors at the highest possible market price, thereby minimizing the dilution in their ownership of the firm. The firm will raise the required financing *I* from those investors who are willing to pay the most for the combination of projects A and B at time 1.

**Lemma 1.** Suppose that the firm chooses to develop the new project B inside the firm and raise an amount I for investment in the new project against the cash flows of the combined firm. If the degree of ex ante correlation between investors' beliefs about projects A and B is equal to  $\rho$ , where  $\rho \in [-1, +1]$ , the time-0 expected values of the marginal investor's time-1 beliefs about project A and project B are given by

(i) If 
$$d_a(X_a^H - X_a^L) \le d_b(X_b^H - X_b^L)$$
, then
$$\mathbb{E}[\hat{\theta}_a] = \theta_a^m + \rho d_a \left(1 - \frac{2I}{W}\right), \tag{11}$$

$$E[\hat{\theta}_b] = \theta_b^m + d_b \left( 1 - \frac{2I}{W} \right). \tag{12}$$

(ii) Otherwise, if 
$$d_a(X_a^H - X_a^L) > d_b(X_b^H - X_b^L)$$
, then 
$$E[\hat{\theta}_a] = \theta_a^m + d_a \left(1 - \frac{2I}{W}\right), \tag{13}$$

$$E[\hat{\theta}_b] = \theta_b^m + \rho d_b \left( 1 - \frac{2I}{W} \right). \tag{14}$$

At the time of the equity issue, there are two different cases in which the marginal outside investor of the combined firm is determined. Suppose first that, at time 1, outside investors' beliefs about project B are perfectly positively correlated with their beliefs about project A. Then, outside investors who are most optimistic about the total firm value are the same investors who are most optimistic about project A or project B as separate, stand-alone entities. In other words, each individual outside investor has the same belief ranking for project A and project B, and therefore for the combined firm (A+B). Since the firm insiders' objective at time 1 is to maximize the share price at which they sell equity to outsiders, the marginal outside investor is determined by starting with the most optimistic outside investor willing to invest in the firm (whose belief about project B is given by  $(\theta_b^m + d)$  and working down the ladder of outside investors' beliefs until the entire amount I is raised by selling equity. Thus,  $\theta_b$  is implicitly given by the following equation:

$$\int_{\hat{\theta}_b}^{\theta_b^m + d_b} \frac{W}{2d_b} d\theta = I. \tag{15}$$

<sup>&</sup>lt;sup>17</sup> Clearly, there may be heterogeneity in investor beliefs about the magnitude of the synergy between projects A and B as well. We choose not to incorporate such heterogeneity into our model, since doing so complicates our analysis considerably without generating commensurate insights.

Solving for the belief  $\hat{\theta}_b$  (about project B) of the marginal investor of the combined firm at time 1, we obtain

$$\hat{\theta}_b = \theta_b^m + d_b \left( 1 - \frac{2I}{W} \right). \tag{16}$$

Then, from (2), it follows that this same investor has the belief  $\hat{\theta}_a = \theta_a^m + d_a(1-2I/W)$  about project  $A^{18}$ 

The second case is where, at time 1, outside investors' beliefs about project B are perfectly negatively correlated with their beliefs about project A. Then, outside investors who are most optimistic about project B will be the investors who are most pessimistic about project A, and vice versa. For any investor, the mapping between his beliefs about project A and project B is uniquely given by (3). In this case, the above lemma shows that, at time 1, the investor who is most optimistic about the combined firm (A+B) is also the investor who is most optimistic about project B if the following condition holds:

$$d_a(X_a^H - X_a^L) \le d_b(X_b^H - X_b^L). \tag{17}$$

In particular, consider any investor with belief  $\theta_b \in (\theta_b^m - d_b, \theta_b^m + d_b)$  about project B at time 1. From (3), it follows that his belief about project A is equal to  $\theta_a^m - (d_a/d_b)(\theta_b - \theta_b^m)$ . Then, consider also the investor with belief  $(\theta_b + \varepsilon)$  about project B, where  $\varepsilon$  is a very small real positive number. By (3), his belief about project A is equal to

$$\theta_a^m - \frac{d_a}{d_b}(\theta_b + \varepsilon - \theta_b^m) = \theta_a^m - \frac{d_a}{d_b}(\theta_b - \theta_b^m) - \frac{d_a}{d_b}\varepsilon.$$

Then, the total firm value imputed by the investor with belief  $\theta_b$  will be less than the total firm value imputed by the investor with belief  $(\theta_b + \varepsilon)$ :

$$V(\theta_{b}) + V\left(\theta_{a}^{m} - \frac{d_{a}}{d_{b}}(\theta_{b} - \theta_{b}^{m})\right)$$

$$\leq V(\theta_{b} + \varepsilon) + V\left(\theta_{a}^{m} - \frac{d_{a}}{d_{b}}(\theta_{b} - \theta_{b}^{m}) - \frac{d_{a}}{d_{b}}\varepsilon\right), \tag{18}$$

$$\varepsilon \frac{d_a}{d_b} (X_a^H - X_a^L) \le \varepsilon (X_b^H - X_b^L), \tag{19}$$

if and only if the inequality in (17) is satisfied. By induction, this implies that if (17) holds, the most optimistic investor about the combined firm (A+B) is also the same investor who is most optimistic about project B, since the total firm value (value of A+B) is monotonically increasing in  $\theta_b$ . By the same token, the total firm value is monotonically decreasing in  $\theta_a$  if (17) holds. Therefore, it is optimal for firm insiders to raise the required financing from those investors who are more optimistic about project B and integrate over the ladder of outsider beliefs as in (16). Thus, the firm will start raising money from the outside investor with belief  $(\theta_b^m + d_b)$ , who is most optimistic about project B, and go down the ladder until the total amount of investment capital I is raised. Then, it follows that the marginal investor's time-1 beliefs  $\hat{\theta}_a$  and

 $\hat{\boldsymbol{\theta}}_b$  about projects A and B, respectively, will be given by

$$\hat{\theta}_a = \hat{\theta}_a^l \equiv \theta_a^m - d_a \left( 1 - \frac{2I}{W} \right), \quad \hat{\theta}_b = \hat{\theta}_b^h \equiv \theta_b^m + d_b \left( 1 - \frac{2I}{W} \right), \quad (20)$$

if (17) holds. Conversely, if (17) does not hold so that  $d_a(X_a^H - X_a^I) > d_b(X_b^H - X_b^I)$ , the combined firm will find it optimal to raise the required funding I from those investors who are most optimistic about project A, since the outside investor who is most optimistic about the combined firm (A+B) will also be the investor who is most optimistic about project A. Then, the marginal investor's time-1 beliefs  $\hat{\theta}_a$  and  $\hat{\theta}_b$  about projects A and B, respectively, will be given by

$$\hat{\theta}_a = \hat{\theta}_a^h \equiv \theta_a^m + d_a \left( 1 - \frac{2I}{W} \right), \quad \hat{\theta}_b = \hat{\theta}_b^l \equiv \theta_b^m - d_b \left( 1 - \frac{2I}{W} \right). \tag{21}$$

One should note that the condition in (17) is more likely

to hold if investor beliefs about the new project B are more dispersed than investor beliefs about project A. and/or the payoff spread of project B is larger than that of project A. 19 The higher the dispersion in outsiders' beliefs about a particular project, the larger is the pool of outside investors who have extremely optimistic beliefs about that project. Therefore, if (17) holds, the integrated firm will be better off by raising the required investment I by starting from those investors who are most optimistic about project B and going down their belief ladder. In this case, the above discussion shows that the marginal investor financing the integrated firm at time 1 will have the belief  $\hat{\theta}_b = \hat{\theta}_b^h$  about project B with probability 1, regardless of whether the correlation in time-1 beliefs about projects A and B is perfectly positive or perfectly negative. Thus, if the firm chooses integration at time 0, all agents in the economy will rationally expect that the marginal investor's belief about project A will be either

optimistic  $(\hat{\theta}_a = \hat{\theta}_a^h)$  with probability  $(1+\rho)/2$ , or pessimistic  $(\hat{\theta}_a = \hat{\theta}_a^l)$  with probability  $(1-\rho)/2$ . Then, it follows that, if (17) holds, the expected time-0 value of the marginal investor's time-1 belief about project A will be given by

$$\begin{split} \mathbf{E}[\hat{\theta}_{a}] &= \frac{(1+\rho)}{2} \left( \theta_{a}^{m} + d_{a} \left( 1 - \frac{2I}{W} \right) \right) + \frac{(1-\rho)}{2} \left( \theta_{a}^{m} - d_{a} \left( 1 - \frac{2I}{W} \right) \right) \\ &= \theta_{a}^{m} + \rho d_{a} \left( 1 - \frac{2I}{W} \right). \end{split} \tag{22}$$

Conversely, the following condition is more likely to hold if the dispersion in outside investors' beliefs,  $d_a$ , about project A and/or its payoff spread,  $(X_a^H - X_a^L)$ , are higher than those of project B:

$$d_a(X_a^H - X_a^L) > d_b(X_b^H - X_b^L).$$
 (23)

In this case, at time 1, it will be optimal for the combined firm to issue new equity worth *I* to those outside investors who are most optimistic about project A and going down

<sup>&</sup>lt;sup>18</sup> If outside investors' beliefs about project B are perfectly positively correlated with their beliefs about project A at time 1, the same results can also be obtained by integrating over the beliefs of investors about project A, since, in this case, each individual outside investor would have the same belief ranking for project A and project B.

<sup>&</sup>lt;sup>19</sup> This will be the case if, for project B, the dispersion in investor beliefs,  $d_b$ , and/or the payoff spread between the high state and low state,  $(X_b^H - X_b^L)$ , are relatively high compared to those of the existing project A.

their belief ladder until the entire amount I is raised. Thus, in this case, the marginal investor of the combined firm will be very optimistic about project A at time 1, so that he will have the belief  $\hat{\theta}_a = \hat{\theta}_a^h$  about project A with probability 1. On the contrary, he will be either pessimistic about project B with probability  $(1-\rho)/2$ , or optimistic about it with probability  $(1+\rho)/2$ . Therefore, it follows that the expected time-0 value of his time-1 belief about project B will be given by

$$\begin{split} \mathbf{E}[\hat{\theta}_b] &= \frac{(1+\rho)}{2} \left(\theta_b^m + d_b \left(1 - \frac{2I}{W}\right)\right) + \frac{(1-\rho)}{2} \left(\theta_b^m - d_b \left(1 - \frac{2I}{W}\right)\right) \\ &= \theta_b^m + \rho d_b \left(1 - \frac{2I}{W}\right). \end{split} \tag{24}$$

#### 2.2.2. The equity value in the case of integration

Since the identity of the marginal investor financing the integrated firm and his beliefs  $\hat{\theta}_a$  and  $\hat{\theta}_b$  about projects A and B, respectively, depend on the correlation in outsiders' beliefs about projects A and B at time 1, the valuation of the combined firm's equity and therefore the cost of raising capital for the new project will also be substantially affected by this correlation in outside investors' beliefs.

If the equity of the combined firm is offered at the price of  $P_{a+b}$  per share when the firm issues new equity to finance project B at time 1, all outside investors whose valuation is higher than  $P_{a+b}$  will participate in the new equity issue. The total market value  $V_{a+b}$  of the combined firm will be equal to the valuation of projects A and B and their synergy by the marginal investor financing the new issue:

$$V_{a+b} = V(\hat{\theta}_a) + V(\hat{\theta}_b) + s, \tag{25}$$

where  $\hat{\theta}_a$  and  $\hat{\theta}_b$  are the beliefs of the marginal investor about project A and project B, respectively.<sup>20</sup> If the firm issues E new shares at the offer price  $P_{a+b}$  to raise an amount of I, the size of the equity issue, I, must be equal to the market value of the new shares offered to outside investors: i.e.,  $I = P_{a+b} \times E$ . The market share price  $P_{a+b}$  is equal to the total market value of the firm  $V_{a+b}$  divided by the number of shares outstanding (1 + E) after the equity issue:  $P_{a+b} = V_{a+b}/(1+E) = (V(\hat{\theta}_a) + V(\hat{\theta}_b) + s)/(1+E)$ .

