ECO 401-002 Problem Set #5 KEY Spring 20[illegible]

(4 pts.) 1. If the two sisters split the work and split the profits, then they must each compare the net revenue they receive after all other economic costs are taken into account to the opportunity cost of their time. If one sister has better job alternatives than the other sister, then the opportunity cost of her time will be higher. She may decide that working in the shop is not worth it, while her sister thinks it is a good deal because her alternatives are not so good.

(4 pts.) 2. 8.2. The following incomplete table shows a firm's various costs of producing up to 6 units of output. Fill in as much of the table as possible. If you cannot determine the number in a box, explain why it is not possible to do so.

Q	TC	TVC	AFC	AC	MC	AVC
1	100	40	60	100	40	40
2	110	50	30	55	10	25
3	120	60	20	40	10	20
4	180	120	15	45	60	30
5	250	190	12	50	70	38
6	330	270	10	55	80	45

Total Fixed Cost = 60 (if $Q = 2$, $AFC = 30$)

Recall that: $TC = TFC + TVC$

$ATC = TC / Q$ $\quad$ $AFC = TFC / Q$

$AVC = TVC / Q$ $\quad$ $MC = \frac{\Delta TC}{\Delta Q}$

$ATC = AFC + AVC$

So the entire table can be filled in.

3.
(6 pts.)

Q	TC	ATC	MC
1	971	971	971
2	1888	944	917
3	2757	919	869
4	3584	896	827
5	4375	875	791
6	5136	856	761
7	5873	839	737
8	6592	824	719
9	7299	811	707
10	8000	800	701
11	8701	791	701
12	9408	784	707
13	10127	779	719
14	10864	776	737
15	11625	775	761
16	12416	776	791
17	13243	779	827
18	14112	784	869
19	15029	791	917
20	16000	800	971
21	17031	811	1031
22	18128	824	1097
23	19297	839	1169
24	20544	856	1247
25	21875	875	1331
26	23296	896	1421
27	24813	919	1517
28	26432	944	1619
29	28159	971	1727
30	30000	1000	1841

