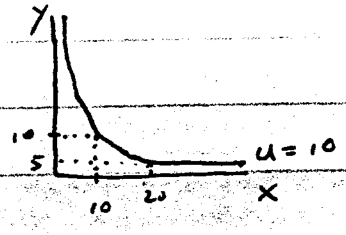
ECO 660 Problem Set #2 Fall 2002

3.2. $U(X, Y) = \sqrt{X \cdot Y}$

(a) if $U = 10 = \sqrt{XY}$, then $XY = 100$ and $Y = \frac{100}{X}$

this is a rectangular hyperbola.



(b) if $X = 5$, $Y = 20$

$Y = \frac{100}{X}$, $\frac{dY}{dX} = -\frac{100}{X^2}$, $MRS = -\frac{dY}{dX} = \frac{100}{X^2} = \frac{100}{25} = 4$

(c) $\frac{\partial U}{\partial X} = \frac{1}{2} X^{-1/2} Y^{1/2} = MU_x$

$\frac{\partial U}{\partial Y} = \frac{1}{2} X^{1/2} Y^{-1/2} = MU_y$

$MRS = \frac{MU_x}{MU_y} = \frac{Y}{X}$

(d) $U' = \log_{10} U = \log_{10}(X^{1/2} Y^{1/2})$

if $U' = 1$, then $\log_{10} U = 1$ and $U = 10$

(recall $\log_{10} 10 = 1$, i.e. $10^1 = 10$)

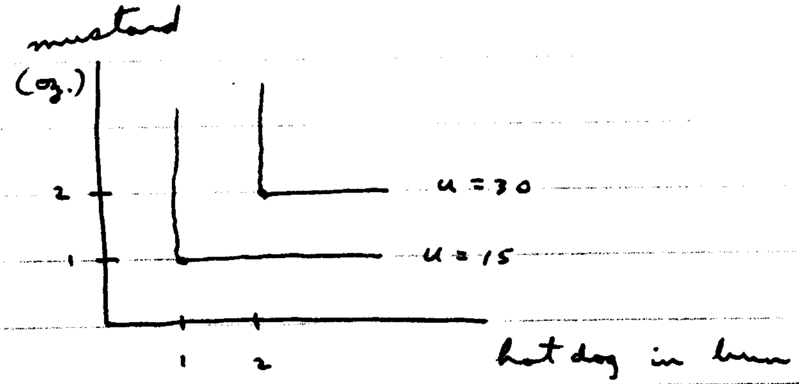
$MU_x' = \frac{dU'}{dX} = \frac{d \log(X^{1/2} Y^{1/2})}{dX} = \frac{\frac{1}{2} X^{-1/2} Y^{1/2}}{X^{1/2} Y^{1/2} \ln 10}$

$MU_y' = \frac{dU'}{dY} = \frac{d \log(X^{1/2} Y^{1/2})}{dY} = \frac{\frac{1}{2} X^{1/2} Y^{-1/2}}{X^{1/2} Y^{1/2} \ln 10}$

$\frac{MU_x'}{MU_y'} = \frac{Y}{X}$ as before

see A.C. Chiang *Fundamental Methods of Mathematical Economics* 3rd ed p 295 for $\log_{10}$ derivative rule.

3.3. (a)

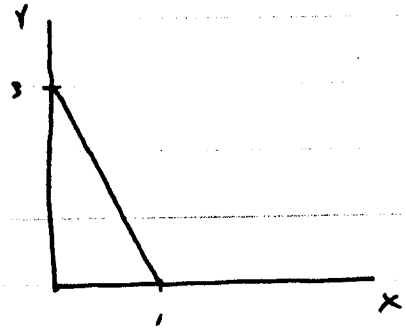


hot dogs, buns, and mustard are perfect complements

(b) 15 units of utility can be had for an expenditure of \$1.50, so $U = 10M$ where M = expenditure on the package of hot dog, bun, and mustard

(c) 15 units of utility per \$2.00 spent, so $U = 7.5M$

3.5 (a)
and 3.6



$$u = 3X + Y$$

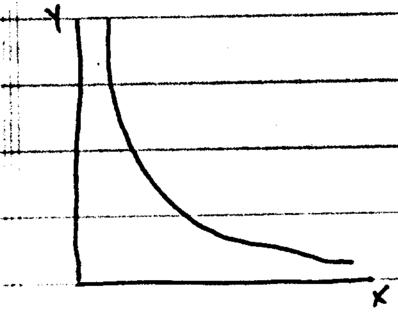
$$f_1 = 3 \quad f_2 = 1$$

$$f_{11} = f_{12} = f_{22} = 0$$

$$\therefore \; f_2^2 f_{11} - 2 f_1 f_2 f_{12} + f_1^2 f_{22} = 0$$

and ICs are not strictly convex

(b)



$$u = \sqrt{X \cdot Y} = X^{1/2} Y^{1/2}$$

$$f_1 = \tfrac{1}{2} X^{-1/2} Y^{1/2} \qquad f_2 = \tfrac{1}{2} X^{1/2} Y^{-1/2}$$