Project B has a positive success probability and has synergies with project A only if the employee-entrepreneur exerts effort (e=1). The employee-entrepreneur's optimal equity compensation scheme is given by the solution to the following problem:

$$\min_{\alpha_c} \quad \alpha_c(V(\theta_a^f) + V(\theta_b^f) + s) 
s.t. \quad \alpha_c(V(\theta_a^f) + V(\theta_b^f) + s) \ge \Psi,$$
(IR2)

$$\alpha_{c}(V(\theta_{a}^{f}) + V(\theta_{b}^{f}) + s) - \Psi \ge \alpha_{c}(V(\theta_{a}^{f}) + X_{b}^{L}),$$
 (IC2)

where  $\alpha_c$  is the fraction of the combined firm's equity held by the employee-entrepreneur. The employee-entrepreneur's participation constraint is given in (IR2), and his incentive

compatibility constraint is given in (IC2). The number of shares of equity offered to the entrepreneur is denoted by E'. Since the total number of shares outstanding after the IPO is (1+E), the fraction of combined firm's equity held by the employee-entrepreneur is given by  $\alpha_c = E'/(1+E)$ . The fraction of equity retained by the firm's current shareholders after the equity issue is  $\beta_c = (1-E')/(1+E)$ .

**Proposition 2** (Equity issue under integration). If the firm chooses to develop the new project B inside the firm and raise an amount I for investment in the new project, it has to issue a total of

$$E = \frac{I}{V(\hat{\theta}_a) + V(\hat{\theta}_b) + s - I} \tag{26}$$

new shares to outside investors at time1, where  $\hat{\theta}_a$  and  $\hat{\theta}_b$  represent the beliefs of the marginal outside investor financing the integrated firm about the success probabilities of projects A and B, respectively. The market value of the combined firm at time1 is  $V_{a+b} = V(\hat{\theta}_a) + V(\hat{\theta}_b) + s$ . Ex ante, the expected market value  $E[V_{a+b}]$  of the combined firm is increasing in the degree of correlation  $\rho$ .

The number of shares that is offered to the employeeentrepreneur in exchange for his effort provision for project B is

$$E' = \left[ \frac{\Psi}{\theta_b^f(X_b^H - X_b^L) + s} \right] \left[ \frac{V(\hat{\theta}_a) + V(\hat{\theta}_b) + s}{V(\hat{\theta}_a) + V(\hat{\theta}_b) + s - I} \right]. \tag{27}$$

The employee-entrepreneur's fraction of equity in the combined firm is  $\alpha_c = \Psi/(\theta_b^f(X_b^H - X_b^L) + s)$ , and he extracts a surplus of  $\Psi(V(\theta_a^f) + X_b^L)/(\theta_b^f(X_b^H - X_b^L) + s)$ . The fraction of equity retained by the firm's current shareholders is given by

$$\beta_{c} = 1 - \frac{I}{V(\hat{\theta}_{a}) + V(\hat{\theta}_{b}) + s} - \frac{\Psi}{\theta_{b}^{f}(X_{b}^{H} - X_{b}^{L}) + s}.$$
 (28)

Proposition 2 shows that the number of shares, E, and the fraction of equity issued to outside investors at time 1,  $I/(V(\hat{\theta}_a)+V(\hat{\theta}_b)+s)$ , depend on the marginal outside investor's beliefs about both project A and project B.<sup>21</sup> The market value of the firm,  $V_{a+b}=V(\hat{\theta}_a)+V(\hat{\theta}_b)+s$ , is maximized when this investor is optimistic about both projects so that the cost of raising the external capital I for investment is minimized. If, however, the marginal outside investor has divergent opinions about the two projects at time 1, the fraction of equity that needs to be issued to outsiders to raise the amount I will be higher.

It is useful to consider the differences in equity valuation in the two extreme cases of correlation in outsiders' beliefs about projects A and B at time 1. Suppose (without loss of

 $<sup>^{20}</sup>$  We have already shown how the marginal investor's beliefs are determined in Lemma 1, so we take them as given here.

<sup>&</sup>lt;sup>21</sup> Clearly, a wealth constraint will prevent current (time-0) share-holders from buying any additional equity in the firm at time 1, under either integration or non-integration. We also assume that current shareholders are affiliated with firm insiders, and thus prevented from selling into the equity issue (e.g., through lock-up provisions). However, it should be noted that, even if there is a limited amount of selling into the equity issue by current shareholders, the qualitative nature of our results do not change, as long as such selling by current shareholders does not constitute a significant fraction of the equity issue. Introducing such selling only introduces additional complexity into our model without generating commensurate insights.

generality) that (17) holds. When we compare the market value of the integrated firm, when outsiders' beliefs about the two projects are perfectly positively correlated at time 1, to the market value of the firm, when outsiders' beliefs about the two projects are perfectly negatively correlated at time 1, the difference is equal to<sup>22</sup>

$$\begin{split} &[V(\hat{\theta}_{a}^{h}) + V(\hat{\theta}_{b}^{h}) + s] - [V(\hat{\theta}_{a}^{l}) + V(\hat{\theta}_{b}^{h}) + s] \\ &= 2d_{a} \left(1 - \frac{2I}{W}\right) (X_{a}^{H} - X_{a}^{L}) > 0. \end{split} \tag{29}$$

This implies that, at time 1, outside investors will own a larger fraction of the combined firm's equity in exchange for financing the new project B when investors' beliefs about the two projects are perfectly negatively correlated than when those beliefs are perfectly positively correlated. In other words, the firm can issue new equity worth *I* at a higher share price if outside investors' beliefs about assets in place and the innovation are perfectly positively correlated. Consequently, in the case of perfectly negative correlation in outsiders' beliefs, the cost of raising capital for the new project will be higher for the combined firm. In contrast, Proposition 1 shows that if the firm chooses to carve out the new project as a separate entity through an IPO, the fraction of the new firm's equity issued to outside investors, and the market value of the new firm, are independent of the correlation in outside investors' beliefs about projects A and B at time 1.

One should also notice that the time-0 expected value of the marginal investor's time-1 belief about project A is increasing in the degree of correlation  $\rho$  in outsider beliefs about projects A and B. When the firm finances its new project using a new equity issue for the integrated firm at time 1, the expected valuation discount on the integrated firm by outside investors will increase as  $\rho$  decreases from +1 to -1. Therefore, the expected market value of the integrated firm at time 0 as a function of the degree of ex ante correlation  $\rho$  will be given by

$$E[V_{a+b}] = \begin{cases} V\left(\theta_{a}^{m} + \rho d_{a}\left(1 - \frac{2I}{W}\right)\right) + V(\hat{\theta}_{b}^{h}) + s : d_{a}(X_{a}^{H} - X_{a}^{L}) \\ \leq d_{b}(X_{b}^{H} - X_{b}^{L}), \\ V(\hat{\theta}_{a}^{h}) + V\left(\theta_{b}^{m} + \rho d_{b}\left(1 - \frac{2I}{W}\right)\right) + s : d_{a}(X_{a}^{H} - X_{a}^{L}) \\ > d_{b}(X_{b}^{H} - X_{b}^{L}), \end{cases}$$
(30)

which is increasing in  $\rho$ , the ex ante correlation in outsider beliefs about projects A and B. This result also implies that the time-0 expected value of the fraction of equity issued to outsiders to finance project B,  $\mathbb{E}[I/(V(\hat{\theta}_a)+V(\hat{\theta}_b)+s)]$ , is decreasing in  $\rho$ .

In the case of integration, if firm insiders use the combined firm's equity to motivate the employee-entrepreneur to exert effort for project B, they will also have to channel a fraction of cash flows from project A to him. Note that the employee-entrepreneur's expected surplus in excess of his effort costs,  $\Psi(V(\theta_a^f) + X_h^I)/(\theta_h^f(X_h^H - X_h^I) + s)$ ,

depends on the value  $V(\theta_a^f)$  of project A based on insiders' beliefs. When the employee-entrepreneur is compensated using the combined firm's equity, he will benefit from the cash flows of project A regardless of whether he exerts effort or not. The fraction of equity given to the employee-entrepreneur,  $\alpha_c$ , is increasing in the employee-entrepreneur's effort cost,  $\Psi$ , and decreasing in his marginal productivity of effort,  $\theta_b^f(X_b^H - X_b^I) + s$ .

Finally, Eq. (28) characterizes the fraction of equity  $\beta_c$  retained by the firm's current shareholders at time 1 in the case of integration. The higher the fraction of equity  $I/(V(\hat{\theta}_a)+V(\hat{\theta}_b)+s)$  issued to outsiders to raise the required investment I and the fraction of equity  $\alpha_c = \Psi/(\theta_b^f(X_b^H - X_b^L) + s)$  used to motivate the employee-entrepreneur, the lower the fraction of equity  $\beta_c$ , i.e., the higher the dilution in the equity ownership of the firm's current shareholders. Since synergy gains from integration increase the combined firm's valuation  $V_{a+b}$  by an amount s, they reduce the fraction of equity distributed to outsiders and the employee-entrepreneur (see the denominator terms above).

#### 3. Analysis of integration versus non-integration

In this section, we analyze and characterize the conditions under which one of the above organizational forms is preferred by firm insiders to manage and finance the firm's new project.

If firm insiders choose to implement project B inside the existing organization along with project A, we showed in Proposition 2 that they will issue E new shares to outside investors to raise the required investment I and distribute E' shares to the employee-entrepreneur to compensate his effort provision, where E and E' are given by (26) and (27), respectively. Thus, the fraction of firm equity retained by current shareholders at time 1 is  $\beta_c = (1-E')/(1+E)$  given in (28) explicitly. If (17) holds, Lemma 1 and Proposition 2 imply that the time-0 expected value  $\lambda$  of  $\beta_c$ , the fraction of firm equity that current shareholders will retain at time 1, is given by E'

$$\lambda = 1 - \frac{\Psi}{\theta_b^f(X_b^H - X_b^I) + s} - I \left[ \frac{\frac{(1+\rho)}{2}}{V(\hat{\theta}_a^h) + V(\hat{\theta}_b^h) + s} + \frac{\frac{(1-\rho)}{2}}{V(\hat{\theta}_a^l) + V(\hat{\theta}_b^h) + s} \right]. \tag{31}$$

Then, the time-0 expected payoff of current shareholders from choosing integration, based on insiders' beliefs  $\theta_a^f$  and  $\theta_b^f$ , is given by

$$EU_{a+b} = \lambda (V(\theta_a^f) + V(\theta_b^f) + s)$$

$$= V(\theta_a^f) + V(\theta_b^f) + s - \Psi \left( 1 + \frac{V(\theta_a^f) + X_b^L}{\theta_a^f(X_a^H - X_b^L) + s} \right)$$
(32)

<sup>&</sup>lt;sup>22</sup> If the condition in (23) holds instead of that in (17), this difference is equal to  $2d_b(1-2I/W)(X_b^H-X_b^I) > 0$ .

