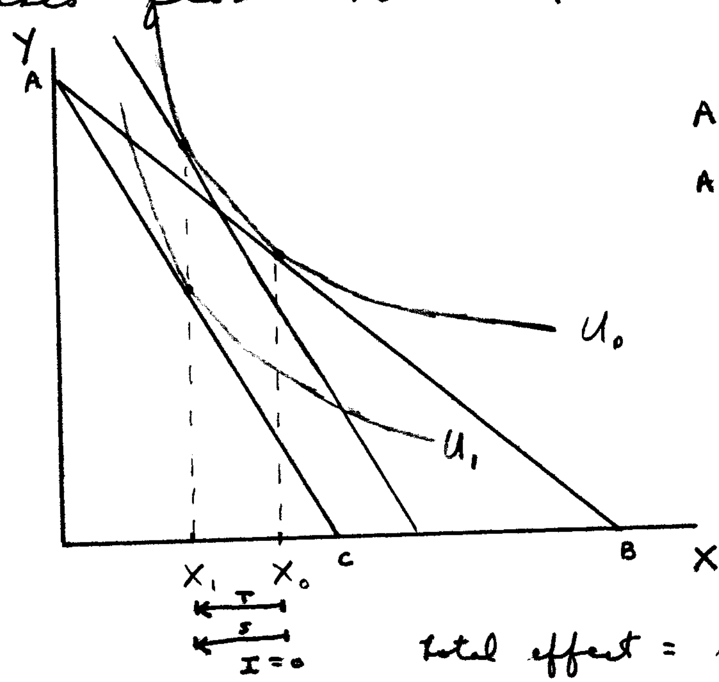
ECO 601 1st Test KEY Fall 1998

1. (a) $D_x = h_x$ between P_0 and P_1, hence $\frac{dX}{dI} = 0$ or there is no income effect, only a substitution effect when the price of X rises from P_0 to P_1:

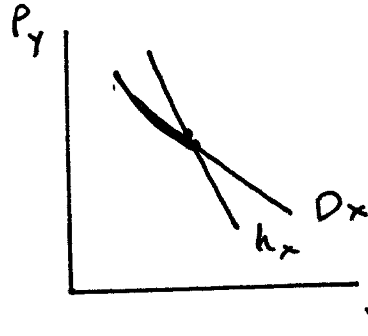


AB = original budget line

AC = new budget line

total effect = substitution effect because income effect = 0

(b) In a two good world, if $\frac{dX}{dI} = 0$ then $\frac{dY}{dI} > 0$, so Y is a normal good. For Y then:



Marshallian demand for good Y is more elastic than the Hicksian demand.

2. <u>Janis</u>: $U = X^{1/4} Y^{3/4} + 8$

$$\mathcal{L} = X^{1/4} Y^{3/4} + 8 + \lambda (I - P_x X - P_Y Y)$$

(1) $\frac{\partial \mathcal{L}}{\partial X} = \frac{1}{4} X^{-3/4} Y^{3/4} - \lambda P_x = 0$

(2) $\frac{\partial \mathcal{L}}{\partial Y} = \frac{3}{4} X^{1/4} Y^{-1/4} - \lambda P_Y = 0$

(3) $\frac{\partial \mathcal{L}}{\partial \lambda} = I - P_x X - P_Y Y = 0$

combining (1) and (2):

$$\frac{\frac{1}{4} X^{-3/4} Y^{3/4}}{\frac{3}{4} X^{1/4} Y^{-1/4}} = \frac{\lambda P_x}{\lambda P_Y}$$

or $\frac{Y}{3X} = \frac{P_x}{P_Y}$, so $Y = \frac{3 X P_x}{P_Y}$

substituting into budget constraint:

$$I - P_x X - P_Y 3 X P_x / P_Y = 0$$

$$I - 4 X P_x = 0$$

so $X^* = \frac{I}{4 P_x}$ and $Y^* = \frac{3 I}{4 P_Y}$

<u>Jim</u>: $U = .5 X^{.5} Y^{1.5}$

$$\mathcal{L} = \tfrac{1}{2} X^{1/2} Y^{3/2} + \lambda (I - P_x X - P_y Y)$$

(1) $\frac{\partial \mathcal{L}}{\partial X} = \frac{1}{4} X^{-1/2} Y^{3/2} - \lambda P_x = 0$

