KEY

ECO 401-002, 003
Spring 2013
Problem Set #4

Due: Wednesday, March 6

1. Labor and capital are used to produce widgets according to the production table below:

	Labor Input					
		1	2	3	4	5
	1	20	40	55	65	75
Capital Input	2	40	60	75	85	90
	3	55	75	90	100	105
	4	65	85	100	110	115
	5	75	90	105	115	120

5 pts.

a) Draw the TP_L and the corresponding AP_L and MP_L curves when K is fixed at 2 in the short run.
b) Is this production process subject to the law of eventually diminishing marginal returns? Pick a set of combinations of inputs to illustrate and explain your answer.
c) Draw two points on the Q=40 isoquant, four points on the Q=75 isoquant, three points on the Q=90 isoquant, and two points on the Q=115 isoquant.
d) Calculate the $MRTS_{L,K}$ along the Q=75 isoquant. Show that it is diminishing.
e) Does this production exhibit increasing, constant, or decreasing returns to scale? Pick a set of combinations of inputs to illustrate and explain your answer. (Hint: start with K=1 and L=1, and then double all inputs. Then double all inputs again, and see what happens to Q.)

6 pts.

2. The production function for a firm that produces pizzas is $Q = 15K^{1/4}L^{3/4}$. Q is the number of pizzas produced per hour, K is the number of pizza ovens, and L is the number of workers. In the short run K is fixed at 3.
a) Write an equation for the firm's short-run production function showing output as a function of labor when K is equal to 3.
b) Calculate total output per hour when L = 0, 1, 2, 3, 4, and 5.
c) Calculate MP_L for L = 1 to L = 5. Is MP_L diminishing?
d) Calculate AP_L for L = 1 to L = 5.
e) Graph TP_L. In a separate diagram, graph AP_L and MP_L.
f) Does this firm experience decreasing, constant, or increasing returns to scale? Hint: set K=3 and L=3, and calculate Q. Then double both K and L, and show what happens to output.

3 pts.

3. Refer to Figure 7.3 in the text. Calculate and graph TP_L, AP_L, and MP_L when K is fixed at K=3.

14 pts.

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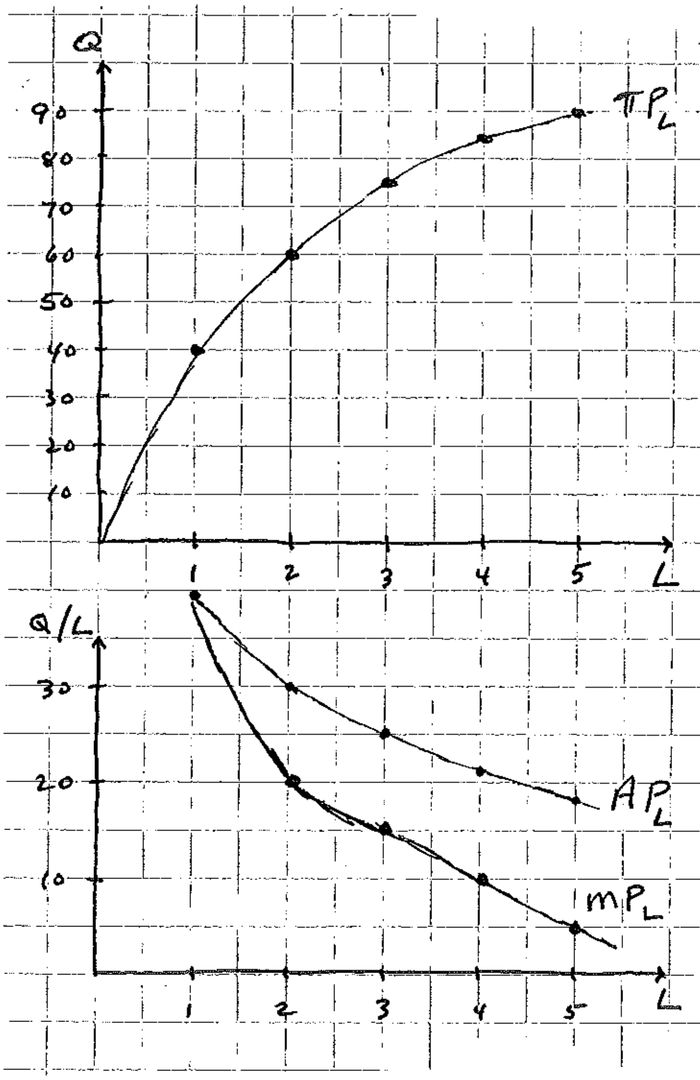
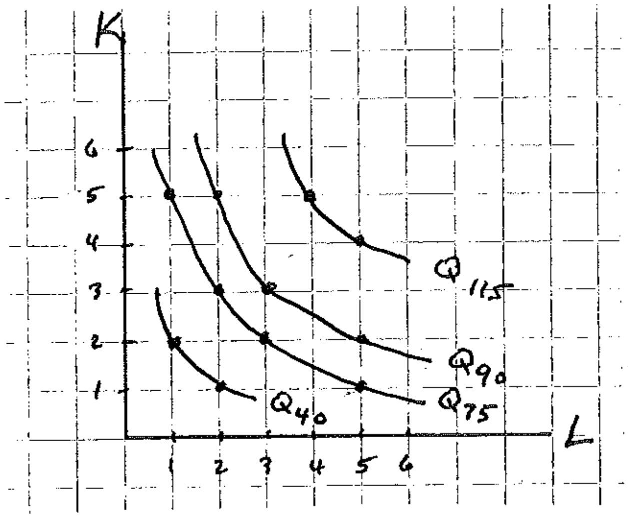
1. (a) when $\bar{K} = 2$;