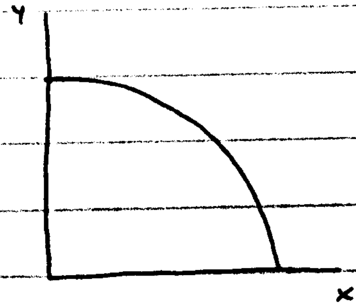
$$f_{11} = -\tfrac{1}{4} X^{-3/2} Y^{1/2} < 0$$

$$f_{22} = -\tfrac{1}{4} X^{1/2} Y^{-3/2} < 0$$

$$f_{12} = \tfrac{1}{4} X^{-1/2} Y^{-1/2} > 0$$

since f_{11} and f_{22} are < 0 and $f_{12} > 0$, $f_2^2 f_{11} - 2 f_1 f_2 f_{12} + f_1^2 f_{22}$ must be negative and the ICs are strictly convex

(c)



$$u = \sqrt{X^2 + Y^2} = (X^2 + Y^2)^{1/2}$$

$$f_1 = \frac{X}{\sqrt{X^2+Y^2}} \qquad f_2 = \frac{Y}{\sqrt{X^2+Y^2}}$$

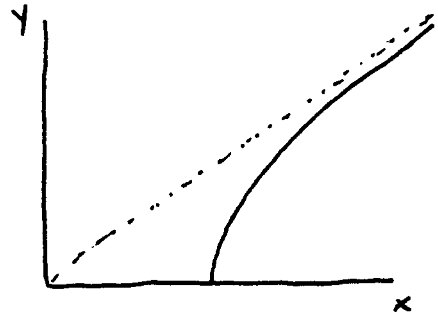
$$f_{11} = Y^2 / (X^2 + Y^2)^{3/2} > 0$$

$$f_{22} = X^2 / (X^2 + Y^2)^{3/2} > 0$$

$$f_{12} = -XY / (X^2 + Y^2)^{3/2} < 0$$

since f_{11} and f_{22} are positive and f_{12} is negative, then $f_2^2 f_{11} - 2 f_1 f_2 f_{12} + f_1^2 f_{22}$ must be positive and the ICs are strictly concave

(d)



$$u = \sqrt{X^2 - Y^2} = (X^2 - Y^2)^{1/2}$$

$$f_1 = X/(X^2 - Y^2)^{1/2}$$

$$f_2 = -Y/(X^2 - Y^2)^{1/2}$$

$$f_{11} = -Y^2/(X^2 - Y^2)^{3/2}$$

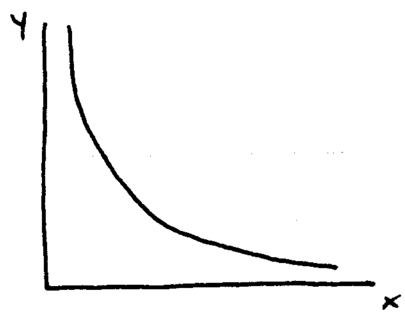
$$f_{22} = -X^2/(X^2 - Y^2)^{3/2}$$

$$f_{12} = XY/(X^2 - Y^2)^{3/2}$$

now evaluating $\underbrace{f_2^2 f_{11}}_{<0} \underbrace{- 2 f_1 f_2 f_{12}}_{>0} + \underbrace{f_1^2 f_{22}}_{<0}$:

so sign is uncertain and we can't conclude anything about convexity of ICs

(e)



$$u = X^{2/3} Y^{1/3}$$

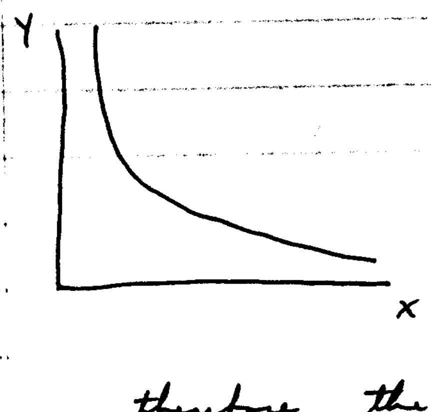
$$f_1 = \tfrac{2}{3} X^{-1/3} Y^{1/3}$$

$$f_2 = \tfrac{1}{3} X^{2/3} Y^{-2/3}$$

and so on similar to (b)

ICs are strictly convex

(f)



$u = \log X + \log Y$

$u = \log (XY)$, which is a monotonic transformation of $u = (XY)^{1/2}$ (i.e. square and then take logarithm)

therefore the proof for (b) suffices.