(2) $\frac{\partial \mathcal{L}}{\partial Y} = 3/4\, X^{1/2} Y^{1/2} - \lambda P_Y = 0$

(3) $\frac{\partial \mathcal{L}}{\partial \lambda} = I - P_x X - P_Y Y = 0$

combining (1) and (2):

$$\frac{\frac{1}{4} X^{-1/2} Y^{3/2}}{\frac{3}{4} X^{1/2} Y^{1/2}} = \frac{\lambda P_x}{\lambda P_Y}$$

or $\frac{Y}{3x} = \frac{P_x}{P_Y}$, which is the same condition as for Janis. Thus Jim's Marshallian demand functions for X and Y are identical to Janis's.

3. (a) sum of the budget-share-weighted income elasticities:

$$s_X \, \varepsilon_{X,I} + s_Y \, \varepsilon_{Y,I} = 1$$

$$s_X > 0 \, , \quad s_Y > 0 \, , \quad \varepsilon_{X,I} < 0$$

$\therefore$ $\varepsilon_{Y,I}$ must exceed 1, so the statement is true.

(b) $$\varepsilon_{X,P_X} + \varepsilon_{X,P_Y} + \varepsilon_{X,I} = 0$$

(sum of all price and income elasticities for a single good)

$$\varepsilon_{X,I} > 1 \, , \quad \varepsilon_{X,P_Y} > 0$$

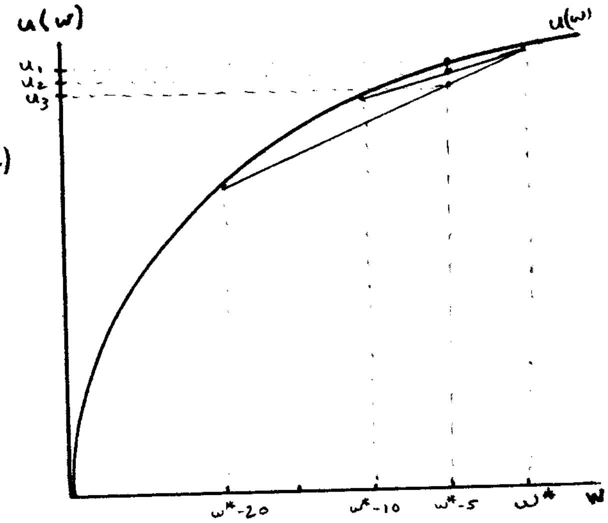
$\therefore$ ε_{X,P_X} must be less than -1, i.e. demand for X is elastic, <u>not</u> inelastic, so the statement is false.

(c) Slutsky eq. in elasticity form:

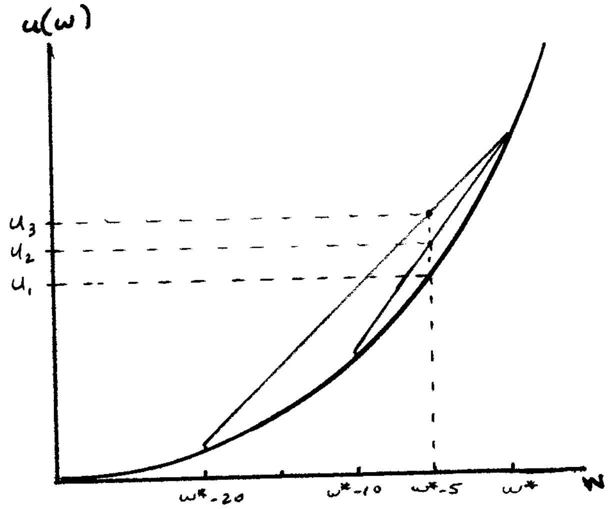
$$\varepsilon_{X,P_X} = \varepsilon^{S}_{X,P_X} - s_X \, \varepsilon_{X,I}$$

ε^{S}_{X,P_X} is the same across countries. $0 < \varepsilon_{X,I} < 1$, hence s_X must be larger in poor countries than in rich countries. Thus we would expect ε_{X,P_X} to be bigger in absolute magnitude (more elastic) where s_X is bigger. Thus the statement is true.

