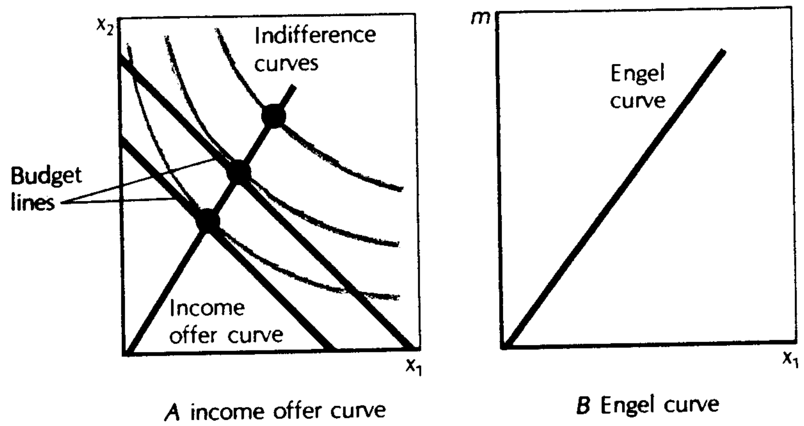
Eco 601 Problem Set #4 KEY Fall 2002

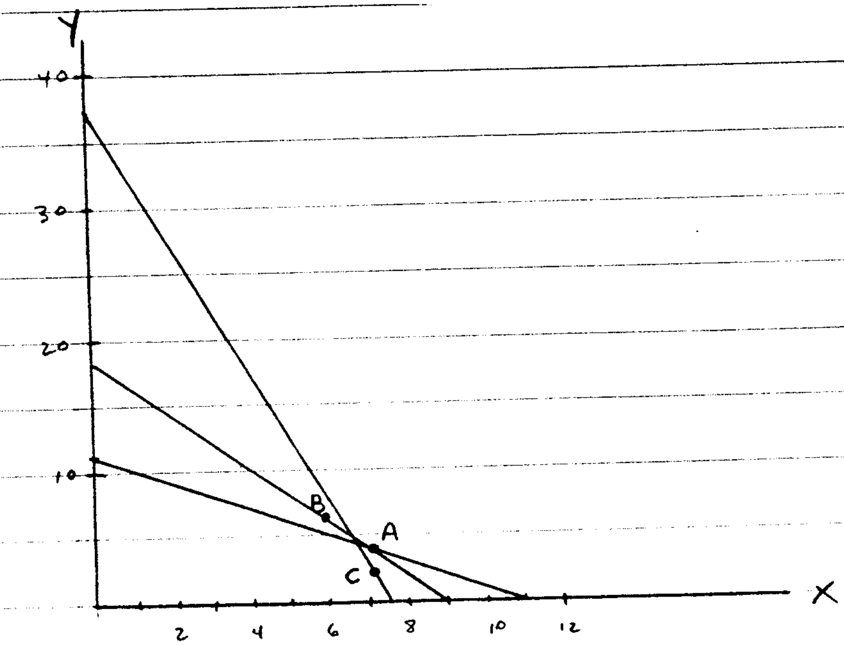
5.5 a. As income increases, the ratio P_X/P_Y stays constant, and the utility-maximization conditions also require that *MRS* stay constant. Thus, if *MRS* depends on the ratio X_2/X_1, this ratio must stay constant as income increases. Therefore, since income is spent only on X_1 and X_2, X_1 and X_2 are proportional to income. Thus, Engel curves are straight lines.

b. Because of part (a), $\dfrac{\partial X}{\partial I} > 0$. Therefore, Giffen's paradox cannot arise.

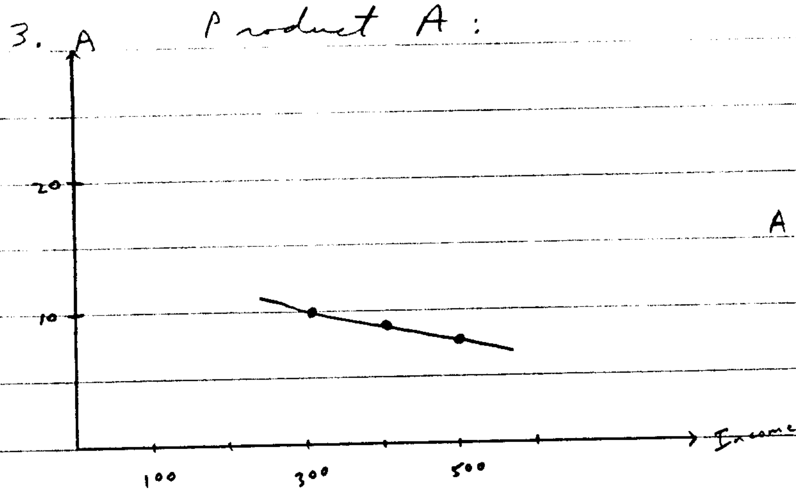


Homothetic preferences. An income offer curve (A) and an Engel curve (B) in the case of homothetic preferences.