$TC = 1000Q - 30Q^2 + Q^3$

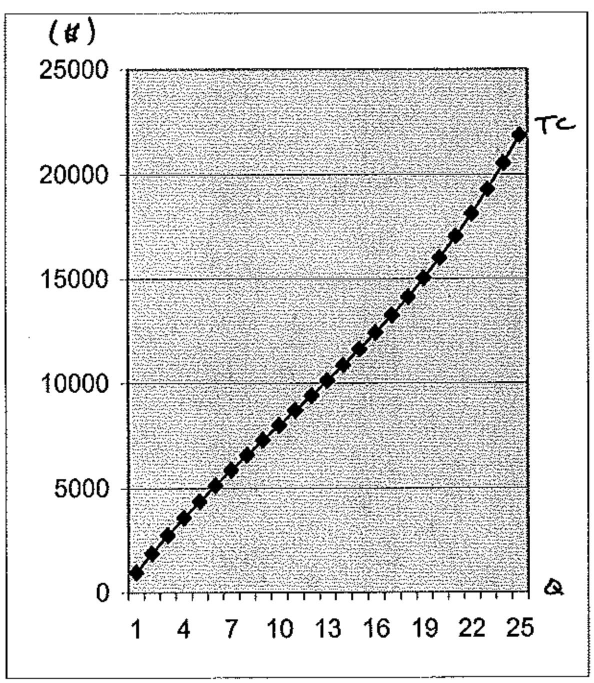
$ATC = TC/Q$

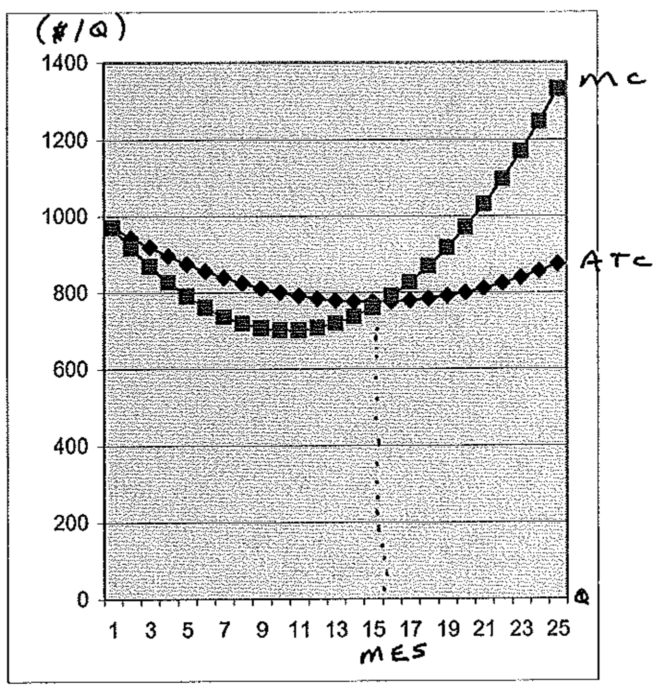
$= 1000 - 30Q + Q^2$

$MC = \frac{dTC}{dQ}$

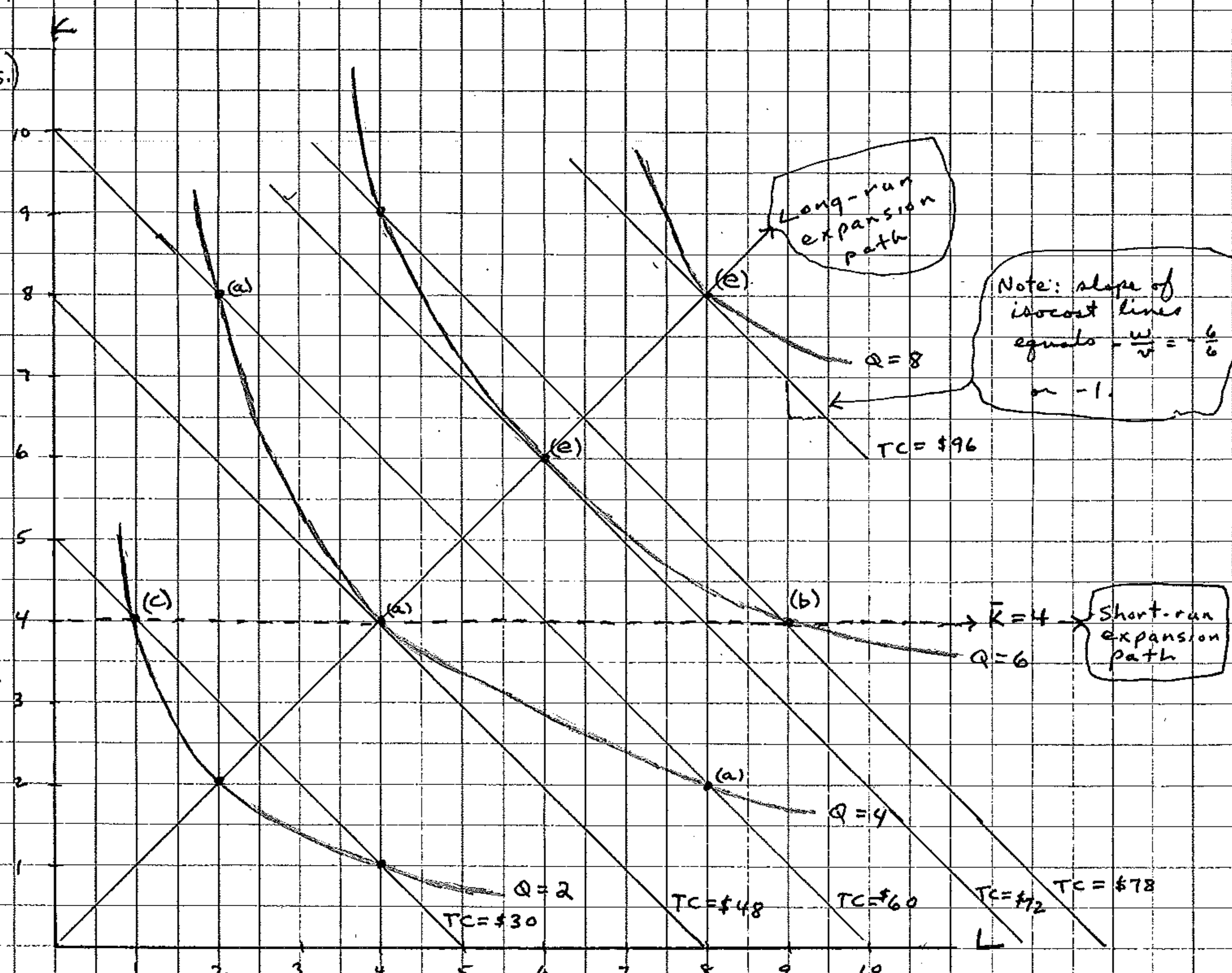
$= 1000 - 60Q + 3Q^2$

ATC reaches its minimum point at an output of Q=15. MC = ATC where ATC is at a minimum, and this is the output we refer to as MES, or minimum efficient scale.





4. (10pts.)



