ECO 601 2nd Test KEY Fall 2002

① $u(W) = W^{1/2}$

$E(w) = \frac{1}{10}(2500) + \frac{9}{10}(40,000) = \$36,250$

(a) $E(u) = \frac{1}{10}u(2500) + \frac{9}{10}u(40000)$

$= .1(2500)^{.5} + .9(40000)^{.5}$

$= .1(50) + .9(200) = 185$

(b) $u(w) = 185$

$w^{1/2} = 185$

$w = \$34,225$

(c) expected loss $= \frac{1}{10}(37,500) = \$3750$

actuarially fair premium $+ \$1000 = \$4750$

$u(40,000 - 4,750) = u(35,250)$

$= (35,250)^{.5} = 187.75$

so she will buy the insurance

② $Q = (K^{1/2} + L^{1/2})^2$ exhibits constant returns to scale:

Increase both K and L by a factor of t. What happens to Q?

$\Rightarrow \left[(tK)^{1/2} + (tL)^{1/2}\right]^2$

$\Rightarrow \left[t^{1/2}K^{1/2} + t^{1/2}L^{1/2}\right]^2$

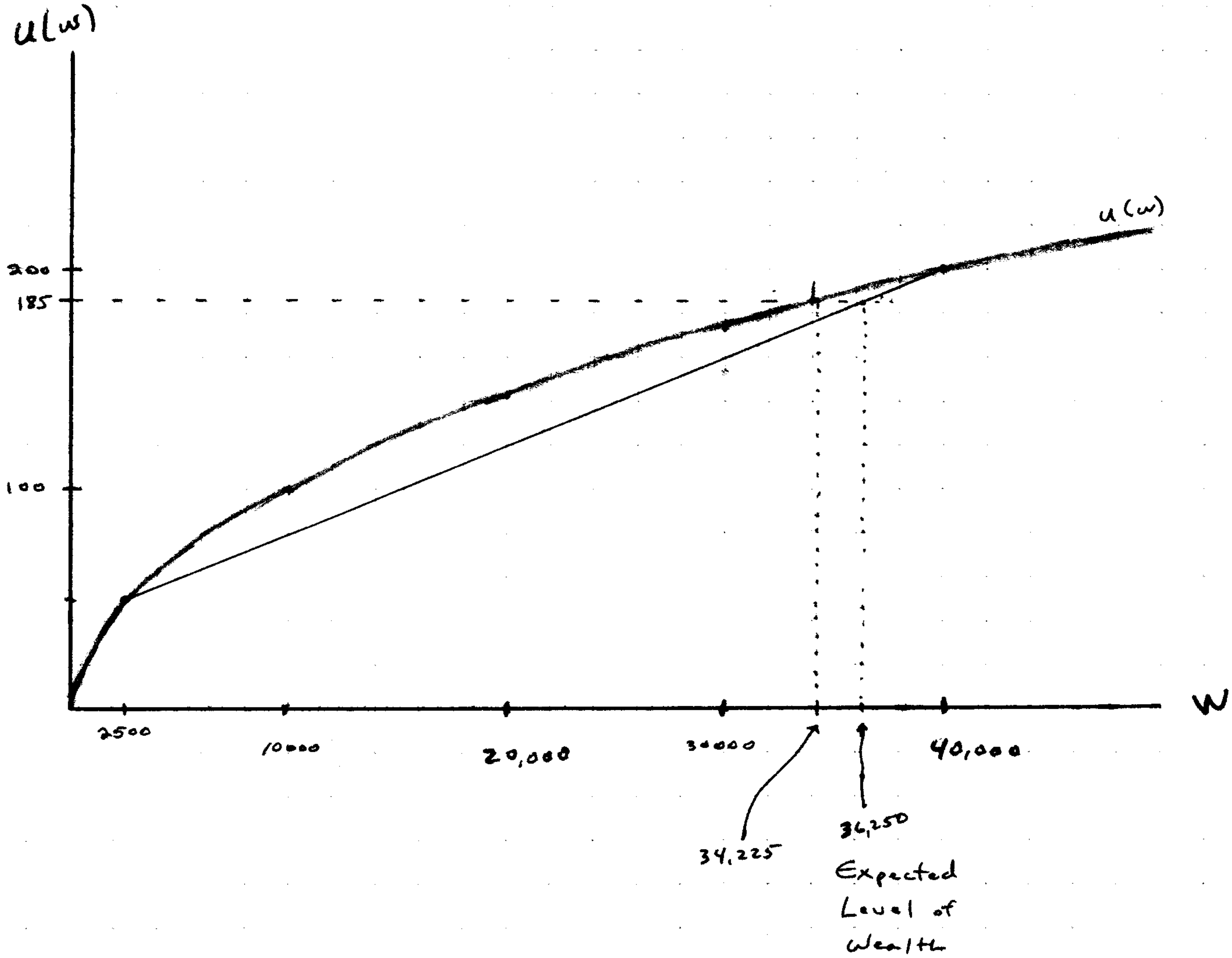
$\Rightarrow \left[t^{1/2}(K^{1/2} + L^{1/2})\right]^2$