5.9 Year 2's bundle is revealed preferred to Year 1's since both cost the same in Year 2's prices. Year 2's bundle is also revealed preferred to Year 3's for the same reason. But in Year 3, Year 2's bundle costs less than Year 3's but is not chosen. Hence, these violate the axiom.

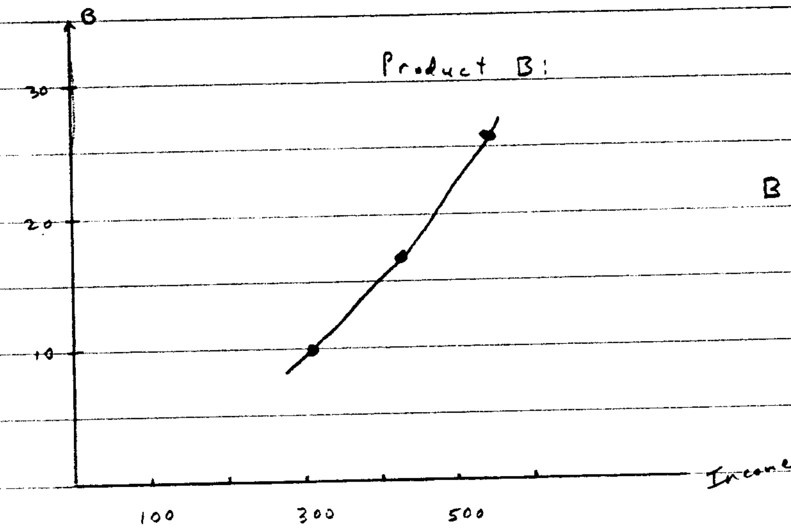


3. Product A:



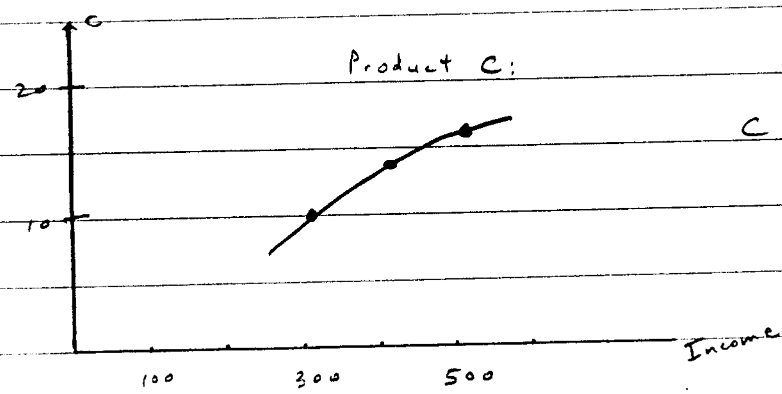
A is inferior

Product B:



B is a luxury good

Product C:



C is a necessity

4. $$U(X, Y) = (X-20)^{1/2}(Y-10)^{1/2}$$

$$\mathcal{L} = (X-20)^{1/2}(Y-10)^{1/2} + \lambda(I - P_x X - P_y Y)$$

(1) $$\frac{d\mathcal{L}}{dX} = \frac{1}{2}(X-20)^{-1/2}(Y-10)^{1/2} - \lambda P_x = 0$$

(2) $$\frac{d\mathcal{L}}{dY} = \frac{1}{2}(X-20)^{1/2}(Y-10)^{-1/2} - \lambda P_y = 0$$

(3) $$\frac{d\mathcal{L}}{d\lambda} = I - P_x X - P_y Y = 0$$

(a) (1) and (2) imply $$\frac{(X-20)^{-1/2}(Y-10)^{1/2}}{(X-20)^{1/2}(Y-10)^{-1/2}} = \frac{P_x}{P_y}$$

or $\dfrac{Y-10}{X-20} = \dfrac{P_x}{P_y}$, or $Y = 10 + \dfrac{(X-20)P_x}{P_y}$

substituting into budget constraint:

$$I - P_x X - P_y\left(10 + \frac{(X-20)P_x}{P_y}\right) = 0$$

solving for X and Y:

$$X^* = \frac{I - 10P_y + 20P_x}{2P_x}$$

$$Y^* = \frac{I + 10P_y - 20P_x}{2P_y}$$

(b) $P_x = 1$, $P_y = 2$, $I = 100$, then $X^* = 50$ and $Y^* = 25$

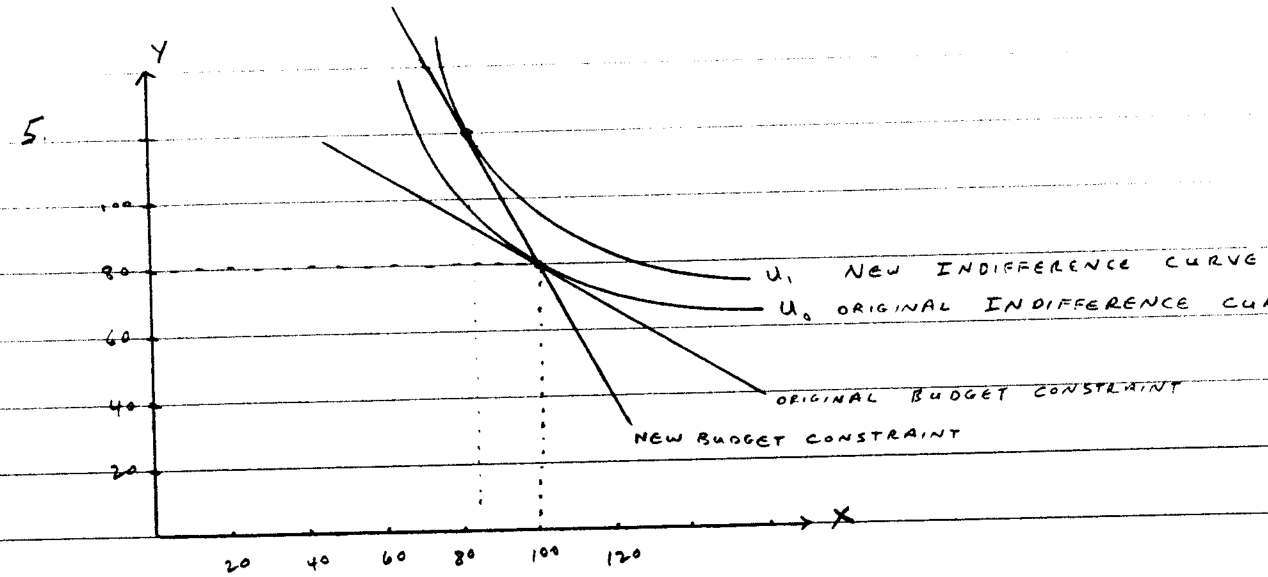
(c) $\dfrac{dX}{dI} = \dfrac{1}{2P_x} > 0$, so X is a normal good

$$\frac{dX}{dP_x} = \frac{20(2P_x) - 2(I - 10P_y + 20P_x)}{4P_x^2}$$

$$= \frac{10P_y - I}{2P_x}$$

$$\frac{dX}{dP_y} = -\frac{5}{P_x}$$

5.



$$\text{Expenditures} = P_x \cdot 100 + P_y \cdot 80$$

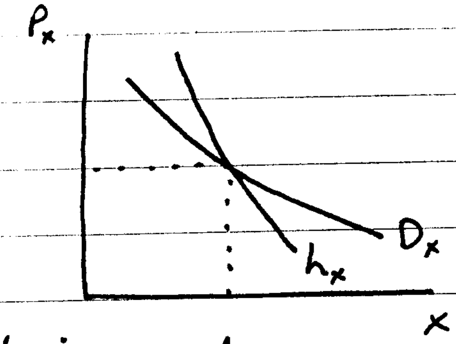
What happens when P_x rises and P_y falls:

$100 \cdot \Delta P_x = 100 \cdot \$.40 = \$40.00 \quad (+)$

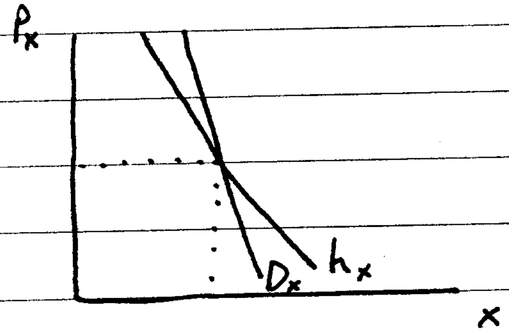
$80 \cdot \Delta P_y = 80 \cdot \$.50 = \$40.00 \quad (-)$

The consumer could buy their original bundle of 100 X and 80 Y after the price changes. In other words, their new budget line passes through the original bundle. The new budget line does have a steeper slope than the original. Thus there will be a substitution effect but no income effect. The consumer will buy less X, regardless of its income elasticity.

6. Normal good:



inferior good:



so UNCERTAIN. It depends on whether the good is normal or inferior.