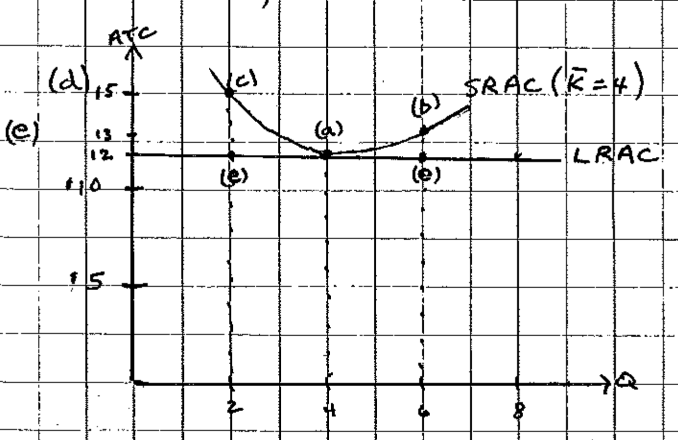
(a) $L=4$, $K=4$ minimizes the total cost of producing $Q=4$. At that point the slope of the isocost = slope of the isoquant, or in other words $MRTS_{L,K} = \frac{w}{v}$.

(b) If K is fixed at $\bar{K}=4$, then the firm will have to increase labor usage to $L=9$ to produce output $Q=6$. Total cost of the input combination $L=9$, $K=4$ is $TC = \$78$.

(c) $\bar{K}=4$, $L=1 \Rightarrow Q=2$. $TC = \$30$

(d)

(e)



(a) SRAC = LRAC, $Q=4$, $TC = \$48$
SRATC = \$12 = LRATC

(b) $\bar{K}=4$, $L=9$, $Q=6$, $TC = \$78$
SRATC = \$13

(c) $\bar{K}=4$, $L=1$, $Q=2$, $TC = \$30$
SRATC = \$15

(e) $L=6$, $K=6$, $Q=6$, $TC = \$72$, LRAC = \$12
$L=8$, $K=8$, $Q=8$, $TC = \$96$, LRAC = \$12

(2 pts.)

BONUS

$$MRTS_{L,K} = \frac{MP_L}{MP_K}$$

$$Q = L^{1/2} K^{1/2}$$

$$MP_L = \frac{dQ}{dL} = \frac{1}{2} L^{-1/2} K^{1/2}$$

$$MP_K = \frac{dQ}{dK} = \frac{1}{2} L^{1/2} K^{-1/2}$$

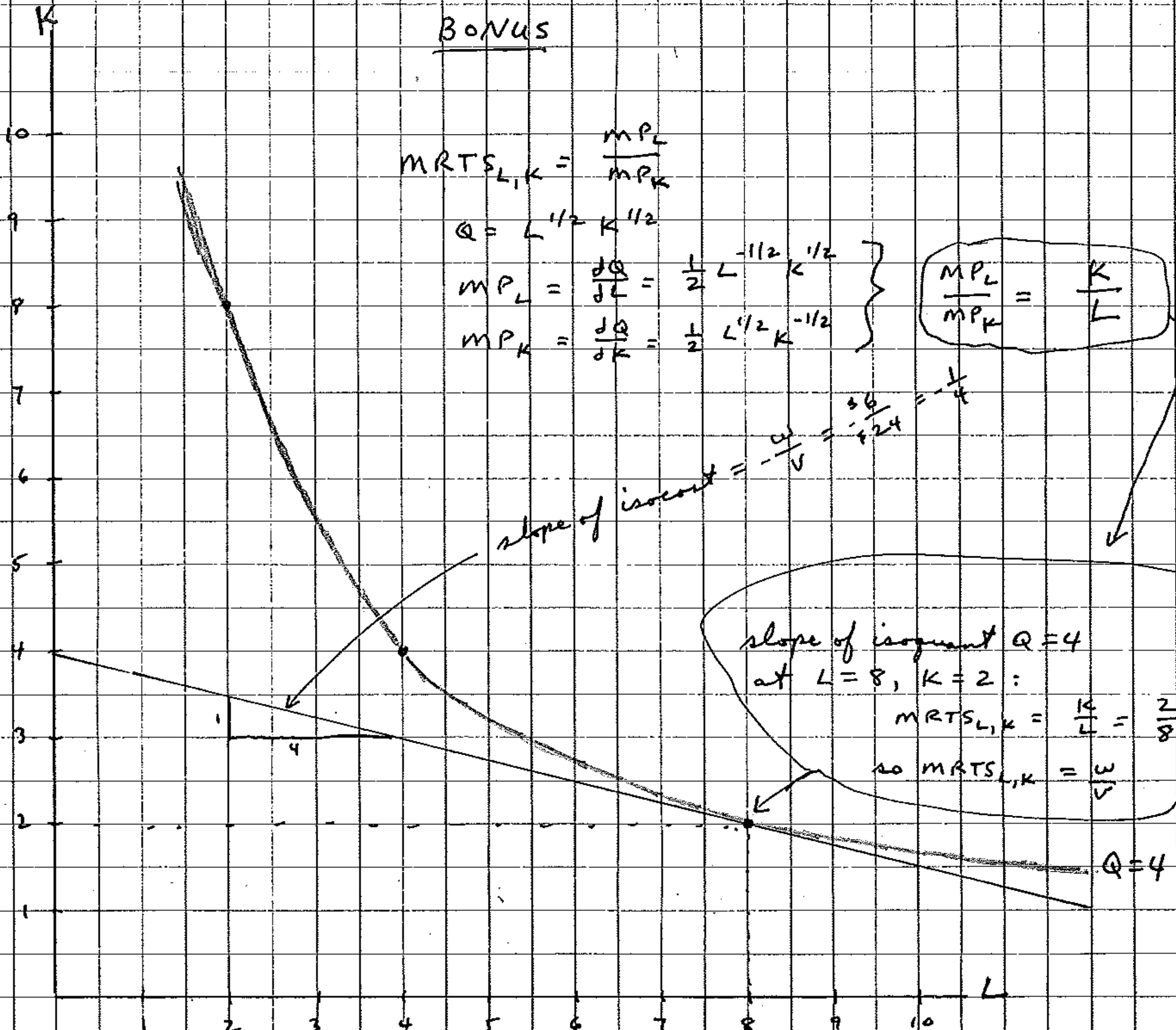
$$\boxed{\frac{MP_L}{MP_K} = \frac{K}{L}}$$