5. From footnote 8 on p 79 of Nicholson:

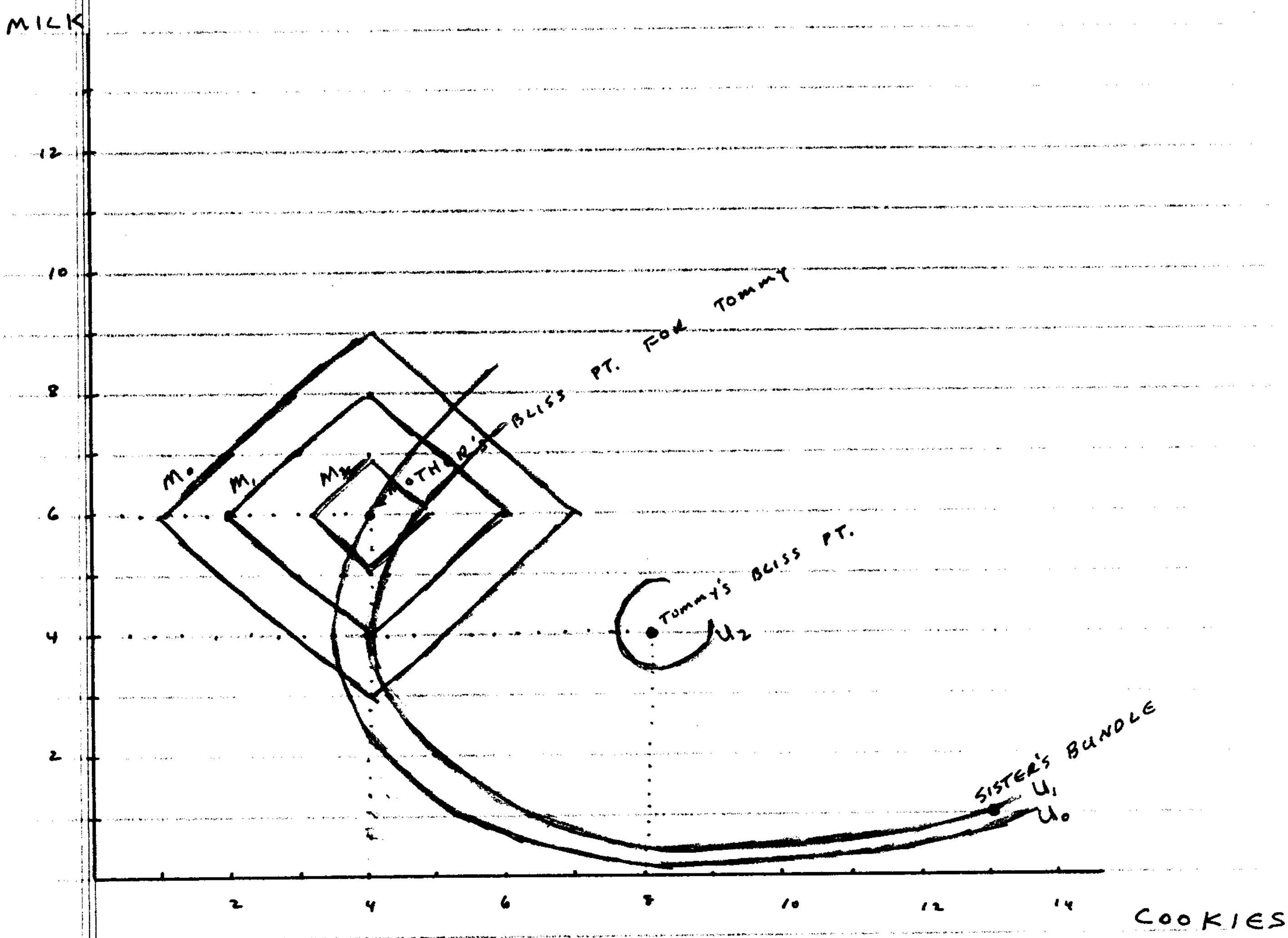
Both X and Y may exhibit diminishing marginal utility, i.e., $\partial^2 U / \partial X^2 < 0$ and $\partial^2 U / \partial Y^2 < 0$, yet indifference curves may be concave. That will happen if

$$f_2^2 f_{11} - 2 f_1 f_2 f_{12} + f_1^2 f_{22} > 0 .$$

If the effect of additional Y on the Marginal Utility of X, i.e. f_{12}, is strongly negative and overwhelms the direct effects f_{11} and f_{22}, then indifference curves may be concave.

so <u>TRUE</u>.

6.


MILK
12
10
8
6
4
2
2
4
6
8
10
12
14
COOKIES
M_0
M_1
M_2
MOTHER'S BLISS PT. FOR TOMMY
TOMMY'S BLISS PT.
U_2
SISTER'S BUNDLE
U_1
U_0

(b) $D = |6-m| + |4-c|$

7. (a) $U = x_1^2 + 2x_1 x_2 + x_2^2$

$$dU = \frac{dU}{dx_1}\,dx_1 + \frac{dU}{dx_2}\,dx_2$$

$$0 = (2x_1 + 2x_2)\,dx_1 + (2x_1 + 2x_2)\,dx_2$$

$$\frac{dx_2}{dx_1} = -\frac{(2x_1 + 2x_2)}{(2x_1 + 2x_2)} = -1$$

(b) $V = x_2 + x_1$

$$dV = \frac{dV}{dx_1}\,dx_1 + \frac{dV}{dx_2}\,dx_2$$

$$0 = 1\,dx_1 + 1\,dx_2$$

$$\frac{dx_2}{dx_1} = -1$$

(c) Yes, the indifference curves are identical because U is a monotonic transformation of V: $U = V^2$