4. (a)



law-abiding students:
risk averse



scofflaws:
risk seekers

w^* = wealth level if you don't put \$5 in the parking meter and don't get caught.

w^*-5 = wealth level if you put \$5 in the meter.

w^*-10 = wealth level if you gamble by not putting money in the meter and lose, having to pay a \$10 fine.

u_1 = certain utility if rental fee (\$5) is paid

u_2 = expected utility if parked illegally in (a)

u_3 = expected utility if parked illegally in (b)

(b) Scofflaws (risk seekers) will like this gamble even more than the one in (a), since $u_3 > u_2$ for them. They will be even more likely to park illegally.

5. $U = X^{1/4} Y^{3/4}$

$$x^* = I/4P_x$$

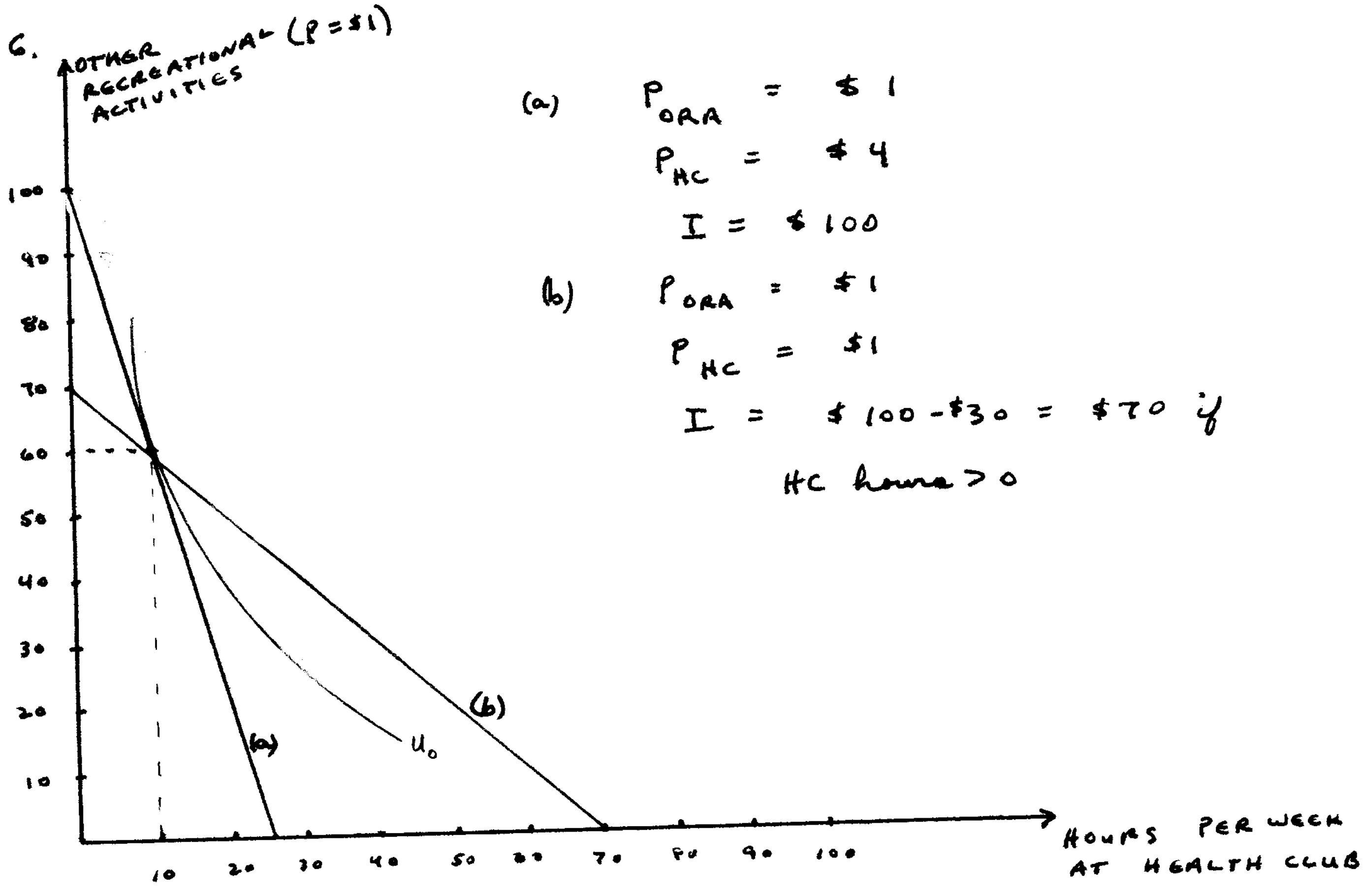
$$\varepsilon_{x,I} = \frac{dx}{dI} \cdot \frac{I}{x}$$