 $<sup>^{23}</sup>$  Lemma 1 implies that, if the parameter condition in (17) holds, the marginal outside investor will be optimistic about project B with probability 1 at time 1, whereas he will be optimistic about project A only with probability (1+ $\rho$ )/2. However, if (17) does not hold, so that (23) holds, the marginal outside investor will be more optimistic about project A than about project B. In this case, the third term in the expression for  $\lambda$  is slightly different, and the equation for  $\lambda$  is (A.16).

$$-I[V(\theta_a^f) + V(\theta_b^f) + s] \left( \frac{\frac{(1+\rho)}{2}}{V(\hat{\theta}_a^h) + V(\hat{\theta}_b^h) + s} + \frac{\frac{(1-\rho)}{2}}{V(\hat{\theta}_a^l) + V(\hat{\theta}_b^h) + s} \right). \tag{33}$$

On the other hand, if firm insiders choose to carve out project B through an IPO, we showed in Proposition 1 that they will issue  $E_e$  new shares of the IPO firm to outside investors, and distribute  $E_{e'}$  shares of the IPO firm to the employee-entrepreneur, where  $E_e$  and  $E_{e'}$  are given by (8) and (9), respectively. The current shareholders of the parent firm will hold the entire equity of the parent firm, which will run project A alone. With probability 1, the fraction of equity they retain in the new firm with project B is equal to  $\beta = (1-E_{e'})/(1+E_e)$  given in (10) explicitly. Thus, the time-0 expected payoff of the parent firm's current shareholders from choosing non-integration (based on insiders' beliefs  $\theta_a^f$  and  $\theta_b^f$ ), is given by

$$\begin{split} EU_{a} + EU_{b} &= V(\theta_{a}^{f}) + \beta V(\theta_{b}^{f}) \\ &= V(\theta_{a}^{f}) + V(\theta_{b}^{f}) - I\left(\frac{V(\theta_{b}^{f})}{V(\overline{\theta}_{b})}\right) - \Psi\left(1 + \frac{X_{b}^{L}}{\theta_{b}^{f}(X_{b}^{H} - X_{b}^{L})}\right). \end{split}$$

Firm insiders will choose non-integration over integration if  $EU_a + EU_b \ge EU_{a+b}$ , and integration otherwise.

**Proposition 3** (Non-integration versus integration).

- (i) Let  $\Psi < (1-I/(V(\hat{\theta}_a^l)+V(\hat{\theta}_b^l)+s))(\theta_b^f(X_b^H-X_b^L)+s)$ . Firm insiders will choose to carve out the new project B rather than implement and fund it inside, if the following non-integration condition holds:
  - (a) If  $d_a(X_a^H X_a^L) \le d_b(X_b^H X_b^L)$ , the non-integration condition is given by:

$$EU_{a} + EU_{b} - EU_{a+b} = I\left(\frac{\frac{(1+\rho)}{2}[V(\theta_{a}^{f}) + V(\theta_{b}^{f}) + s]}{V(\hat{\theta}_{a}^{h}) + V(\hat{\theta}_{b}^{h}) + s}\right) + \frac{\frac{(1-\rho)}{2}[V(\theta_{a}^{f}) + V(\theta_{b}^{f}) + s]}{V(\hat{\theta}_{a}^{f}) + V(\hat{\theta}_{b}^{h}) + s} - \frac{V(\theta_{b}^{f})}{V(\overline{\theta}_{b})}\right) + \Psi\left(\frac{V(\theta_{a}^{f}) + X_{b}^{f}}{\theta_{b}^{f}(X_{b}^{H} - X_{b}^{f}) + s} - \frac{X_{b}^{f}}{\theta_{b}^{f}(X_{b}^{H} - X_{b}^{f})}\right) - s \ge 0. \quad (35)$$

(b) If, on the other hand,  $d_a(X_a^H - X_a^L) > d_b(X_b^H - X_b^L)$ , then the non-integration condition is given by

$$EU_{a} + EU_{b} - EU_{a+b} = I \left( \frac{\frac{(1+\rho)}{2} [V(\theta_{a}^{f}) + V(\theta_{b}^{f}) + s]}{V(\hat{\theta}_{a}^{h}) + V(\hat{\theta}_{b}^{h}) + s} + \frac{\frac{(1-\rho)}{2} [V(\theta_{a}^{f}) + V(\hat{\theta}_{b}^{f}) + s]}{V(\hat{\theta}_{a}^{h}) + V(\hat{\theta}_{b}^{f}) + s} - \frac{V(\theta_{b}^{f})}{V(\overline{\theta}_{b})} \right) + \Psi \left( \frac{V(\theta_{a}^{f}) + X_{b}^{f}}{\theta_{b}^{f}(X_{b}^{h} - X_{b}^{f}) + s} - \frac{X_{b}^{f}}{\theta_{b}^{f}(X_{b}^{h} - X_{b}^{f})} \right) - s \ge 0. \quad (36)$$

They choose integration if one of the non-integration conditions (35) and (36) above does not hold.

(ii) The non-integration condition above is more likely to be satisfied if (a) outside investors' average belief about the new project and the dispersion of beliefs about its cash flows are high, (b) insiders' belief about the new project is low, (c) insiders' belief about the assets in place is high, (d) the new project's potential synergies with assets in place are low, (e) the employee-entrepreneur's effort costs are high, (f) the relative size of the new project B with respect to the existing project A is small, and (g) the ex ante correlation in outsider beliefs about the cash flows of the new project and the assets in place is not highly positive.

If the marginal outside investor is much more optimistic than firm insiders about the cash flow prospects of the new project relative to those of the existing assets of the firm, current shareholders can capture the market's optimism about the new project B better if it is implemented in a new organization rather than within the existing firm along with project A. From the perspective of firm insiders, whose belief about the success probability of project B is  $\theta_b^f$ , the value of new equity issued to outside investors in an equity carve-out is less than the amount of funds raised  $I_s$ , so that we have

$$I\left(\frac{V(\theta_b^f)}{V(\overline{\theta}_b)}\right) < I,\tag{37}$$

when the belief of the marginal investor about project B is more optimistic than that of firm insiders: i.e.,  $\overline{\theta}_b = \hat{\theta}_b^h = \theta_b^m + d(1-2I/W) > \theta_b^f$ , and therefore,  $V(\overline{\theta}_b) > V(\theta_b^f)$ . Thus, based on insiders' beliefs, the firm can sell overvalued equity in an equity carve-out IPO, and capture substantial incremental value from outside investors' relative optimism.

Suppose that the condition given in (17) holds as in part (i(a)) of Proposition 3, so that the marginal investor of the combined firm is determined from the pool of outside investors who are most optimistic about project B, i.e.,  $\hat{\theta}_b = \hat{\theta}_b^h = \theta_b^m + d(1-2I/W)$ . Similar to the case of nonintegration, the new shares sold to outside investors can also be overvalued based on the insiders' belief  $\theta_b^f$  if the marginal investor of the combined firm is more optimistic than firm insiders about project B: i.e.,  $\hat{\theta}_b^h > \theta_b^f$ . In this case, the following condition will be satisfied, provided that the marginal investor is not much more pessimistic about the existing project A than firm insiders (i.e., if it is not the case that  $E[\hat{\theta}_a] \ll \theta_a^f$ ):

$$I\left(\frac{\frac{(1+\rho)}{2}[V(\theta_{a}^{f})+V(\theta_{b}^{f})+s]}{V(\hat{\theta}_{a}^{h})+V(\hat{\theta}_{b}^{h})+s}+\frac{\frac{(1-\rho)}{2}[V(\theta_{a}^{f})+V(\theta_{b}^{f})+s]}{V(\hat{\theta}_{a}^{l})+V(\hat{\theta}_{b}^{h})+s}\right) < I. \tag{38}$$

However, it is important to notice that the left-hand side (LHS) of the above inequality is decreasing in  $\rho$ , which is our measure of ex ante correlation in outsiders' beliefs about projects A and B. If  $\rho$  is close to -1, the marginal investor of the combined firm will be expected to be very pessimistic (relative to insiders) about the prospects of the firm's assets in place: i.e.,  $\mathbb{E}[\hat{\theta}_a] \ll \theta_a^f$ . In this case, the LHS of the above inequality, which is the

cost of raising I from the perspective of firm insiders, can even be greater than I (so that the inequality does not hold). This can be particularly significant if the firm's assets in place dominate the new project in terms of size.

If (17) holds, the marginal investor in the combined firm (in the case of integration) has the same time-1 belief about project B as the marginal investor in the carved-out firm (in the case of non-integration), i.e.,  $\hat{\theta}_b = \hat{\theta}_b^h = \overline{\theta}_b$ . Nevertheless, the expected cost of raising capital I can be higher in the combined firm than in the carved-out firm, even if the ex ante correlation in outsiders' beliefs about projects A and B is highly positive (e.g., when  $\rho$  is close to +1). If outside investors have only a slightly higher (or lower) average opinion about the existing project A than firm insiders, and if the dispersion in outsiders' beliefs about project A is low, the wedge between the subjective equity valuation of insiders and the valuation of the marginal outside investor will be lower in the case of integration than in the case of non-integration. Thus, if the marginal investor of the combined firm is much more optimistic about project B relative to firm insiders, and this incremental optimism of the marginal investor over firm insiders is greater for project B than for project A (i.e., the difference between  $(\hat{\theta}_b^h - \theta_b^f)$  and  $(\hat{\theta}_a^h - \theta_a^f)$  is positive), then the following inequality will hold:

$$I\left(\frac{\frac{(1+\rho)}{2}[V(\theta_{a}^{f})+V(\theta_{b}^{f})+s]}{V(\hat{\theta}_{a}^{h})+V(\hat{\theta}_{b}^{h})+s}+\frac{\frac{(1-\rho)}{2}[V(\theta_{a}^{f})+V(\theta_{b}^{f})+s]}{V(\hat{\theta}_{a}^{l})+V(\hat{\theta}_{b}^{h})+s}\right)$$

$$>I\left(\frac{V(\theta_{b}^{f})}{V(\overline{\theta}_{b})}\right). \tag{39}$$

In this case, the cost of raising capital I based on firm insiders' belief will be lower if they issue equity in the newly carved-out IPO firm than in the combined firm. Therefore, the first term on the LHS of (35) will be positive, and this will tilt the decision of the firm insiders towards non-integration. As the incremental optimism of the marginal investor (relative to firm insiders) about the innovation becomes larger compared to the marginal investor's incremental optimism about the firm's assets in place, the cost of capital advantage of non-integration will be higher. Note also that if firm insiders have very optimistic beliefs about project A ( $\theta^f_a$  is high), and they simultaneously have very pessimistic beliefs about project B ( $\theta^f_b$  is low), the first term on the LHS of (35) will be likely to be positive, making insiders prefer non-integration to integration.

In the case of integration, the expected market valuation  $\mathrm{E}[V_{a+b}]$  of the combined firm is increasing in the ex ante correlation  $\rho$  in outsiders' beliefs about projects A and B. If  $\rho$  is not highly positive, insiders will expect that outsiders will apply a significant discount to their valuation of the combined firm, even if they are very optimistic about the new project itself. Thus, as  $\rho$  decreases, the marginal outside investor in the combined

firm is expected to become less optimistic about the firm's assets in place, so that the inequality (39) will be more likely to be satisfied, making firm insiders favor non-integration over integration.

If (17) does not hold, so that (23) holds as in part (i(b)) of Proposition 3, our results on the choice of organizational form are very similar, except that the marginal investor in the combined firm is determined from the pool of outside investors who are most optimistic about project A rather than project B. In this case, if the correlation in outsider beliefs about projects A and B is very negative at time 1, the marginal investor in the combined firm will have a very pessimistic opinion about the prospects of project B. In general, as the ex ante correlation  $\rho$  in outsider beliefs about projects A and B decreases from +1 to -1, the combined firm's expected cost of raising the investment capital I increases as the marginal investor in the combined firm is expected to become much less optimistic about project B than the marginal investor in the carved-out firm. Thus, if  $\rho$ is sufficiently low, the first term on the LHS of (36) will be positive. Hence, to capture outside investors' optimism about project B and issue new equity at a higher price, firm insiders will prefer non-integration to integration. In addition, if firm insiders have very optimistic beliefs about project A ( $\theta_a^f$  is high), and they simultaneously have very pessimistic beliefs about project  $B_h(\theta_b^f)$  is low), so that the difference between  $(\hat{\theta}_b^h - \theta_b^f)$  and  $(\hat{\theta}_a^h - \theta_b^f)$  is higher, the first term on the LHS of (36) is more likely to be positive, making firm insiders prefer non-integration to integration.