$\Rightarrow (t^{1/2})^2(K^{1/2} + L^{1/2})^2$

$\Rightarrow t(Q)$

so Q changes by a factor of t

1.



u(w)
200
185
100
u(w)
W
2500
10000
20,000
30000
40,000
34,225
36,250
Expected Level of Wealth

③ $Q = KL^2 - L^3$

K is fixed at $K = 600$ in short run.

so $Q = 600L^2 - L^3$

$AP_L = \frac{Q}{L} = 600L - L^2$

$MP_L = \frac{dQ}{dL} = 1200L - 3L^2$

- $MP_L = 0$ at $L = 0$ and $L = 400$
- $MP_L = AP_L \Rightarrow 1200L - 3L^2 = 600L - L^2$

  so $MP_L = AP_L$ at $L = 0$ and $L = 300$
- MP_L reaches maximum where $\frac{dMP_L}{dL} = 0$

  $\frac{dMP_L}{dL} = 1200 - 6L$ or $L = 200$

④ $Q = (K^{1/2} + L^{1/2})^2 = K + 2K^{1/2}L^{1/2} + L$

(a) $MP_L = \frac{dQ}{dL} = K^{1/2}L^{-1/2} + 1 = L^{-1/2}(K^{1/2} + L^{1/2})$

$MP_K = \frac{dQ}{dK} = K^{-1/2}L^{1/2} + 1 = K^{-1/2}(K^{1/2} + L^{1/2})$

(b) min $TC = wL + vK$ S.T. $Q = 12{,}100$; $w = 10$, $v = 1$

$\mathcal{L} = 10L + K + \lambda\left[12{,}100 - (K^{1/2} + L^{1/2})^2\right]$

$\frac{d\mathcal{L}}{dL} = 10 - \lambda\left[L^{-1/2}(K^{1/2} + L^{1/2})\right] = 0$

$\frac{d\mathcal{L}}{dK} = 1 - \lambda\left[K^{-1/2}(K^{1/2} + L^{1/2})\right] = 0$

$\frac{d\mathcal{L}}{d\lambda} = 12{,}100 - (K^{1/2} + L^{1/2})^2 = 0$

combining first two 1st order conditions:

$\frac{10}{1} = \frac{L^{-1/2}(K^{1/2} + L^{1/2})}{K^{-1/2}(K^{1/2} + L^{1/2})}$

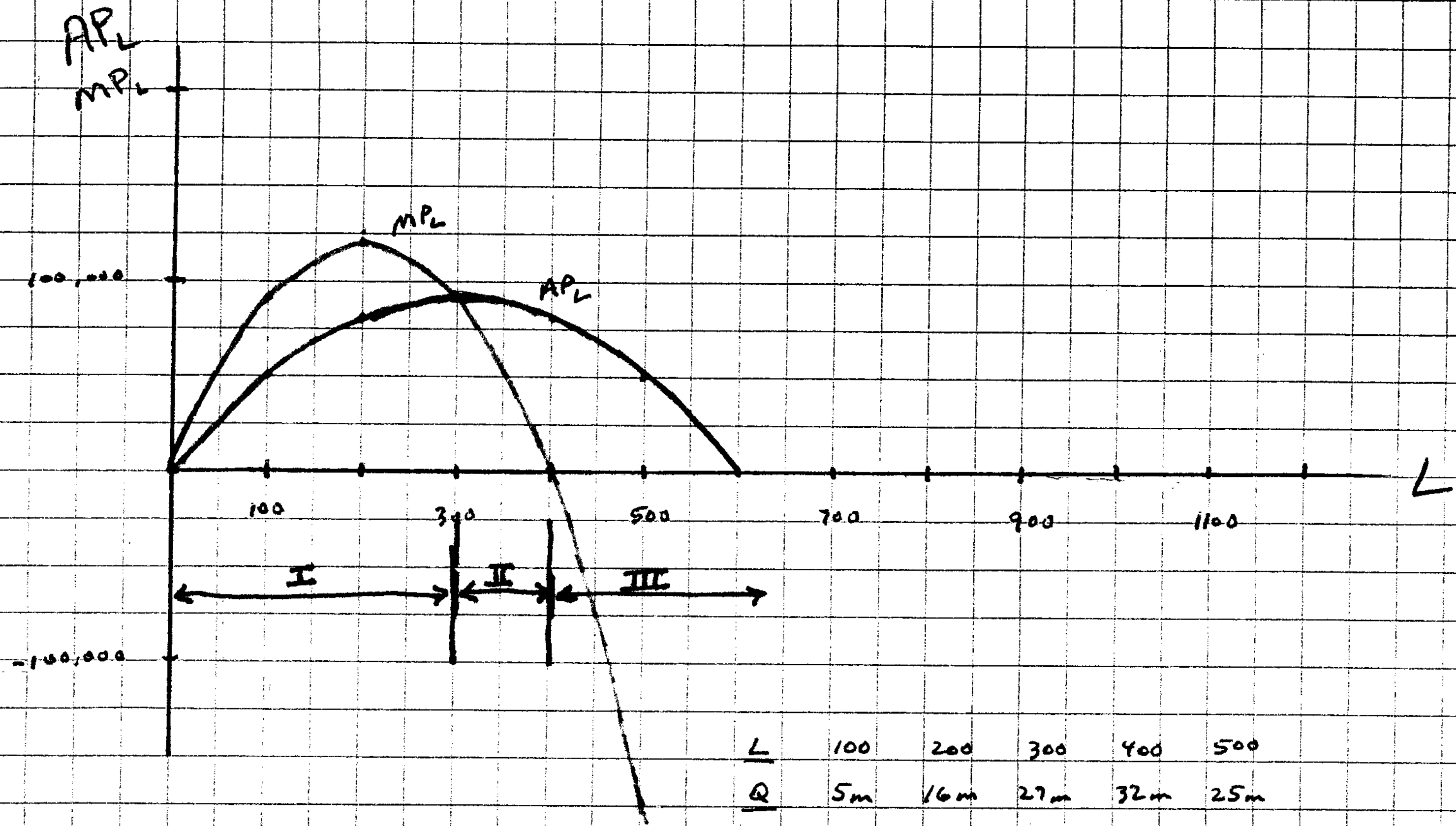
or $10 = \sqrt{\frac{K}{L}}$, $\frac{K}{L} = 100$, $K = 100L$

plug back into production ftn. constraint:

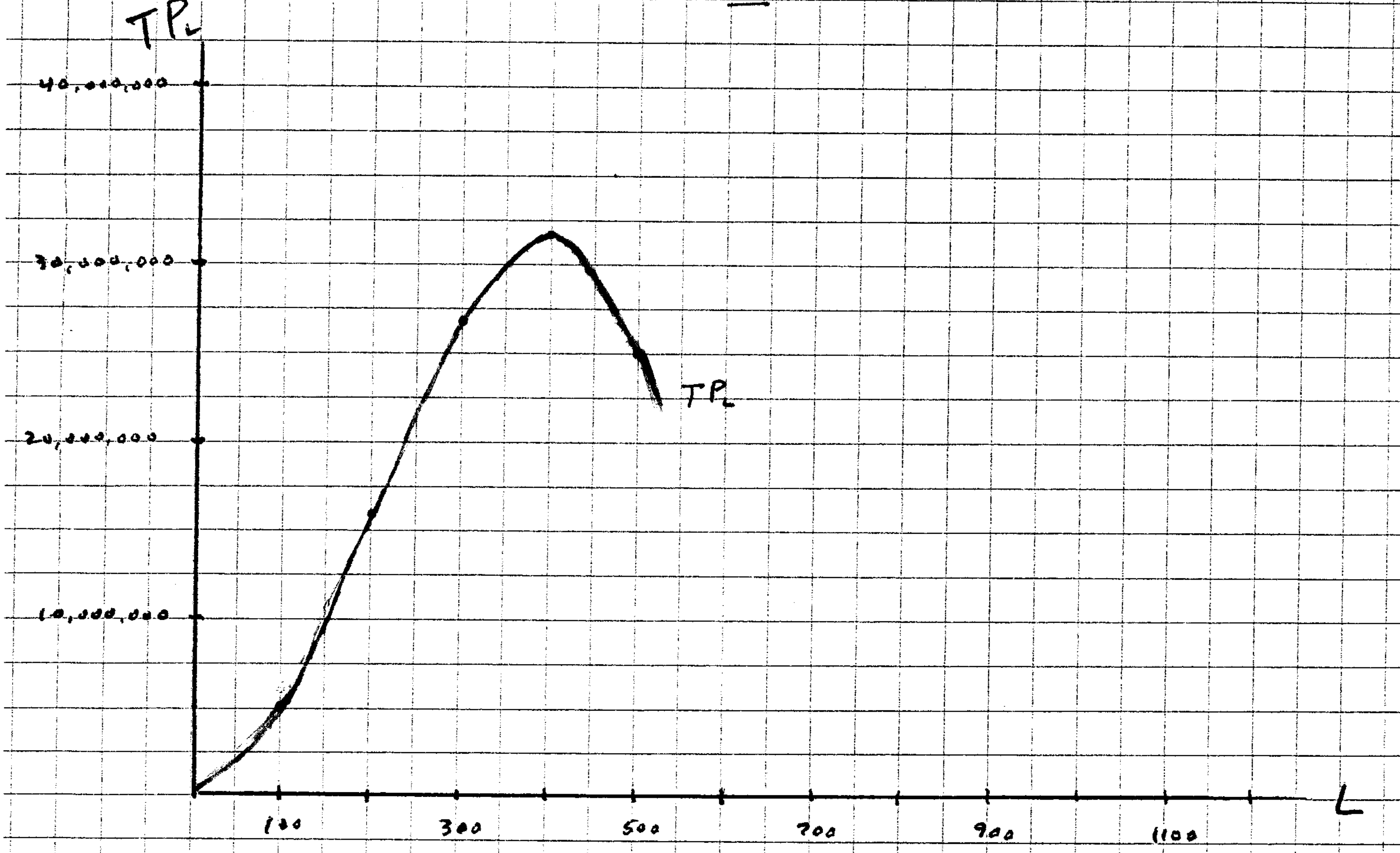
$\left[(100L)^{1/2} + L^{1/2}\right]^2 = 12{,}100$

$(10L^{1/2} + L^{1/2})^2 = 12{,}100$; $(11L^{1/2})^2 = 12{,}100$

$11L^{1/2} = 110$, $L^{1/2} = 10$, $L = 100$ and $K = 10{,}000$



L	100	200	300	400	500
Q	5m	16m	27m	32m	25m
AP_L	50k	80k	90k	80k	50k
MP_L	90k	120k	90k	0	-150k



⑤ (a) $Q = K^{1/2} L^{1/4} M^{1/4}$

minimize $TC = wL + vK + uM$ s.t. $Q \geq Q_0$

$$\mathcal{L} = wL + vK + uM + \lambda(Q_0 - K^{1/2} L^{1/4} M^{1/4})$$

1st order conditions:

(1) $\frac{\partial \mathcal{L}}{\partial L} = w - \frac{1}{4}\lambda K^{1/2} L^{-3/4} M^{1/4} = 0$