$$\varepsilon_{x,I} = \frac{1}{4P_x} \cdot \frac{I}{I/4P_x} = 1$$

$$\varepsilon_{x,P_Y} = \frac{dx}{dP_Y} \cdot \frac{P_Y}{X}$$

but $\frac{dx}{dP_Y} = 0$, so $\varepsilon_{x,P_Y} = 0$

6.



(a) $P_{ORA} = \$1$
$P_{HC} = \$4$
$I = \$100$

(b) $P_{ORA} = \$1$
$P_{HC} = \$1$
$I = \$100 - \$30 = \$70$ if HC hours > 0

Since the original bundle can be purchased under the new pricing scheme, Mick can clearly reach a higher indifference curve than U_0.

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1. $U = x^2 y^3$

$f_1 = \frac{du}{dx} = 2xy^3$

$f_2 = \frac{du}{dy} = 3x^2y^2$

$f_{11} = \frac{d^2u}{dx^2} = 2y^3$

$f_{22} = \frac{d^2u}{dy^2} = 6x^2y$

$f_{12} = f_{21} = \frac{d^2u}{dx\,dy} = 6xy^2$

does $f_2^2 f_{11} - 2f_1f_2f_{12} + f_1^2 f_{22} < 0$?

$= (3x^2y^2)^2\, 2y^3 - 2(2xy^3)(3x^2y^2)(6xy^2) + (2xy^3)^2\, 6x^2y$

$= 18x^4y^7 - 72x^4y^7 + 24x^4y^7$

$= 30x^4y^7 < 0$

so diminishing MRS.