So far, we have been discussing cases under which outside investors' beliefs favor non-integration. However, there may be some cases where outsider beliefs favor integration. One such scenario is the case where outsiders are more pessimistic than insiders about the future prospects of the new project B, but they are more optimistic than insiders about the firm's assets in place (project A). Then, if the firm does an equity carve-out and sells equity in project B, this equity will be undervalued with respect to firm insiders' valuation of the new firm's equity. Consequently, it will be optimal for insiders to sell equity in the combined firm since this equity reflects outsiders' valuation of both projects A and B, so that this combined firm equity will be overvalued with respect to insider beliefs. Under these conditions, the parent firm will choose to implement the new project under integration. Another scenario is the case where the ex ante correlation  $\rho$  between outsider beliefs about the two projects is positive, and outsiders are more optimistic than firm insiders about both assets in place and the new project, but they are, on average, more optimistic about the firm's assets in place than about the new project. In this case, the incremental optimism of the marginal investor in the combined firm's equity (relative to the beliefs of firm insiders) in the case of integration will be greater than the incremental optimism of the marginal investor in the equity of the carved-out firm in the case of non-integration. The firm will therefore again choose to implement the new project under integration.

Another important economic factor that drives the choice of integration versus non-integration is the employee-entrepreneur's incentives. Firm insiders can induce the employee-entrepreneur to exert a high level

 $<sup>^{24}</sup>$  This inequality can be satisfied even if the combined firm can expect to raise equity at the highest possible share price when  $\rho=+1.$  That would be the case if the difference  $(\hat{\theta}_b^{\hbar}-\theta_b^f)$  is significantly larger than the difference  $(\hat{\theta}_b^{\hbar}-\theta_a^f)$ .

of effort for the new project by offering an incentive compatible compensation scheme, paying him with the equity of the firm he works for. The employee-entrepreneur's surplus (i.e., the value of his equity holdings is in excess of his effort cost  $\Psi$ ) in the case of integration will be greater than his surplus in the case of non-integration as long as the value  $V(\theta_a^f)$  of the firm's assets in place (based on insiders' belief) exceeds the synergy value s between projects A and B. In other words

$$\Psi \frac{V(\theta_{a}^{f}) + X_{b}^{L}}{\theta_{b}^{f}(X_{b}^{H} - X_{b}^{L}) + s} > \Psi \frac{X_{b}^{L}}{\theta_{b}^{f}(X_{b}^{H} - X_{b}^{L})}, \tag{40}$$

if  $V(\theta_a^f) > s$ . To motivate the employee-entrepreneur to exert high effort for project B, the firm has to promise him a certain fraction of the future cash flows from project B to compensate his marginal effort cost  $\Psi$ . However, under integration, promising  $\alpha_c$  of the future cash flows from project B also entails promising  $\alpha_c$  of the future cash flows from project A, since the firm is paying the employee-entrepreneur with the combined firm's equity. Therefore, it will prove more costly to the firm's current shareholders to incentivize the employee-entrepreneur within the existing organization than in a new firm consisting of project B only. This extra compensation cost is increasing with the employee-entrepreneur's effort cost  $\Psi$ , and the size of the parent firm's assets in place (project A). In summary, this second economic factor always favors non-integration.

The final economic factor determining a firm's choice between integration and non-integration is the synergy between projects A and B. This synergy will be eliminated if the firm chooses to implement the new project under non-integration. Consequently, this third economic factor always favors integration. If these potential synergy benefits, s, from integrating the innovation with the existing assets of the firm are sufficiently large, firm insiders may still prefer to implement and fund the new project under integration, even though the cost of raising new capital and the cost of compensating the employee-entrepreneur is higher in the case of integration due to the economic factors mentioned before.

The threshold value of the ex ante correlation  $\rho^*$  in outsider beliefs about projects A and B, at which firm insiders are indifferent between integration and non-integration, is determined by the following indifference equation obtained from (35) in Proposition 3 when (17) holds:

$$\begin{split} F &= I \left( \frac{\frac{(1+\rho^*)}{2} [V(\theta_a^f) + V(\theta_b^f) + s]}{V(\hat{\theta}_a^h) + V(\hat{\theta}_b^h) + s} + \frac{\frac{(1-\rho^*)}{2} [V(\theta_a^f) + V(\theta_b^f) + s]}{V(\hat{\theta}_a^l) + V(\hat{\theta}_b^h) + s} - \frac{V(\theta_b^f)}{V(\hat{\theta}_b^h)} \right) \\ &+ \Psi \left( \frac{V(\theta_a^f) + X_b^L}{\theta_b^f (X_b^H - X_b^L) + s} - \frac{X_b^L}{\theta_b^f (X_b^H - X_b^L)} \right) - s = 0. \end{split} \tag{41}$$

On the other hand, if (23) holds instead of (17), this threshold value is determined by the indifference equation obtained from (36) in Proposition 3:

$$\begin{split} F &= I \left( \frac{\frac{(1+\rho^{*})}{2} [V(\theta_{a}^{f}) + V(\theta_{b}^{f}) + s]}{V(\hat{\theta}_{a}^{h}) + V(\hat{\theta}_{b}^{h}) + s} + \frac{\frac{(1-\rho^{*})}{2} [V(\theta_{a}^{f}) + V(\theta_{b}^{f}) + s]}{V(\hat{\theta}_{a}^{h}) + V(\hat{\theta}_{b}^{f}) + s} - \frac{V(\theta_{b}^{f})}{V(\hat{\theta}_{b}^{h})} \right) \\ &+ \Psi \left( \frac{V(\theta_{a}^{f}) + X_{b}^{L}}{\theta_{b}^{f}(X_{b}^{H} - X_{b}^{L}) + s} - \frac{X_{b}^{L}}{\theta_{b}^{f}(X_{b}^{H} - X_{b}^{L})} \right) - s = 0. \end{split} \tag{42}$$

The following proposition provides a more complete characterization of the effect of the ex ante correlation in outside investors' beliefs about the new project B and the firm's assets in place (project A) on the firm's choice between integration and non-integration and the interaction of this correlation with the other ingredients of our model.

**Proposition 4** (Comparative statics on  $\rho^*$ ). Let  $\Psi < (1-l/(V(\hat{\theta}_a^l) + V(\hat{\theta}_b^l) + s))(\theta_b^f(X_b^H - X_b^L) + s)$  and denote by  $\rho^*$  the threshold value of the ex ante correlation in outsider beliefs about projects A and B above which the firm implements and funds the new project under integration. This threshold value is: (a) increasing in outside investors' average belief about the new project,  $\theta_b^m$ ; (b) increasing in the dispersion of outside investors' beliefs about project B's cash flows,  $d_b$ ; (c) decreasing in insiders' belief about the new project,  $\theta_b^f$ ; (d) increasing in insiders' belief about assets in place,  $\theta_a^f$ ; (e) increasing in the entrepreneur's effort cost,  $\Psi$ ; (f) decreasing in the value of the potential synergies between the new project and assets in place, s; (g) decreasing in the size of the new project,  $\chi_b^H$ .

In the above proposition, we first show that, ceteris paribus, as the outside investors' average belief about the new project,  $\theta_b^m$ , or the dispersion in their beliefs about the new project,  $d_b$ , increases, the marginal investor financing the new project in an equity carve-out becomes relatively more optimistic about the new project. Consequently, the choice of non-integration becomes optimal for a greater range of the values of the ex ante correlation  $\rho$  in outside investors' beliefs about the firm's assets in place and its new project. This is so, because it becomes relatively more costly for firm insiders to raise capital against cash flows from both project A and project B versus raising capital against the cash flows of project B only, if the marginal investor's beliefs about project B and the existing project A are not highly positively correlated. As  $\theta_b^m$  or  $d_b$  increases, for any value of  $\rho$ , the incremental optimism of the marginal investor (relative to firm insiders) in the equity of the carved-out firm about the new project increases (compared to the incremental optimism of the marginal investor financing the combined firm about the firm's assets in place). This extra dash of outsider optimism can be better captured by firm insiders through an equity carve-out rather than be diluted with the firm's assets in place through integration. Hence, the firm's optimal decision tilts toward non-integration for a greater range of values of the ex ante correlation parameter  $\rho$ . Similarly, as insiders become more pessimistic about the new project, so that  $\theta_h^f$  decreases, the parent firm will be more likely to choose non-integration through an equity carve-out rather than integration, since the incremental optimism of the marginal outside investor (relative to firm insiders) about the new project will be greater than that about assets in place. In summary, firm insiders' choice between integration and non-integration is determined by the difference in optimism between insiders and outsiders about the future prospects of the new project B.

Second, as the employee-entrepreneur's effort cost,  $\Psi$ , to implement the new project increases, it becomes more costly for the firm's existing shareholders to pay the

employee-entrepreneur with the equity of the combined firm rather than with the equity of the carved-out new project for any level of the ex ante correlation parameter  $\rho$ . Hence, the choice of non-integration becomes optimal for a greater range of values of the ex ante correlation parameter  $\rho$  as  $\Psi$  increases, so that the threshold value  $\rho^*$  above which integration becomes optimal increases.

Third, as the synergy between project A and project B increases, firm insiders will tolerate a larger valuation discount due to the difference in outside investors' beliefs about projects A and B that occurs under integration. Hence, as the synergy created by integrating the two projects increases, the choice of integration becomes optimal for a greater range of values of the ex ante correlation parameter  $\rho$ , so that the threshold value  $\rho^*$  above which integration becomes optimal decreases.

Finally, the above proposition shows how a change in the size of the project B relative to that of project A affects the choice between integration and non-integration, and thereby the threshold value  $\rho^*$  of the ex ante correlation parameter. As the size of project B gets larger (for a given size of project A), it becomes relatively less costly to develop it inside and integration becomes the optimal choice of organization for a greater range of  $\rho$ . This is due to two effects. First, the cost of issuing equity to outsiders to finance the new project under integration (i.e., by issuing equity in the combined firm) decreases, since investor optimism about project B is less diluted with the presence of the existing project A as the size of project B increases. Second, as project B is relatively larger, the marginal cost of compensating the employee-entrepreneur to motivate him to exert optimal effort under integration decreases.

#### 4. Negative stub values in equity carve-outs

In this section, we analyze the conditions under which a negative stub value can arise after an equity carve-out in a world with rational agents under heterogeneous beliefs and short-sale constraints. Let us first define what the terms "stub value" and "negative stub value" mean in the context of our model. If firm insiders optimally choose to carve out the new project and set up a new firm through an IPO, the parent firm will have some equity holdings in the new IPO firm; in particular, the parent firm will hold a fraction  $\beta$  of project B, where  $\beta$  is given in (10). Thus, after an equity carve-out, the shareholders of the parent firm will hold 100% of the equity in project (firm) A and a fraction  $\beta$  of the equity in project (firm) B, and therefore, they are entitled to all cash flows from project A and a fraction  $\beta$  of the cash flows from project B. In such a setting, the stub value is defined as the difference between the market value of the parent firm,  $V_{parent}$ , and the market value of the equity holdings of the parent firm in the newly carved-out firm. If this difference is negative, then we define the carve-out as being characterized by a negative stub value.

As we mentioned in our model setup before, the market value of the parent firm is determined by the parent firm's marginal investor's valuation of project A and his valuation of the parent firm's equity holdings in the newly carved-out firm. Since the parent firm's outstanding shares are already held by those outside

investors who are most optimistic about project A, the marginal investor in the parent firm's equity (currently) has the belief  $\theta_a^{parent} = \theta_a^m + d_a$  about the success probability of project A. If we let the belief of this investor about project B's cash flows be denoted by  $\theta_b^{parent}$ , the market value of the parent firm is given by

$$V_{parent} = V(\theta_a^m + d_a) + \beta V(\theta_b^{parent}). \tag{43}$$

On the other hand, the market value of the equity in the new firm is purely determined by the marginal investor financing the new firm, who has the belief  $\overline{\theta}_b = \theta_b^m + d_b(1-2I/W)$  about project B (see Proposition 1). Thus, the market value of the parent firm's equity holdings in the carved-out firm is equal to  $\beta V(\overline{\theta}_b)$ . The stub value is then given by

$$V_{stub} = V(\theta_a^m + d_a) + \beta V(\theta_b^{parent}) - \beta V(\overline{\theta}_b). \tag{44}$$

The parent firm's marginal investor will have the following belief about the success probability of the new project at time 1, depending on whether the correlation in his beliefs about the future prospects of project A and project B is +1 or -1:

$$\theta_b^{parent} = \begin{cases} \theta_b^m + d_b & \text{if the correlation is } +1 \text{ at time 1,} \\ \theta_b^m - d_b & \text{if the correlation is } -1 \text{ at time 1.} \end{cases}$$
(45)

**Proposition 5** (Negative versus positive stub value). Suppose that firm insiders choose to carve out the new project and raise the investment capital I through an equity carve-out IPO. Then<sup>25</sup>:

 If outside investors' beliefs about project A and project B are perfectly negatively correlated at time1, the stub value V<sub>stub</sub> will be negative if and only if the following condition holds:

$$V(\theta_{a}^{m} + d_{a}) < 2\left(1 - \frac{I}{V(\overline{\theta}_{b})} - \frac{\Psi}{\theta_{b}^{f}(X_{b}^{H} - X_{b}^{L})}\right)d_{b}\left(1 - \frac{I}{W}\right)(X_{b}^{H} - X_{b}^{L}).$$
(46)

(ii) If outside investors' beliefs about project A and project B are perfectly positively correlated at time1, the stub value V<sub>stub</sub> will always be positive.