slope of isocost $= -\frac{w}{v} = -\frac{\$6}{\$24} = -\frac{1}{4}$

slope of isoquant $Q=4$ at $L=8$, $K=2$:

$$MRTS_{L,K} = \frac{K}{L} = \frac{2}{8}$$

so $MRTS_{L,K} = \frac{w}{v}$



so $L=8$ and $K=2$ minimizes the cost of producing $Q=4$ if $w = \$6$ and $v = \$24$.

ECO 401-002, 003
Spring 2011
Problem Set #4

Due: Wednesday, March 2, 2011

You own and operate a lawn-care service in Lexington. Your company supplies lawn care to up-scale homeowners whose yards are typically one acre in size. You use labor and capital in the production process. Your production function is given by the formula $Q = L^{.5}K^{.5}$, where L represents worker days and K represents machine days. The number of yards that you can mow and trim in a day thus is given in the following table.

		Labor Input									
		1	2	3	4	5	6	7	8	9	10
Capital Input	1	1.00	1.41	1.73	2.00	2.24	2.45	2.65	2.83	3.00	3.16
	2	1.41	2.00	2.45	2.83	3.16	3.46	3.74	4.00	4.24	4.47
	3	1.73	2.45	3.00	3.46	3.87	4.24	4.58	4.90	5.20	5.48
	4	2.00	2.83	3.46	4.00	4.47	4.90	5.29	5.66	6.00	6.32
	5	2.24	3.16	3.87	4.47	5.00	5.48	5.92	6.32	6.71	7.07
	6	2.45	3.46	4.24	4.90	5.48	6.00	6.48	6.93	7.35	7.75
	7	2.65	3.74	4.58	5.29	5.92	6.48	7.00	7.48	7.94	8.37
	8	2.83	4.00	4.90	5.66	6.32	6.93	7.48	8.00	8.49	8.94
	9	3.00	4.24	5.20	6.00	6.71	7.35	7.94	8.49	9.00	9.49
	10	3.16	4.47	5.48	6.32	7.07	7.75	8.37	8.94	9.49	10.00

1. Write an expression for your short-run production function when capital is fixed at K=4. Now write an expression for the marginal product of labor when K=4. Plot the total product of labor curve for K=4. Does your production function exhibit diminishing returns?

2. Suppose that you contract with seven workers to work the entire summer for your company. You rent your equipment by the day, so capital is variable on a day-to-day basis. Are diminishing returns still an issue? Explain, using a diagram illustrating the marginal product and average product of capital curves.

3. Now consider your long-range planning horizon (i.e. next summer) when both labor and capital are variable. Sketch four or five of the isoquants associated with your production function, based on the information in the table above.

4. Choose one of the isoquants that you have sketched and verify that it exhibits a diminishing marginal rate of technical substitution. Show what you are talking about in your diagram.

5. Does your production process exhibit increasing, constant, or decreasing returns to scale? Explain your answer.