2. Slutsky equation in elasticities:

$\varepsilon_{x,p_x} = \varepsilon^s_{x,p_x} - s_x\,\varepsilon_{x,I}$

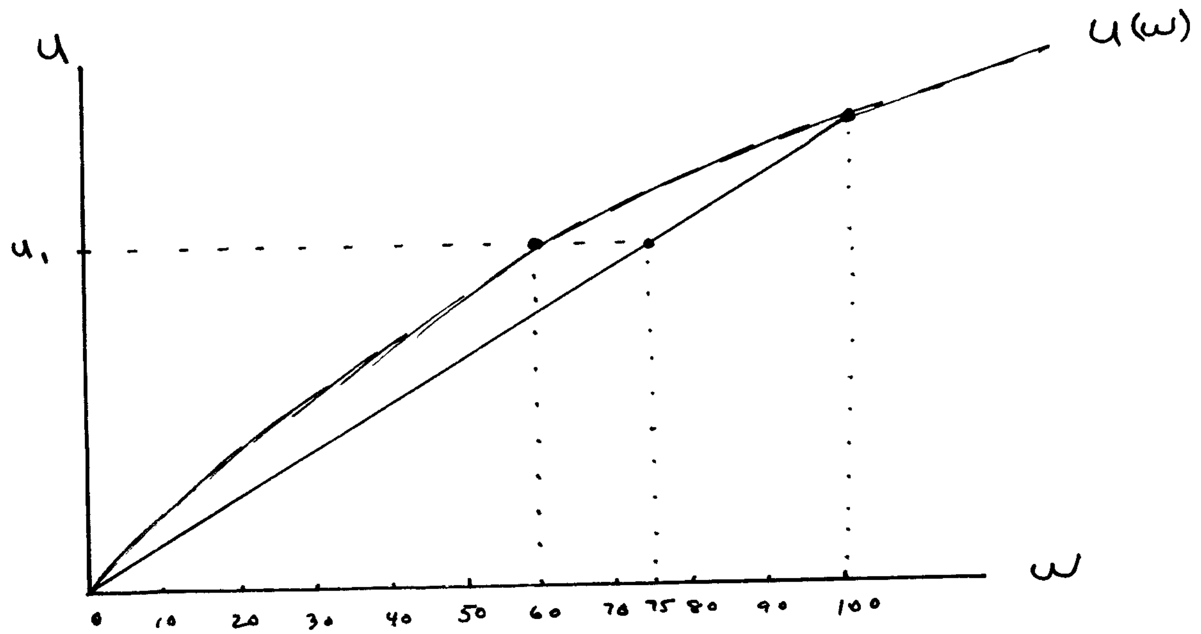
$\varepsilon^s_{x,p_x} < 0$

$\varepsilon_{x,I} < 0$ for inferior good

Since $s_x > 0$, the income effect $(-s_x\,\varepsilon_{x,I})$ works counter to the substitution effect for an inferior good. The bigger s_x is, the less elastic is demand, so the statement is FALSE.

3.

(a)

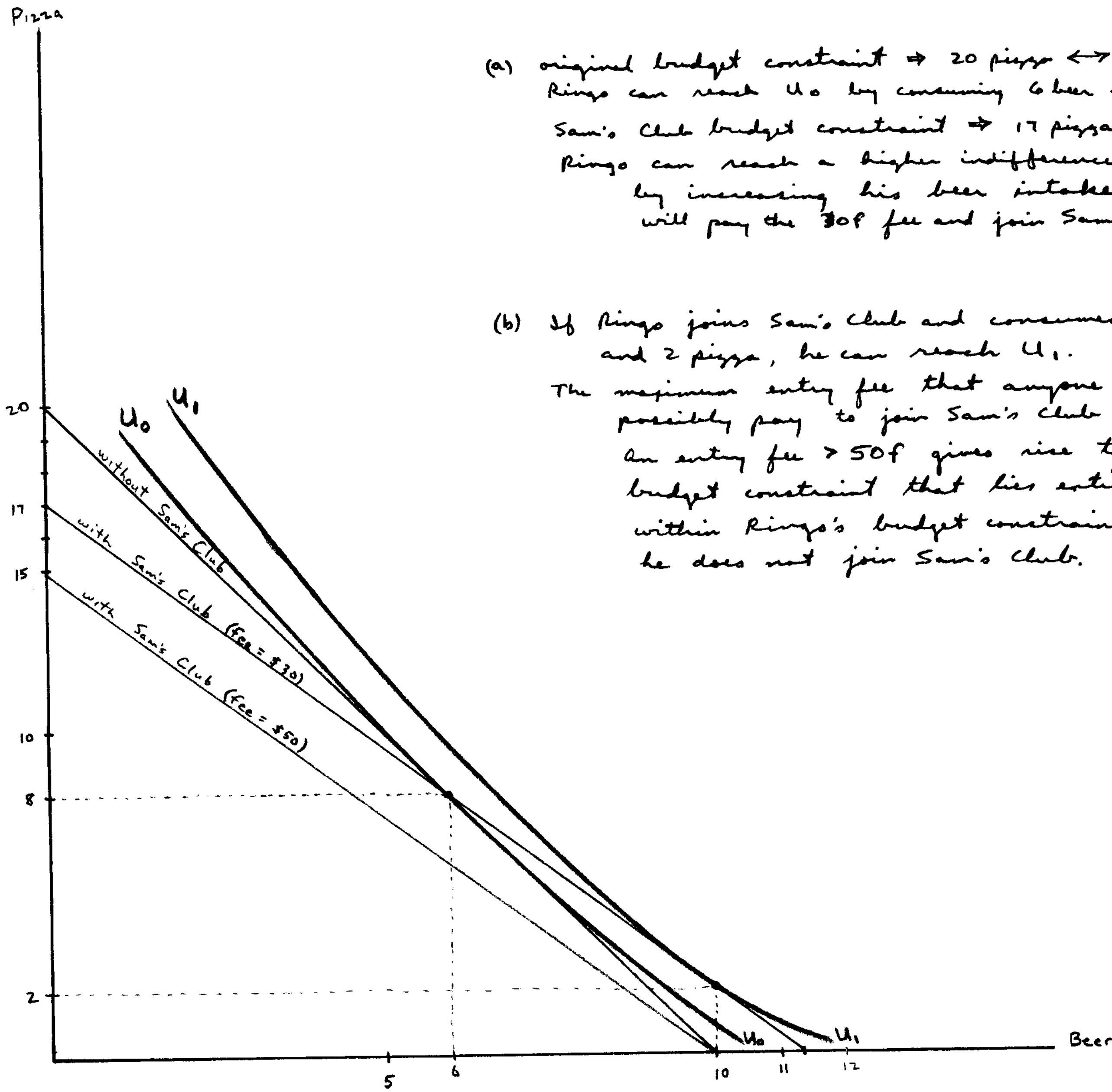


u_1 = utility of \$60 with certainty

u_1 = expected utility of gamble involving \$0 with probability 1/4 and \$100 with probability 3/4 ~~Ein~~.