The preceding discussion about the definition of stub value implies that a negative stub value can arise if and only if the marginal investor of the carved-out firm and the marginal investor of the parent firm differ in their beliefs about the cash flows from project B. Indeed, the above proposition shows that the parameter restrictions under which such an equilibrium outcome can arise are least restrictive when outside investors' beliefs about project A and project B are perfectly negatively correlated. Since the marginal investor of the parent firm is the most optimistic investor about project A (with belief  $\theta_p^{arent} = \theta_p^m + d_q$ ), he will be the most pessimistic investor about project B, when the correlation between his beliefs

 $<sup>^{25}</sup>$  Since we are analyzing the stub values at time 1, the correlation in outside investors' beliefs is already publicly realized, and is known to equal either +1 or -1 (as discussed before).

about the two projects is perfectly negative. In this case, the parent firm's marginal investor will have the belief  $\theta_b^{parent} = \theta_b^m - d_b$  about the probability of success of the new project B, and he will value the new firm's cash flows at the lowest value possible, which is equal to  $V(\theta_b^m - d_b)$ . However, since the equity of the stand-alone firm containing project B is valued by outside investors in the market who are more optimistic about project B (with belief  $\overline{\theta}_b = \theta_b^m + d_b(1-2I/W)$ ), there will be a considerable discrepancy between the imputed value of the equity holdings of the parent firm in the carved-out firm based on the carved-out firm's market-determined share price and the parent firm shareholders' own valuation of their equity holdings in the carved-out firm based on their beliefs about project B:

$$\beta V(\overline{\theta}_b) - \beta V(\theta_b^m - d_b) = \left(1 - \frac{I}{V(\overline{\theta}_b)} - \frac{\Psi}{\theta_b^f(X_b^H - X_b^L)}\right) 2d_b \left(1 - \frac{I}{W}\right) > 0.$$
(47)

If the discrepancy given in (47) is large enough so that (46) holds, the stub value will be negative. <sup>26</sup> The condition (46) is more likely to be satisfied when the dispersion in investor beliefs  $d_b$  about the new project is large, the payoff spread  $(X_b^H - X_b^L)$  of the new project is also large, the investment required for the new project, I, is not too large, and the size of the new project B is also not too small relative to the firm's assets in place (project A).

In the case of a perfectly positive correlation between outsiders' beliefs about projects A and B, the parent firm's marginal investor is also the most optimistic investor about project B. Therefore, the imputed value of the parent firm's holdings in the carved-out firm based on that firm's equity market value cannot be greater than the market value of the parent firm, based on the parent firm's marginal investor's belief  $\theta_b^{parent} = \theta_b^m + d_b > \overline{\theta}_b$ . In

other words, the stub value will always be positive in this scenario

**Proposition 6** (Comparative statics on stub value). Suppose that firm insiders choose to carve out the new project and raise the investment capital I through an equity carveout IPO, and let the correlation in outside investors' beliefs about the two projects be -1. The magnitude of the stub value  $V_{stub}$  of an equity carve-out given in (46) is: (a) decreasing in the dispersion of investors' beliefs about project B's cash flows,  $d_b$ ; (b) decreasing in outside investors' average belief about the new project,  $d_b^m$ ; (c) decreasing in the size of the new project,  $X_b^H$ ; (d) increasing in the level of investment required for the new project, I.

As the dispersion in investor beliefs,  $d_b$ , about project B increases, the heterogeneity of investor valuations about project B increases. More importantly, the market valuation of the carved-out firm increases, because the marginal investor financing firm B becomes more optimistic and therefore attaches a higher market valuation to the new firm. Simultaneously, the parent firm owns a larger fraction  $\beta$  of the carved-out firm. Therefore, as  $d_b$  increases, the stub value decreases (i.e., if it is positive, its magnitude goes down; if it is negative, it becomes even more negative).

A similar effect is realized if the average investor belief about project B,  $\theta^{b}_{b}$ , increases, keeping everything else constant. The marginal investor financing the carved-out firm becomes more optimistic about the project B's cash flows and therefore attaches a higher market valuation to the new firm, and at the same time, the parent firm owns a larger fraction  $\beta$  of the carved-out firm. This, again, leads to the stub value decreasing in  $\theta^{m}_{b}$  (i.e., if it is positive, its magnitude goes down; if it is negative, it becomes even more negative).

As the size of project B,  $X_b^H$ , increases, the effect of the heterogeneity in investor beliefs on stub values becomes more pronounced. In other words, the fraction of the parent firm value accounted for by the value of its equity holdings in the carved-out firm has a larger effect in the determination of the stub value defined in (46). Hence, as  $X_b^H$  increases, the stub value decreases (i.e., if it is positive, its magnitude goes down; if it is negative, it becomes even more negative).

Finally, as the amount of external financing required for the new project, I, increases, the fraction of equity held by the parent firm,  $\beta$ , in the carved-out firm decreases, which reduces the fraction of the parent firm value accounted for by the value of its equity holdings in the carved-out firm. Therefore, the stub value increases as the level of investment required for the new project increases (i.e., if it is positive, its magnitude goes up; if it is negative, it becomes less negative).

#### 5. Empirical implications

We now highlight the testable predictions of our model.
(i) Dispersion in investor beliefs: First, our model predicts that new projects involving radically new technologies (characterized by greater uncertainty and therefore higher dispersion in investor beliefs) are more likely to be implemented outside the firm under an equity carve-out.

<sup>&</sup>lt;sup>26</sup> We assume that current shareholders of the parent firm are affiliated with firm insiders, so that they are not allowed to sell their existing shares in the parent firm to shareholders of the carved-out firm. However, negative stub values will not be eliminated even if we go outside our model and allow for some reselling of shares by shareholders of the parent firm (recall that parent firm shares have claims to both the parent firm's assets in place and its equity ownership stake in the carved-out firm), as long as we allow the outside shareholders in the carved-out firm (and other investors who are optimistic about the carved-out firm) to have beliefs that assign a negative net present value to the parent firm's assets in place. This is because even outside shareholders who are highly optimistic about the prospects of the carved-out firm will not pay a high price for the parent firm's shares (even though these shares have a long-run claim to the significant equity ownership stake in the carved-out firm held by the parent firm), since they are not a "pure-play" on the cash flows of the carved-out firm, and these outsiders are highly pessimistic about the prospects of the parent firm's assets in place. Evidence indicating that this difficulty in separating out claims to the new project from that to the firm's assets in place is at the heart of negative stub values is provided by Mitchell, Pulvino, and Stafford (2002). In a study of attempted arbitrage in 82 cases of negative stub values, they show that uncertainty over the distribution of subsidiaries' shares to parent company shareholders is a significant contributor to the persistence of negative stub values, leading such arbitrageurs to earn a rate of return lower than the risk-free rate. Once such a distribution is announced, the value of the arbitrageur's position increased substantially.

Projects involving technologies which are increments of older technologies, for which the dispersion in outsider beliefs is relatively less, are more likely to be financed and implemented under the existing organizational structure of the firm. In testing this prediction, the dispersion in investor beliefs can be proxied by the dispersion in analysts' earnings forecasts as in Diether, Malloy, and Scherbina (2002), or by trading volume and turnover as suggested by the trading activity model of Harris and Raviv (1993).

(ii) Investor optimism: Our model predicts that projects involving technologies about which outsiders are currently more optimistic, on average, are more likely to be "carved-out" and therefore implemented under non-integration. Examples of recent carve-outs of firms adopting such technologies include many firms in the alternative energy industry and the bio-tech industry associated with sequencing the human genome and developing medical therapies targeting small groups of individuals based on their genetic makeup. Two such carve-outs were the carve-out of Sunpower, which makes solar panels, from Cypress Semiconductor (the parent firm) and CoGenesys (which develops Cardeva, a long-acting version of a drug given to heart-failure patients) from the parent firm, Human Genome Sciences. Similar phenomena occurred during the Internet bubble period, where many firms operating "mainstream" technologies carved out their Internet divisions as separate firms. In testing this prediction, empiricists can measure investor optimism by using the investor sentiment proxies developed and implemented by Baker and Wurgler (2006) and Kamstra, Kramer, and Levi (2009).

(iii) Correlation in investor beliefs: If an innovative technology appeals to an investor base different from the current investor base of the existing firm, it is likely that these investor bases will have differences in opinion regarding the new technology. Thus, our model predicts that projects involving new technologies with a different investor base from the firm's existing projects (i.e., low correlation in outside investors' beliefs across the assets in place and the innovation) are more likely to be implemented outside. The examples given in (ii) above also apply here.

(iv) Employee-entrepreneur incentives: If the effort of certain employees is crucial for implementing a new project, non-integration will likely be the optimal organizational choice under which the new project is funded and implemented. In the equity carve-out of CoGenesys from Human Genome Sciences, one of the reasons cited was to retain and motivate Craig Rosen and Steven C. Mayer, experts in genomic research, who were given a 13% equity stake in the new firm (see the article, "A biotech firm's new formula," Washington Post, July 31, 2006). Indirect evidence supporting this prediction is also provided by Allen (1998), who examines Thermo Electron and its 11 equity carve-outs. He finds that carveouts subject units of the company to the scrutiny of the capital markets and allow the compensation contracts of unit managers to be based on equity market performance. In particular, he documents that the majority of options granted to unit managers were tied to the stock performance of the subsidiary unit; further, for 1983–1995, compensation received by them from the exercise of stock options was roughly twice that of salary and bonus compensation during that period. Additional evidence is provided by Wruck and Wruck (2002), who find that, in the context of spinoffs, the probability of selecting a parent firm manager as the spinoff top manager was positively associated with the parent firm's pre-spinoff industry-adjusted profitability. Using the parent firm's profitability as a proxy for the human capital of its managers, this indicates that, when managers are more valuable to the firm, they are put in charge of running subsidiaries that are spun off by the parent firm.

(v) *Synergy*: Our model predicts that integration will more likely be the organizational structure under which a new project will be implemented, if the synergy between that project and the firm's assets in place is greater. Such synergies are likely to be larger when the new project creates value in the same industry as the parent firm. In other words, equity carve-outs are more likely when the new project is unrelated to the main business activity of the parent firm, so that it does not have much synergy with the existing projects of the firm.

(vi) Relative size of the new project: If the size of the new project is relatively small compared to the size of assets in place, the parent firm is more likely to carve it out to outside investors and choose non-integration as the preferred form of organization. This arrangement allows the parent firm to better capture the optimism of outside investors when raising investment capital for the implementation of the project and to better motivate the employee-entrepreneur implementing the project.