	$L=1$	2	3	4	5		
	40	60	75	85	90	⇔	TP_L
	40	30	25	21.25	18	⇔	$AP_L = \frac{TP_L}{L}$
	40	20	15	10	5	⇔	$MP_L = \frac{\Delta TP_L}{\Delta L}$

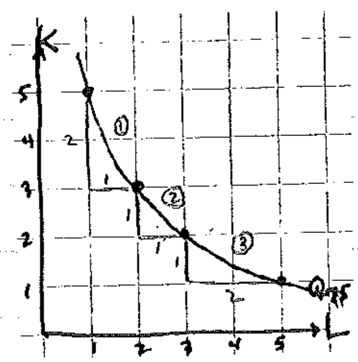


(b) when $\bar{K} = 2$ and $L = 1, 2, 3, 4,$ and 5, output goes up as L goes up, but the increase in output gets smaller and smaller — the marginal product of labor falls.

(c)



(d)



$$MRTS_{L,K} = -\frac{\Delta K}{\Delta L}\bigg|_{\bar{Q}}$$

① $MRTS_{L,K} = \frac{2}{1}$

② $MRTS_{L,K} = \frac{1}{1}$

③ $MRTS_{L,K} = \frac{1}{2}$

diminishing

(e) Returns to scale:

L	K	Q
1	1	20
2	2	60
4	4	110

↓ increasing returns to scale

↓ decreasing returns to scale

When $L=1$ and $K=1$, $Q=20$. Doubling both L and K leads to a more than doubling of output, so increasing returns to scale.

When $L=2$ and $K=2$, $Q=60$. Doubling both L and K leads to a less than doubling of output, so decreasing returns to scale.

2.

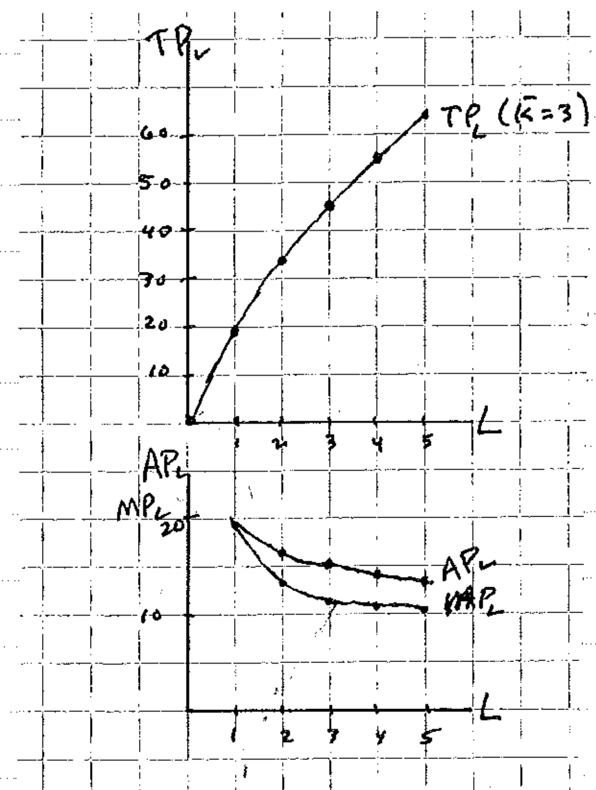
a. To write the production function for the short run, we plug $\overline{K} = 3$ into the production function to create an equation that shows output as a function of labor:

$$Q = f(\overline{K}, L) = 15\overline{K}^{0.25}L^{0.75}$$
$$= 15(3^{0.25})L^{0.75} = 15(1.316)L^{0.75} = 19.74L^{0.75}$$

b. To calculate total output, we plug in the different values of L and solve for Q:

$L = 0$	$Q = 19.74(0)^{0.75} = 19.74(0) = 0$
$L = 1$	$Q = 19.74(1)^{0.75} = 19.74(1) = 19.74$
$L = 2$	$Q = 19.74(2)^{0.75} = 19.74(1.682) = 33.20$
$L = 3$	$Q = 19.74(3)^{0.75} = 19.74(2.280) = 45.01$
$L = 4$	$Q = 19.74(4)^{0.75} = 19.74(2.828) = 55.82$
$L = 5$	$Q = 19.74(5)^{0.75} = 19.74(3.344) = 66.01$

(e)



c. The marginal product of labor is the additional output generated by an additional unit of labor, holding capital constant. We can use our answer from (b) to calculate the marginal product of labor for each worker:

$L = 1$	$MP_L = 19.74 - 0 = 19.74$
$L = 2$	$MP_L = 33.20 - 19.74 = 13.46$
$L = 3$	$MP_L = 45.01 - 33.20 = 11.81$
$L = 4$	$MP_L = 55.82 - 45.01 = 10.81$
$L = 5$	$MP_L = 66.01 - 55.82 = 10.19$

Note that, because MP_L falls as L rises, there is a diminishing marginal product of labor. This implies that output rises at a decreasing rate when labor is added to the fixed level of capital.

d. The average product of labor is calculated by dividing total output (Q) by the quantity of labor input (L):

$L = 1$	$AP_L = 19.74/1 = 19.74$
$L = 2$	$AP_L = 33.20/2 = 16.60$
$L = 3$	$AP_L = 45.01/3 = 15.00$
$L = 4$	$AP_L = 55.82/4 = 13.96$
$L = 5$	$AP_L = 66.01/5 = 13.20$

(f) Returns to scale:

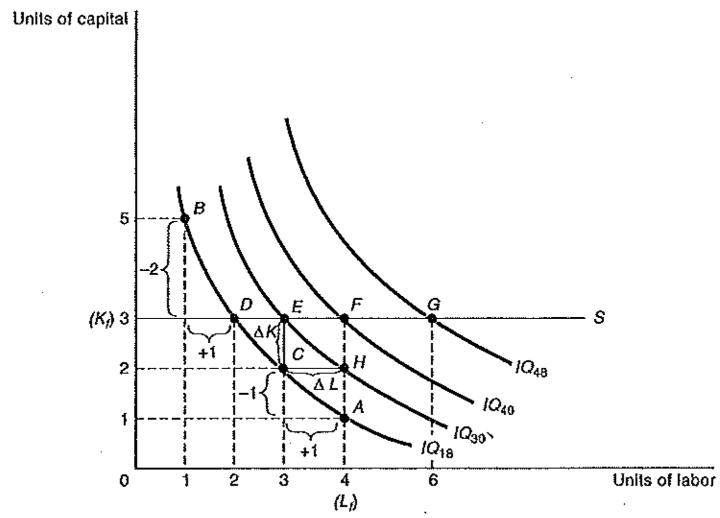
if $K = 3$ and $L = 3$,
$Q = 15(3)^{1/4}(3)^{3/4} = \underline{45}$ output doubles
if $K = 6$ and $L = 6$,
$Q = 15(6)^{1/4}(6)^{3/4} = \underline{90}$
so constant returns to scale

3.

Figure 7.3

Production Isoquants

Production isoquants show how much output a firm can produce with various combinations of inputs. A set of isoquants graphs the production function of the firm. Isoquants have geometric properties that are similar to those of indifference curves: they are downward sloping, nonintersecting, and convex. The slope of an isoquant measures the marginal rate of technical substitution between the inputs. Between points *B* and *D* the $MRTS_{LK}$ equals $2K/1L$, implying that 1 unit of labor can replace 2 units of capital without reducing the firm's output.



K	L	Q	Q/L	ΔQ/ΔL
3	2	18	9	
3	3	30	10	12
3	4	40	10	10
3	6	48	8	4