Since the expected payoff from the gamble, \$75, exceeds the amount offered with certainty, \$60, the person is risk averse.

(b) The gamble, \$20 with probability 1/4 and \$60 with probability 3/4, has an expected payoff of \$50. A risk averse person will prefer \$50 with certainty to a gamble that has the same expected payoff.



(a) original budget constraint ⇒ 20 pizza ⟷ 10 beer
Ringo can reach U_0 by consuming 6 beer + 8 pizza.
Sam's Club budget constraint ⇒ 17 pizza ⟷ 11.33 beer
Ringo can reach a higher indifference curve by increasing his beer intake, so he will pay the 30¢ fee and join Sam's Club.

(b) If Ringo joins Sam's Club and consumes 10 beer and 2 pizza, he can reach U_1.
The maximum entry fee that anyone might possibly pay to join Sam's Club is 50¢.
An entry fee > 50¢ gives rise to a budget constraint that lies entirely within Ringo's budget constraint when he does not join Sam's Club.

5. $U = \ln(X) + Y$

(a) $\mathcal{L} = \ln(X) + Y + \lambda(I - P_X X - P_Y Y)$

(1) $\dfrac{d\mathcal{L}}{dX} = \dfrac{1}{X} - \lambda P_X = 0$

(2) $\dfrac{d\mathcal{L}}{dY} = 1 - \lambda P_Y = 0$

(3) $\dfrac{d\mathcal{L}}{d\lambda} = I - P_X X - P_Y Y = 0$

combining (1) and (2) : $\dfrac{1}{X} = \dfrac{P_X}{P_Y}$, or $P_X X = P_Y$

solve for X : $X^* = \dfrac{P_Y}{P_X}$

substitute $P_X X = P_Y$ into (3)

$I - P_Y - P_Y Y = 0$, $I - P_Y = P_Y Y$,

solve for Y : $Y^* = \dfrac{I - P_Y}{P_Y} = \dfrac{I}{P_Y} - 1$

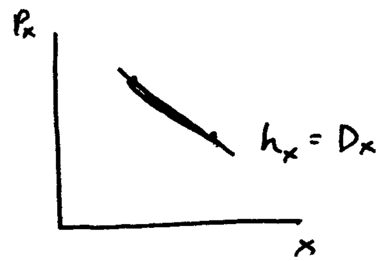
(b) $X^* = \dfrac{P_Y}{P_X}$ $Y^* = \dfrac{I}{P_Y} - 1$

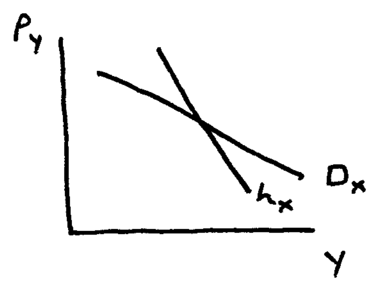
$\varepsilon_{X,I} = \dfrac{dX}{dI} \cdot \dfrac{I}{X} = 0$

$\varepsilon_{Y,I} = \dfrac{dY}{dI} \cdot \dfrac{I}{Y} = \dfrac{1}{P_Y} \cdot \dfrac{I}{Y} = \dfrac{I}{P_Y \cdot Y} > 0$

so Y is a normal good

(c)





(d) $\varepsilon_{X,P_Y} = \dfrac{dX}{dP_Y} \cdot \dfrac{P_Y}{X} = \dfrac{1}{P_X} \cdot \dfrac{P_Y}{X} = \dfrac{P_Y}{P_X \cdot X} = 1$

$\varepsilon_{Y,P_X} = \dfrac{dY}{dP_X} \cdot \dfrac{P_X}{Y} = 0$