(vii) Negative stub values: Our model has two predictions regarding negative stub values. First, our model predicts that negative stub values in the equity carve-outs of certain firms are more likely to arise if (a) the dispersion in investor beliefs about the new project is higher and larger in magnitude than the dispersion in investor beliefs about the parent firm; (b) the investors are more optimistic about the new project, on average; (c) the correlation between investor beliefs about the new project and the firm's assets in place is negative (i.e., the investor bases for the new and old technologies used by the firm are different); and (d) the relative size of the new project is not too small. Second, our model predicts that, whenever negative stub values are present, the heterogeneity in investor beliefs about the value of the subsidiary firm will be much higher than the heterogeneity in beliefs about the value of the parent firm. Therefore, our model predicts that, when the stub value is negative, the turnover in the shares of the subsidiary firm will be much higher than that in the shares of the parent firm since differences of opinion lead to trade: see, e.g., Harris and Raviv (1993). Evidence consistent with this is presented by Lamont and Thaler (2003), who study mispricing in tech-stock carve-outs and find that, when the stub value is negative, the higher priced security has turnover that is many times higher than the turnover of the lower priced security. They find that, in the case of the well known Palm-3Com carve-out, the turnover in the shares of Palm (the carved-out firm's security) was vastly higher than the turnover in the shares of 3Com (the parent firm's security). Ofek and Richardson (2001) also present evidence consistent with a heterogeneous beliefs explanation of negative stub values.

(viii) Equity carve-outs of existing projects: While, in our formal analysis, we focus on the carve-outs of new projects related to raising the external financing required to implement them, our analysis can be easily extended to generate predictions also for the carve-outs of ongoing projects that are already funded, which often occur in practice. One situation in which such carve-outs occur is when there is a change in outside investors' beliefs about project B so that they are much more optimistic about this project relative to firm insiders, while their beliefs about the prospects of project A remain unchanged. In this situation, parent firm insiders may carve-out project B to outsiders even if there is no immediate external financing required for that project, to take advantage of outsider optimism or to meet other funding requirements of the parent firm (such as retiring debt of the parent firm). Another situation where such a carve-out can occur is when the synergy between projects A and B changes dynamically over time. Thus, while at the initiation of project B this synergy may be very high (for example, due to its sharing production or other facilities with the parent firm), this synergy may be reduced over time, at which point the parent firm may choose to carve out project B.

#### 6. Conclusion

We have developed a theory of new-project financing and equity carve-outs under heterogeneous beliefs among investors in the equity market. We considered a setting where an employee of a firm generates an idea for a new project that can be financed either by issuing equity against the future cash flows of the entire firm, i.e., both assets in place and the new project ("integration"), or by undertaking an equity carve-out of the new project ("non-integration"). The patent underlying the new project is owned by the firm. However, the employee generating the idea needs to be motivated to exert optimal effort for the project to be successful. The most important ingredient driving the firm's choice between integration and non-integration is heterogeneity in beliefs among outside investors (each of whom has limited wealth to invest in the equity market) and between firm insiders and outsiders. If outsider beliefs are such that the marginal outsider financing the new project is more optimistic about the prospects of the project than firm insiders, and this incremental optimism of the marginal outsider over firm insiders is greater regarding the new project than about the firm's assets in place, then the firm will implement the project under non-integration rather than integration. Two other ingredients driving the choice between integration and non-integration are the cost of motivating the employee to exert optimal effort for project implementation, and the synergy between the new project and the firm's assets in place, which is eliminated under non-integration. We derived a number of testable predictions regarding a firm's equilibrium choice between integration and non-integration. We also provided a rationale for the "negative stub values" documented in the equity carve-outs of certain firms (e.g., the carve-out of Palm from 3Com) and developed predictions for the magnitude of these stub values.

#### Appendix A

**Proof of Proposition 1.** Solving the equations  $I = P_b \times E_e$  and  $P_b = V_b/(1 + E_e) = V(\overline{\theta}_b)/(1 + E_e)$  for  $P_b$  and  $E_e$ , we obtain

$$P_b = V(\overline{\theta}_b) - I, \quad E_e = \frac{I}{V(\overline{\theta}_b) - I}.$$
 (A.1)

Note that, for the new firm, the incentive compatibility constraint in (IC1) is binding, but the individual rationality constraint in (IR1) is not binding. Since the IC constraint is binding for the optimal linear contract, it follows from (IC1) that we obtain

$$\alpha = \frac{\Psi}{\theta_b^f(X_b^H - X_b^L)}. \tag{A.2}$$

Since  $\alpha = E_e'/(1+E_e)$  by definition, it follows from (A.1) and (A.2) that

$$E_{e'} = \frac{\Psi}{\theta_b^f(X_b^H - X_b^L)} \frac{V(\overline{\theta}_b)}{V(\overline{\theta}_b) - I}.$$
(A.3)

The employee-entrepreneur's expected payoff is given by

$$EU_{Entrepreneur} = \alpha V(\theta_b^f) - \Psi. \tag{A.4}$$

Substituting  $\alpha$  from (A.2) in the above equation, we find that the employee-entrepreneur obtains a surplus of

$$EU_{Entrepreneur} = \Psi \frac{X_b^L}{\theta_b^f(X_b^H - X_b^L)} > 0. \tag{A.5}$$

The fraction of equity held by the parent firm in the carved-out firm is equal to

$$\beta = \frac{1 - E_{e}'}{1 + E_{e}} = 1 - \frac{I}{V(\overline{\theta}_{b})} - \frac{\Psi}{\theta_{b}^{f}(X_{b}^{H} - X_{b}^{I})}. \qquad \Box$$
 (A.6)

**Proof of Lemma 1.** Note that at time 1, outside investors' beliefs about project B will be either perfectly positively or perfectly negatively correlated with their beliefs about project A. Therefore, in the case of integration, as the discussion in the text after Lemma 1 points out, if (17) holds, the marginal investor financing the new equity issue of the combined firm at time 1 is determined by integrating over the beliefs of investors who are most optimistic about the cash flow prospects of project B, regardless of whether the correlation between time-1 beliefs is perfectly positive or perfectly negative. Hence, the marginal investor's belief about project B at time 1 and its time-0 expectation will be given by (12). Then, it follows that the expected value of this marginal investor's time-1 belief about project A's cash flow will be given by

$$E[\hat{\theta}_a] = \frac{(1+\rho)}{2} \left(\theta_a^m + d_a \left(1 - \frac{2I}{W}\right)\right) + \frac{(1-\rho)}{2} \left(\theta_a^m - d_a \left(1 - \frac{2I}{W}\right)\right),\tag{A.7}$$

which simplifies to (11).

On the other hand, if the condition in (17) is not satisfied, so that (23) holds, the marginal investor financing the new equity issue of the combined firm at time 1 is determined by integrating over the beliefs of investors who are most optimistic about the cash flow prospects of

project A, regardless of the realized time-1 value of the correlation in outsiders' beliefs about projects A and B. Then, the marginal investor's belief about project A at time 1 and its time-0 expectation will be given by (13). Then, it follows that the expected value of this marginal investor's time-1 belief about project B's cash flow will be given by

$$\mathbb{E}[\hat{\theta}_b] = \frac{(1+\rho)}{2} \left( \theta_b^m + d_b \left( 1 - \frac{2I}{W} \right) \right) + \frac{(1-\rho)}{2} \left( \theta_b^m - d_b \left( 1 - \frac{2I}{W} \right) \right), \tag{A.8}$$

which simplifies to (14).

**Proof of Proposition 2.** Solving  $I = P_{a+b} \times E$  and  $P_{a+b} = V_{a+b}/(1+E) = (V(\hat{\theta}_a) + V(\hat{\theta}_b) + s)/(1+E)$  for  $P_{a+b}$  and E, we obtain

$$P_{a+b} = V(\hat{\theta}_a) + V(\hat{\theta}_b) + s - I, \quad E = \frac{I}{V(\hat{\theta}_a) + V(\hat{\theta}_b) + s - I}.$$
 (A.9)

From Lemma 1, it follows that the expected market value  $E[V_{a+b}] = E[V(\hat{\theta}_a) + V(\hat{\theta}_b) + s]$  of the integrated firm at time 0 will be given by (30), which is decreasing in  $\rho$ , since the expected value of  $\hat{\theta}_a$  ( $\hat{\theta}_b$ ) is decreasing in  $\rho$ , if the condition in (17) (condition in (23)) holds.

Simplifying the incentive compatibility constraint given in (IC2) to

$$\alpha_{c}(\theta_{b}^{f}(X_{b}^{H}-X_{b}^{L})+s) \geq \Psi, \tag{IC2'}$$

we find that the incentive compatibility constraint in (IC2) is binding, but that the individual rationality constraint in (IR2) is not binding. Thus, given E from (A.9), the binding IC constraint in (IC2'), and  $\alpha_c = E'/(1+E)$ , it follows that

$$E' = \frac{\Psi}{\theta_b^f(X_b^H - X_b^L) + s} \times \frac{V(\hat{\theta}_a) + V(\hat{\theta}_b) + s}{V(\hat{\theta}_a) + V(\hat{\theta}_b) + s - I}. \tag{A.10}$$

The employee-entrepreneur's expected payoff is given by

$$EU_{Fntrengeneur} = \alpha_c(V(\theta_a^f) + V(\theta_b^f) + s) - \Psi. \tag{A.11}$$

Substituting  $\alpha_c$  from the binding IC constraint in (IC2′) above, we find that the employee-entrepreneur extracts a surplus of

$$EU_{Entrepreneur} = \Psi \frac{V(\theta_a^f) + X_b^L}{\theta_b^f(X_b^H - X_b^L) + s} > 0. \tag{A.12}$$

The fraction of equity retained by the firm's current shareholders is then equal to

$$\beta_{c} = \frac{1 - E'}{1 + E} = 1 - \frac{I}{V(\hat{\theta}_{a}) + V(\hat{\theta}_{b}) + s} - \frac{\Psi}{\theta_{b}^{f}(X_{b}^{H} - X_{b}^{I}) + s}.$$
(A.13)

**Proof of Proposition 3.** We first assume that (17) holds. Then, Lemma 1 and Proposition 2 imply that the time-0 expected value  $\lambda$  of the fraction of firm equity ( $\beta_c$ ) that current shareholders will retain at time 1 (in the case of

integration) is given by

$$\lambda = \omega \left[ 1 - \frac{I}{V(\hat{\theta}_a^h) + V(\hat{\theta}_b^h) + s} - \frac{\Psi}{\theta_b^f(X_b^H - X_b^L) + s} \right]$$

$$+ (1 - \omega) \left[ 1 - \frac{I}{V(\hat{\theta}_a^l) + V(\hat{\theta}_b^h) + s} - \frac{\Psi}{\theta_b^f(X_b^H - X_b^L) + s} \right],$$
(A.14)

where  $\omega = (1+\rho)/2$ . This expression can be simplified to (31). The restriction

$$\Psi < \left(1 - \frac{I}{V(\hat{\theta}_a^l) + V(\hat{\theta}_b^l) + s}\right) (\theta_b^f(X_b^H - X_b^L) + s)$$

ensures that the firm's current shareholders can compensate the employee-entrepreneur from their existing equity holdings.

Given  $(EU_a+EU_b)$  in (34) and  $EU_{a+b}$  in (33), when we compare the expected payoffs of current shareholders of the parent firm from the two choices of financing and implementing the new project, the expected incremental net benefit of non-integration over integration is given by

$$EU_a + EU_b - EU_{a+b}$$

$$\begin{split} &= I \Biggl( \frac{\omega[V(\theta_a^f) + V(\theta_b^f) + s]}{V(\hat{\theta}_a^h) + V(\hat{\theta}_b^h) + s} + \frac{(1 - \omega)[V(\theta_a^f) + V(\theta_b^f) + s]}{V(\hat{\theta}_a^l) + V(\hat{\theta}_b^h) + s} - \frac{V(\theta_b^f)}{V(\overline{\theta}_b)} \Biggr) \\ &+ \Psi \Biggl( \frac{V(\theta_a^f) + X_b^L}{\theta_b^f(X_b^H - X_b^L) + s} - \frac{X_b^L}{\theta_b^f(X_b^H - X_b^L)} \Biggr) - s. \end{split} \tag{A.15}$$

The parent firm's insiders choose a carve-out of project B over integration with project A if this expression is positive as given in (35).

