Problem Set #1 **Answer Key**
Eco 601
Fall 2002

2.1 a. $f(x) = 4x^3 - 12x, \quad f'(x) = 12x^2 - 12,$
$x^2 - 1 = 0 \quad x = \pm 1$
For $x = 1$, $f(x) = -8$ local minimum since $f''(x) = 24x > 0$

For $x = -1$ $f(x) = +8$ local maximum since $f''(x) = 24x < 0$

b. $f(x) = 4x - x^2, \quad f'(x) = 4 - 2x,$

$f'(x) = 0$ when $x = 2$, $f''(x) = -2$ so this is a local maximum. Here this is also a global maximum.

c. $f(x) = x^3, f'(x) = 3x^2, f''(x) = 6x$

$f'(x) = 0$ when $x = 0$, $f''(x) = 0$, test fails.

At 0, $f''(x)$ changes sign. Thus it is a point of inflection.

2.3 a. $H = -16t^2 + 40t$

$$\frac{dH}{dt} = -32t + 40 = 0 \text{ when } t = 5/4 \text{ sec.}$$

$$H(5/4) = -16(25/16) + 40(5/4) = -25 + 50 = 25 \text{ ft.}$$

b. $H = -2.75t^2 + 40t$

$$\frac{dH}{dt} = -5.5t + 40$$

$$\frac{dH}{dt} = 0 \text{ when } t = 7.3$$

$$H(7.3) = 145 \text{ feet}$$

c. $\frac{\partial H}{\partial g} = -\frac{1}{2}(t^*)^2$ depends on g because t^* depends on g.

$$-gt + 40 = 0 \qquad t^* = \frac{40}{g}$$

so $\frac{\partial H}{\partial g} = -\frac{800}{g^2}$

2.5 $U = f(x, y) = 4x^2 + 3y^2$

a. $\frac{\partial U}{\partial x} = 8x, \quad \frac{\partial U}{\partial y} = 6y$

b. 8, 12

c. $\frac{\partial U}{\partial x} dx + \frac{\partial U}{\partial y} dy = 8x\,dx + 6y\,dy$

d. $\frac{dy}{dx}$ for $dU = 0 \qquad 8x\,dx + 6y\,dy = 0$

$$\therefore \frac{dy}{dx} = \frac{-8x}{6y} = \frac{-4x}{3y}$$

e. $x = 1, \quad y = 2 \qquad U = 4 \cdot 1 + 3 \cdot 4 = 16$

f. $\frac{dy}{dx} = \frac{-4(1)}{3(2)} = -2/3$

g. $U = 16$ contour line is an ellipse centered at the origin. With equation $4x^2 + 3y^2 = 16$, slope of the line at (x, y) is $\frac{dy}{dx} = -\frac{4x}{3y}$

2.6 Substitution: $f(x, y) = xy \quad x + y = 1 \quad y = 1 - x$

$f = x(1 - x)$

$\frac{\partial f}{\partial x} = 1 - 2x = 0$

Note: $f''(x = ½) = -2.$ . . This is a local and global maximum.

Lagrangian: $\mathcal{L} = xy + \lambda\,(1 - x - y)$

$\frac{\partial \mathcal{L}}{\partial x} = y - \lambda = 0$

$x = y$

$\frac{\partial \mathcal{L}}{\partial y} = x - \lambda = 0$

$-\frac{\partial \mathcal{L}}{\partial \lambda} = x + y - 1 = 0, \qquad x + y = 1$

$x = y = ½ \quad f = ¼$

2.7 a. Profits $= \pi = TR - TC = -2q^2 + 40q - 100$

$$\frac{d\pi}{dq} = -4q + 40 \qquad q^* = 10$$

$$\pi^* = -2(10)^2 + 40(10) - 100 = 100$$

b. $\frac{d^2\pi}{dq^2} = -4$ so profits are maximized

c. $$MR = \frac{dTR}{dQ} = 70 - 2q$$

$$MC = \frac{dTC}{dQ} = 2q + 30$$

so $q^* = 10$ obeys $MR = MC$.