(2) $\frac{\partial \mathcal{L}}{\partial K} = v - \frac{1}{2}\lambda K^{-1/2} L^{1/4} M^{1/4} = 0$

(3) $\frac{\partial \mathcal{L}}{\partial M} = u - \frac{1}{4}\lambda K^{1/2} L^{1/4} M^{-3/4} = 0$

(4) $\frac{\partial \mathcal{L}}{\partial \lambda} = Q_0 - K^{1/2} L^{1/4} M^{1/4} = 0$

combining equations and simplifying:

(1) and (2) $\Rightarrow \frac{w}{v} = \frac{K}{2L} \Rightarrow vK = 2wL$

(1) and (3) $\Rightarrow \frac{w}{u} = \frac{M}{L} \Rightarrow wL = uM$

(2) and (3) $\Rightarrow \frac{v}{u} = \frac{2M}{K} \Rightarrow vK = 2uM$

now solve for L, K, and M in terms of Q, w, v, and u:

$$Q = \left(\frac{2wL}{v}\right)^{1/2} L^{1/4} \left(\frac{wL}{u}\right)^{1/4}$$

$$Q = L\left(\frac{2w}{v}\right)^{1/2}\left(\frac{w}{u}\right)^{1/4}$$

$$L = \left(Q v^{1/2} u^{1/4} w^{-3/4}\right) / \sqrt{2}$$

$$wL = \frac{1}{\sqrt{2}} Q v^{1/2} w^{1/4} u^{1/4}$$

$$vK = 2wL = \frac{2}{\sqrt{2}} Q v^{1/2} w^{1/4} u^{1/4}$$

$$uM = wL = \frac{1}{\sqrt{2}} Q v^{1/2} w^{1/4} u^{1/4}$$

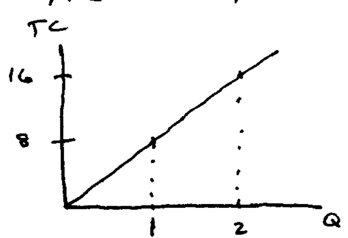
$$TC = \frac{1}{\sqrt{2}} Q v^{1/2} w^{1/4} u^{1/4} + \frac{2}{\sqrt{2}} Q v^{1/2} w^{1/4} u^{1/4} + \frac{1}{\sqrt{2}} Q v^{1/2} w^{1/4} u^{1/4}$$

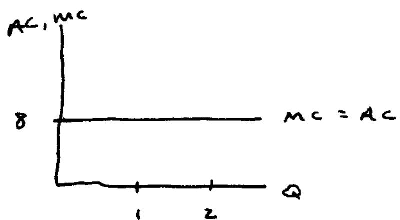
$$TC = \frac{4}{\sqrt{2}} Q v^{1/2} w^{1/4} u^{1/4} = 2.828\, Q v^{1/2} w^{1/4} u^{1/4}$$

(b) if $v = 2$, $w = 16$, $u = 1$, then

$$TC = 2.828\, Q\, (2)^{1/2} (16)^{1/4} (1)^{1/4} = 8Q$$

$AC = TC/Q = 8$ $\qquad MC = \frac{dTC}{dQ} = 8$





⑥ (a) $TC = 1200 + .5Q^2$

$MC = Q$

$P(Q) = 300 - Q$

$TR = 300Q - Q^2$

$MR = 300 - 2Q$

$MR = MC \Rightarrow 300 - 2Q = Q$

so $Q = 100$ and $P = 200$

$\pi = 20{,}000 - 1200 - .5(10{,}000)$

$\pi = \$13{,}800$

(b) inverse elasticity rule: $\frac{P - MC}{P} = -\frac{1}{\varepsilon}$

$\varepsilon = \frac{dQ}{dP} \cdot \frac{P}{Q} = (-1)\left(\frac{200}{100}\right) = -2$

$-\frac{1}{\varepsilon} = -\frac{1}{-2} = \frac{1}{2}$

$\frac{P - MC}{P} = \frac{200 - 100}{200} = \frac{1}{2}$

so IER holds.