If the condition in (17) is not satisfied as assumed above, then the inequality (23) must hold. Then, Lemma 1 implies that the time-0 expected value  $\lambda$  of the fraction of firm equity that current shareholders will hold at time 1 is given by

$$\lambda = \omega \left[ 1 - \frac{I}{V(\hat{\theta}_{a}^{h}) + V(\hat{\theta}_{b}^{h}) + s} - \frac{\Psi}{\theta_{b}^{f}(X_{b}^{H} - X_{b}^{I}) + s} \right] + (1 - \omega) \left[ 1 - \frac{I}{V(\hat{\theta}_{a}^{h}) + V(\hat{\theta}_{b}^{I}) + s} - \frac{\Psi}{\theta_{b}^{f}(X_{b}^{H} - X_{b}^{I}) + s} \right]. \tag{A.16}$$

After following the above steps once again (but now assuming that the condition in (23) holds), we find that the parent firm's insiders choose a carve-out of project B over integration with project A, if the condition in (36) holds.

**Proof of Proposition 4.** We first note that  $\overline{\theta}_b = \hat{\theta}_b^h$  in the case of non-integration. If (17) holds, the value of  $\rho^*$  at which firm insiders are indifferent between integration and non-integration is determined by the indifference Eq. (41). On the other hand, if (23) holds (instead of (17)), this indifference value is determined by the indifference Eq. (42). For simplicity, we define  $\omega \equiv (1+\rho)/2$ .

Note that if we partially differentiate F with respect to  $\omega$ , we obtain

$$\frac{\partial F}{\partial \omega} = \begin{cases} I(V(\theta_a^f) + V(\theta_b^f) + s) \\ \times \left[ \frac{1}{V(\hat{\theta}_a^h) + V(\hat{\theta}_b^h) + s} - \frac{1}{V(\hat{\theta}_a^l) + V(\hat{\theta}_b^h) + s} \right] < 0 & \text{if (17) holds,} \\ I(V(\theta_a^f) + V(\theta_b^f) + s) \\ \times \left[ \frac{1}{V(\hat{\theta}_a^h) + V(\hat{\theta}_b^h) + s} - \frac{1}{V(\hat{\theta}_a^h) + V(\hat{\theta}_b^h) + s} \right] < 0 & \text{if (23) holds,} \end{cases}$$
(A 17)

since  $\hat{\theta}_a^h > \hat{\theta}_a^l$  and  $\hat{\theta}_b^h > \hat{\theta}_b^l$ . By the implicit differentiation rule, for any parameter p, the following relationship holds:

$$\frac{d\omega^*}{dp} = \frac{\frac{\partial F}{\partial p}}{\frac{\partial F}{\partial \omega}}.$$
(A.18)

Thus, if (17) holds, we determine that

$$\begin{split} \frac{\partial F}{\partial d_b} &= I \left( 1 - \frac{2I}{W} \right) (X_b^H - X_b^L) \\ &\times \left[ \frac{V(\theta_b^f)}{V(\hat{\theta}_b^h)^2} - \left[ \frac{\omega(V(\theta_a^f) + V(\theta_b^f) + s)}{(V(\hat{\theta}_a^h) + V(\hat{\theta}_b^h) + s)^2} \right. \\ &\left. + \frac{(1 - \omega)(V(\theta_a^f) + V(\theta_b^f) + s)}{(V(\hat{\theta}_a^f) + V(\hat{\theta}_b^h) + s)^2} \right] \right] > 0, \end{split} \tag{A.19}$$

and therefore that  $d\omega^*/d(d_b) > 0$ . If (17) does not hold so that (23) holds, we find that

$$\begin{split} \frac{\partial F}{\partial d_b} &= I \left( 1 - \frac{2I}{W} \right) (X_b^H - X_b^I) \left[ \frac{V(\theta_b^f)}{V(\hat{\theta}_b^h)^2} + \frac{(1 - \omega)(V(\theta_a^f) + V(\theta_b^f) + s)}{(V(\hat{\theta}_a^h) + V(\hat{\theta}_b^I) + s)^2} \right] \\ &- \frac{\omega(V(\theta_a^f) + V(\theta_b^f) + s)}{(V(\hat{\theta}_a^h) + V(\hat{\theta}_b^I) + s)^2} \right] > 0. \end{split} \tag{A.20}$$

Thus,  $d\omega^*/d(d_b) > 0$  if (23) holds. Similarly, we obtain

$$\begin{split} \frac{\partial F}{\partial \theta_b^m} &= I(X_b^H - X_b^L) \left[ \frac{V(\theta_b^f)}{V(\hat{\theta}_b^f)^2} - \left[ \frac{\omega(V(\theta_a^f) + V(\theta_b^f) + s)}{(V(\hat{\theta}_a^h) + V(\hat{\theta}_b^h) + s)^2} \right. \\ &\left. + \frac{(1 - \omega)(V(\theta_a^f) + V(\theta_b^f) + s)}{(V(\hat{\theta}_a^f) + V(\hat{\theta}_b^h) + s)^2} \right] \right] > 0, \end{split} \tag{A.21}$$

and thus, we find that  $d\omega^*/d\theta_b^m > 0$  if (17) holds. If (17) does not hold so that (23) holds, we obtain:

$$\begin{split} \frac{\partial F}{\partial \theta_b^m} &= I(X_b^H - X_b^I) \left[ \frac{V(\theta_b^I)}{V(\hat{\theta}_b^I)^2} - \left[ \frac{\omega(V(\theta_a^I) + V(\theta_b^I) + s)}{(V(\hat{\theta}_a^I) + V(\hat{\theta}_b^I) + s)^2} \right. \\ &\left. + \frac{(1 - \omega)(V(\theta_a^I) + V(\theta_b^I) + s)}{(V(\hat{\theta}_a^I) + V(\hat{\theta}_b^I) + s)^2} \right] \right] > 0, \end{split} \tag{A.22}$$

so that  $d\omega^*/d\theta_h^m > 0$  if (23) holds. We also find that

$$\frac{\partial F}{\partial \theta_b^f} = (X_b^H - X_b^I)I \left[ \frac{\omega}{V(\hat{\theta}_a^h) + V(\hat{\theta}_b^h) + s} + \frac{1 - \omega}{V(\hat{\theta}_a^l) + V(\hat{\theta}_b^h) + s} - \frac{1}{V(\hat{\theta}_b^h)} \right]$$

$$-\Psi(X_{b}^{H}-X_{b}^{l})\left[\frac{V(\theta_{a}^{f})+X_{b}^{l}}{(\theta_{b}^{f}(X_{b}^{h}-X_{b}^{l})+s)^{2}}-\frac{X_{b}^{l}}{(\theta_{b}^{f}(X_{b}^{h}-X_{b}^{l}))^{2}}\right]<0,$$
(A 23)

if (17) holds. If (17) does not hold so that (23) holds, we obtain

$$\frac{\partial F}{\partial \theta_b^f} = (X_b^H - X_b^I) I \left[ \frac{\omega}{V(\hat{\theta}_a^h) + V(\hat{\theta}_b^h) + s} + \frac{1 - \omega}{V(\hat{\theta}_a^h) + V(\hat{\theta}_b^l) + s} - \frac{1}{V(\hat{\theta}_b^h)} \right] 
- \Psi(X_b^H - X_b^I) \left[ \frac{V(\theta_a^f) + X_b^I}{(\theta_b^f(X_b^h - X_b^I) + s)^2} - \frac{X_b^I}{(\theta_b^f(X_b^h - X_b^I))^2} \right] < 0.$$
(A.24)

Thus, it follows that  $d\omega^*/d\theta_b^f < 0$ . If we partially differentiate F with respect to  $\theta_a^f$ , we also find that, if (17) holds

$$\begin{split} \frac{\partial F}{\partial \theta_{a}^{f}} &= (X_{a}^{H} - X_{a}^{L}) \left[ I \left[ \frac{\omega}{V(\hat{\theta}_{a}^{h}) + V(\hat{\theta}_{b}^{h}) + s} + \frac{1 - \omega}{V(\hat{\theta}_{a}^{h}) + V(\hat{\theta}_{b}^{l}) + s} \right] \\ &+ \frac{\Psi(V(\theta_{a}^{f}) + X_{b}^{l})}{\theta_{b}^{f}(X_{b}^{h} - X_{b}^{l}) + s} \right] > 0. \end{split} \tag{A.25}$$

One can easily check that this partial derivative is also positive if (17) does not hold so that (23) holds as well. Therefore,  $d\omega^*/d\theta_a^f > 0$ .

Next, we also note that

$$\frac{\partial F}{\partial \Psi} = \frac{V(\theta_a^f) + X_b^l}{\theta_b^f(X_b^h - X_b^l) + s} - \frac{X_b^l}{\theta_b^f(X_b^h - X_b^l)} > 0, \tag{A.26}$$

and we obtain

$$\begin{split} \frac{\partial F}{\partial s} &= -\frac{\Psi(V(\theta_{a}^{f}) + X_{b}^{l})}{(\theta_{b}^{f}(X_{b}^{h} - X_{b}^{l}) + s)^{2}} - 1 + I \left[ \omega \frac{(V(\hat{\theta}_{a}^{h}) + V(\hat{\theta}_{b}^{h})) - (V(\theta_{a}^{f}) + V(\theta_{b}^{f}))}{(V(\hat{\theta}_{a}^{h}) + V(\hat{\theta}_{b}^{h}) + s)^{2}} \right. \\ &+ (1 - \omega) \left. \frac{(V(\hat{\theta}_{a}^{l}) + V(\hat{\theta}_{b}^{h})) - (V(\theta_{a}^{f}) + V(\theta_{b}^{f}))}{(V(\hat{\theta}_{a}^{l}) + V(\hat{\theta}_{b}^{h}) + s)^{2}} \right] < 0, \end{split}$$

$$(A.27)$$

if (17) holds. If (17) does not hold so that (23) holds, we obtain

$$\begin{split} \frac{\partial F}{\partial s} &= -\frac{\Psi(V(\theta_{a}^{f}) + X_{b}^{l})}{(\theta_{b}^{f}(X_{b}^{h} - X_{b}^{l}) + s)^{2}} - 1 + I \left[ \omega \frac{(V(\hat{\theta}_{a}^{h}) + V(\hat{\theta}_{b}^{h})) - (V(\theta_{a}^{f}) + V(\theta_{b}^{f}))}{(V(\hat{\theta}_{a}^{h}) + V(\hat{\theta}_{b}^{h}) + s)^{2}} \right. \\ &+ (1 - \omega) \left. \frac{(V(\hat{\theta}_{a}^{h}) + V(\hat{\theta}_{b}^{l})) - (V(\theta_{a}^{f}) + V(\theta_{b}^{f}))}{(V(\hat{\theta}_{a}^{h}) + V(\hat{\theta}_{b}^{l}) + s)^{2}} \right] < 0. \end{split}$$

$$(A.28)$$

Hence, we conclude that  $d\omega^*/d\Psi > 0$  and  $d\omega^*/ds < 0$ . Finally, we obtain

$$\begin{split} \frac{\partial F}{\partial X_b^H} &= -\theta_b^f \Psi \Bigg[ \frac{V(\theta_a^f) + X_b^I}{(\theta_b^f(X_b^h - X_b^I) + s)^2} - \frac{X_b^I}{(\theta_b^f(X_b^h - X_b^I))^2} \Bigg] \\ &+ I\theta_b^f \Bigg[ \frac{\omega}{V(\hat{\theta}_a^h) + V(\hat{\theta}_b^h) + s} + \frac{(1 - \omega)}{V(\hat{\theta}_a^I) + V(\hat{\theta}_b^h) + s} - \frac{1}{V(\hat{\theta}_b^h)} \Bigg] \\ &+ I\hat{\theta}_b^h \Bigg[ \frac{V(\theta_b^f)}{V(\hat{\theta}_b^h)^2} - \omega \frac{V(\theta_a^f) + V(\theta_b^f) + s}{(V(\hat{\theta}_a^h) + V(\hat{\theta}_b^h) + s)^2} \end{split}$$

$$-(1-\omega)\frac{V(\theta_a^f) + V(\theta_b^f) + s}{(V(\hat{\theta}_a^f) + V(\hat{\theta}_b^f) + s)^2} \right| < 0, \tag{A.29}$$

if (17) holds. If (17) does not hold so that (23) holds, we obtain

$$\begin{split} \frac{\partial F}{\partial X_{b}^{H}} &= -\theta_{b}^{f} \Psi \left[ \frac{V(\theta_{a}^{f}) + X_{b}^{l}}{(\theta_{b}^{f}(X_{b}^{h} - X_{b}^{l}) + s)^{2}} - \frac{X_{b}^{l}}{(\theta_{b}^{f}(X_{b}^{h} - X_{b}^{l}))^{2}} \right] \\ &+ I\theta_{b}^{f} \left[ \frac{\omega}{V(\hat{\theta}_{a}^{h}) + V(\hat{\theta}_{b}^{h}) + s} + \frac{(1 - \omega)}{V(\hat{\theta}_{a}^{h}) + V(\hat{\theta}_{b}^{l}) + s} - \frac{1}{V(\hat{\theta}_{b}^{h})} \right] \\ &+ I\hat{\theta}_{b}^{h} \left[ \frac{V(\theta_{b}^{f})}{V(\hat{\theta}_{b}^{h})^{2}} - \omega \frac{V(\theta_{a}^{f}) + V(\theta_{b}^{f}) + s}{(V(\hat{\theta}_{a}^{h}) + V(\hat{\theta}_{b}^{h}) + s)^{2}} \right] \\ &- I\hat{\theta}_{b}^{l} (1 - \omega) \frac{V(\theta_{a}^{f}) + V(\theta_{b}^{f}) + s}{(V(\hat{\theta}_{a}^{h}) + V(\hat{\theta}_{b}^{h}) + s)^{2}} < 0, \end{split} \tag{A.30}$$

and therefore,  $d\omega^*/dX_b^H < 0$ . The results for  $\rho^*$  follow by the chain rule and by the definition of  $\omega$ , where  $\omega \equiv (1+\rho)/2$ . Note that  $d\rho/d\omega = 2$ .

**Proof of Proposition 5.** In the case of non-integration, we know that the marginal investor of the carved-out firm has the belief  $\overline{\theta}_b = \theta_b^m + d_b(1-2I/W)$  about project B. Thus, the market value of the carved-out firm is equal to  $V(\overline{\theta}_b)$ . If outsiders' beliefs about projects A and B are perfectly negatively correlated at time 1, the marginal investor of the parent firm will have the belief  $\theta_b^{parent} = \theta_b^m - d_b$  about project B at time 1 given in (45). Substituting  $\theta_b^{parent}$  and the expression for  $\beta$  from (10) into (44), we obtain the following expression for the stub value  $V_{stub}$  in the case of perfectly negative correlation in outsider beliefs about projects A and B:

$$\begin{split} V_{stub} &= V(\theta_a^m + d_a) - 2\left(1 - \frac{I}{V(\overline{\theta}_b)} - \frac{\Psi}{\theta_b^f(X_b^H - X_b^L)}\right) \\ &\times d_b \left(1 - \frac{I}{W}\right)(X_b^H - X_b^L). \end{split} \tag{A.31}$$

Then, it follows that the stub value will be negative in the case of perfectly negative correlation at time 1, if and only if the condition (46) holds.

If outsiders' beliefs about projects A and B are perfectly positively correlated at time 1, the marginal investor of the parent firm has the belief  $\theta_b^{parent} = \theta_b^m + d_b$ , which is given in (45). Note that in this case,  $\theta_b^{parent}$  is greater than  $\overline{\theta}_b$ , and therefore,  $V(\theta_b^{parent}) > V(\overline{\theta}_b)$ . Hence, from (44), it follows that the stub value will always be positive if the correlation in outside investors' beliefs about projects A and B is +1 at time 1.  $\square$ 

**Proof of Proposition 6.** If the correlation in outside investors' beliefs about projects A and B is -1 at time 1, it follows that the stub value is given by (A.31) above. By partially differentiating the stub value  $V_{stub}$  given in (A.31) with respect to the various parameters mentioned in

Proposition 6, we obtain

$$\begin{split} \frac{\partial V_{stub}}{\partial d_b} &= \left[ -\frac{I}{V(\overline{\theta}_b)^2} (X_b^H - X_b^L) \left( 1 - \frac{2I}{W} \right) d_b - \left( 1 - \frac{I}{V(\overline{\theta}_b)} - \frac{\Psi}{\theta_b^f (X_b^H - X_b^L)} \right) \right] \\ &\times \left( 2 - \frac{2I}{W} \right) (X_b^H - X_b^L) < 0, \end{split} \tag{A.32}$$

$$\frac{\partial V_{stub}}{\partial \theta_b^m} = -\frac{I}{V(\overline{\theta}_b)^2} (X_b^H - X_b^L)^2 d_b \left(2 - \frac{2I}{W}\right) < 0, \tag{A.33}$$

$$\begin{split} \frac{\partial V_{stub}}{\partial X_{b}^{H}} &= d_{b} \left( 2 - \frac{2I}{W} \right) \times \left( - \left[ \frac{I\overline{\theta}_{b}}{V(\overline{\theta}_{b})^{2}} + \frac{\Psi \theta_{b}^{f}}{(\theta_{b}^{f}(X_{b}^{H} - X_{b}^{L}))^{2}} \right] (X_{b}^{H} - X_{b}^{L}) \\ &- \left( 1 - \frac{I}{V(\overline{\theta}_{b})} - \frac{\Psi}{\theta_{b}^{f}(X_{b}^{H} - X_{b}^{L})} \right) \right) < 0, \end{split} \tag{A.34}$$

$$\begin{split} \frac{\partial V_{stub}}{\partial I} &= d_b (X_b^H - X_b^L) \\ &\times \left[ \frac{(2 - \frac{2I}{W})}{V(\overline{\theta}_b)} + \frac{\left(1 - \frac{I}{V(\overline{\theta}_b)} - \frac{\Psi}{\theta_b^f (X_b^H - X_b^L)}\right)}{W} \right] > 0. \quad \Box \end{split}$$

$$(A.35)$$

#### References

Abel, A.B., Mailath, G.J., 1994. Financing losers in competitive markets. Journal of Financial Intermediation 3, 139–165.

Aghion, P., Tirole, J., 1994. The management of innovation. Quarterly Journal of Economics 109, 1185–1209.

Allen, F., Gale, D., 1999. Diversity of opinion and financing of new technologies. Journal of Financial Intermediation 8, 68–89.

Allen, J., 1998. Capital markets and corporate structure: the equity carve-outs of thermo electron. Journal of Financial Economics 48, 99–124.

Amador, M., Landier, A., 2003. Entrepreneurial pressure and innovation. Unpublished working paper, New York University.

Aron, D., 1991. Using the capital market as a monitor: corporate spinoffs in an agency framework. RAND Journal of Economics 22, 505–518.

Audretsch, D.B., 1991. New-firm survival and the technological regime. Review of Economics and Statistics 73, 441–450.

Baker, M., Wurgler, J., 2006. Investor sentiment and the cross-section of stock returns. Journal of Finance 61, 1645–1680.

Bayar, O., Chemmanur, T.J., Liu, M.H., 2010. A theory of capital structure, price impact, and long-run stock returns under heterogeneous beliefs. Unpublished working paper, University of Texas at San Antonio, Boston College, and University of Kentucky.

Biais, B., Perotti, E.C., 2008. Entrepreneurs and new ideas. RAND Journal of Economics 39, 1105–1125.

Boot, A.W.A., Gopalan, R., Thakor, A., 2006. The entrepreneur's choice between private and public ownership. Journal of Finance 61, 803–836.

Cassiman, B., Ueda, M., 2006. Optimal project rejection and new firm start-ups. Management Science 52, 262–275.

Chen, J., Hong, H., Stein, J.C., 2002. Breadth of ownership and stock returns, Journal of Financial Economics 66, 171–205.

Diether, K.B., Malloy, C.J., Scherbina, A., 2002. Differences of opinion and the cross section of stock returns. Journal of Finance 57, 2113–2141. Dittmar, A., Thakor, A., 2007. Why do firms issue equity? Journal of Finance 62, 1–54.

Duffie, D., Gârleanu, N., Pedersen, L.H., 2002. Securities lending, shorting, and pricing. Journal of Financial Economics 66, 307–339.

Fulghieri, P., Sevilir, M., 2009. Organization and financing of innovation, and the choice between corporate and independent venture capital. Journal of Financial and Quantitative Analysis 44, 1291–1321.

Gans, J., Hsu, D., Stern, S., 2002. When does start-up innovation spur the gale of creative destruction? RAND Journal of Economics 33, 571–586. Garmaise, M.J., 2001. Rational beliefs and security design. Review of

Financial Studies 14, 1183-1213.

- Gompers, P., Lerner, J., Scharfstein, D., 2005. Entrepreneurial spawning: public corporations and the genesis of new ventures: 1986–1999. Journal of Finance 60, 577–614.
- Gromb, D., Scharfstein, D., 2002. Entrepreneurship in equilibrium. NBER Working Paper No. 9001.
- Harris, M., Raviv, A., 1993. Differences of opinion make a horse race. Review of Financial Studies 6, 473–506.
- Harrison, J.M., Kreps, D.M., 1978. Speculative investor behavior in a stock market with heterogeneous expectations. Quarterly Journal of Economics 92. 323–336.
- Henderson, R.B., Clark, K.B., 1990. Architectural innovation: the reconfiguration of existing product technologies and the failure of established firms. Administrative Science Quarterly 35, 9–30.
- Kamstra, M.J., Kramer, L.A., Levi, M.D., 2009. Opposing seasonalities in treasury versus equity returns. Unpublished working paper, York University and University of Toronto.
- Kreps, D., 1990. A Course in Microeconomics. Princeton University Press, Princeton, NJ.
- Kurz, M., 1994. On the structure and diversity of rational beliefs. Economic Theory 4, 877–900.
- Lamont, O., Thaler, R., 2003. Can the market add and subtract? Mispricing in tech-stock carve-outs. Journal of Political Economy 111, 227–268.
- Lintner, J., 1969. The aggregation of investor's diverse judgments and preferences in purely competitive security markets. Journal of Financial and Quantitative Analysis 4, 347–400.
- Mathews, R., 2006. Strategic alliances, equity stakes, and entry deterrence. Journal of Financial Economics 80, 35–79.
- Miller, E.M., 1977. Risk, uncertainty, and divergence of opinion. Journal of Finance 32, 1151–1168.

- Mitchell, M., Pulvino, T., Stafford, E., 2002. Limited arbitrage in equity markets. Journal of Finance 57, 551–584.
- Morris, S., 1995. The common prior assumption in economic theory. Economics and Philosophy 11, 227–253.
- Morris, S., 1996. Speculative investor behavior and learning. Quarterly Journal of Economics 111, 1111–1133.
- Myers, S., Majluf, N., 1984. Corporate financing and investment decisions when firms have information that investors do not have. Journal of Financial Economics 9, 187–221.
- Nanda, V., 1991. On the good news in equity carve-outs. Journal of Finance 46, 1717–1737.
- Ofek, E., Richardson, M., 2001. Dotcom mania: a survey of market efficiency in the internet sector. Unpublished working paper, New York University.
- Palia, D., Ravid, S.A., Reisel, N., 2008. Choosing to cofinance: analysis of project-specific alliances in the movie industry. Review of Financial Studies 21, 483–511.
- Robinson, D., 2008. Strategic alliances and the boundaries of the firm. Review of Financial Studies 21, 649–681.
- Scheinkman, J., Xiong, W., 2004. Heterogeneous beliefs, speculation and trading in financial markets. Unpublished working paper, Princeton University.
- Schipper, K., Smith, A., 1986. A comparison of equity carve-outs and seasoned equity offerings: share price effects and corporate restructurings. Journal of Financial Economics 15, 153–186.
- Wruck, E.G., Wruck, K.H., 2002. Restructuring top management: evidence from corporate spinoffs. Journal of Labor Economics 20, 176–218.