KEY

ECO 601
Fall 2002
Problem Set #7

DUE: Monday, November 4

1. Nicholson 11.2

2. TFUE: The production function in the above problem has no region I or region III.

3. Nicholson 11.3

4. TFUE: The production function Q=min(aK,bL) has an elasticity of substitution equal to 0, but the combination of several similar production functions with different values for a and b can yield a positive elasticity of substitution.

5. What if the production function for secondary education (E) is: $E=.5T^{.7}B^{.4}$, where T is teachers and B is buildings and materials.
a) Find the marginal product of T and the marginal product of B.
b) Does the production function exhibit diminishing marginal productivity of inputs? Explain.
c) What is the marginal rate of technical substitution for this production function? Is the function homothetic?
d) Does the production function exhibit diminishing MRTS? Explain.
e) Find the output elasticity of education with respect to teachers.
f) What returns to scale does this production function exhibit? Tell a short,short story why the production might exhibit these returns to scale.

1.

11.2 $q = KL - .8K^2 - .2L^2$

a. **When** $K = 10,\ q = 10L - 80 - .2L^2$.

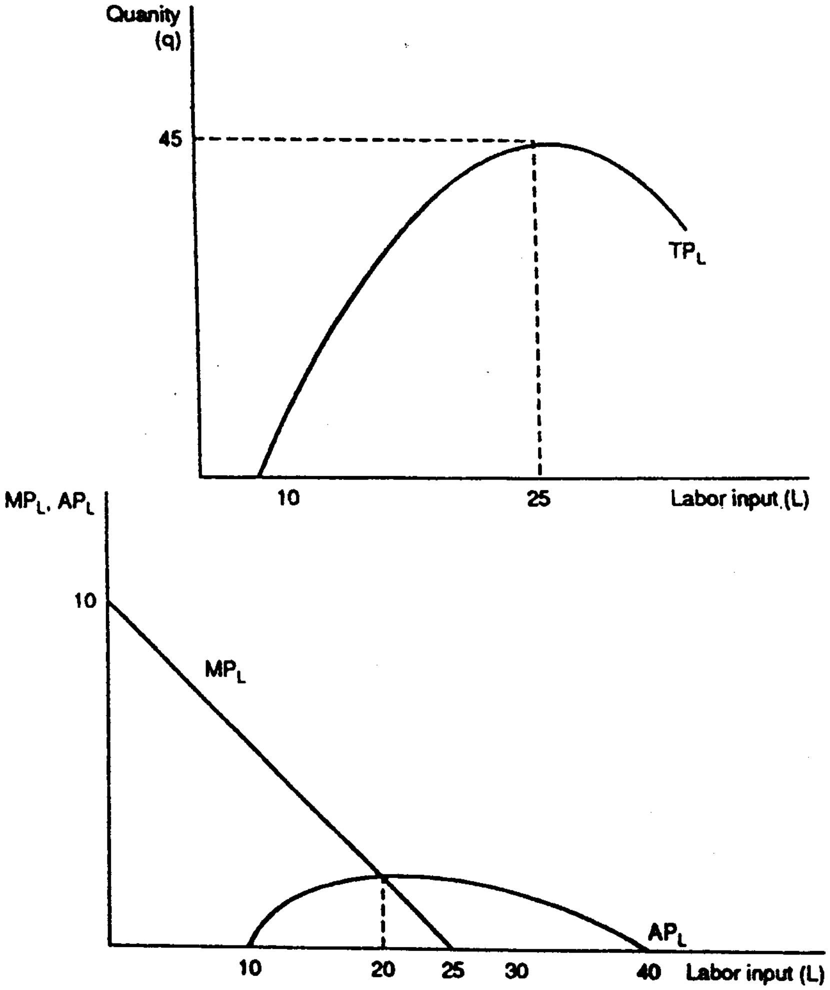
To graph: $\dfrac{dq}{dL} = 10 - .4L = 0$, **maximum at** $L = 25$

$\dfrac{d^2q}{dL} = -.4$, $\therefore$ **Total product curve is concave.**

$AP_L = q/L = 10 - 80/L - .2L$

To graph: $\dfrac{dAP_L}{dL} = \dfrac{80}{L} - .2 = 0$, **maximum at** $L = 20$.

When $L = 20,\ q = 40,\ AP_L = 0$ **where** $L = 10, 40$.



b. $MP_L = 10 - .4L, \quad 10 - .4L = 0, \quad L = 25$

See above graph.

c. $K = 20 \quad q = 20L - 320 - .2L^2$

$AP_L = 20 - \dfrac{320}{L} - .2L$; reaches max. at $L = 40$, $q = 160$

$MP_L = 20 - .4L$, $= 0$ at $L = 50$.

d. Doubling of K and L here multiplies output by 4 (compare a and c). Hence the function exhibits increasing returns to scale.

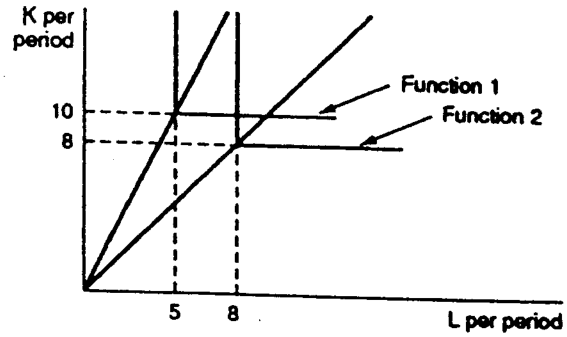
2. $MP_L > AP_L$ in region I.

$MP_L < 0$ in region III.

The above production function has both region I and region III, so the statement is <u>FALSE</u>.

3.

11.3 a., b.



function 1: use $10K$, $5L$

function 2: use $8K$, $8L$

c. Function 1:

$$
\begin{aligned}
2K + L &= 8{,}000 \\
2.5(2K + L) &= 20{,}000 \\
5.0K + 2.5L &= 20{,}000
\end{aligned}
$$

Function 2:

$$
\begin{aligned}
K + L &= 5{,}000 \\
4(K + L) &= 20{,}000 \\
4K + 4L &= 20{,}000
\end{aligned}
$$

Thus, $9.0K$, $6.5L$ is on the 40,000 isoquant

Function 1:

$$
\begin{aligned}
3.75(2K + L) &= 30{,}000 \\
7.50K + 3.75L &= 30{,}000
\end{aligned}
$$

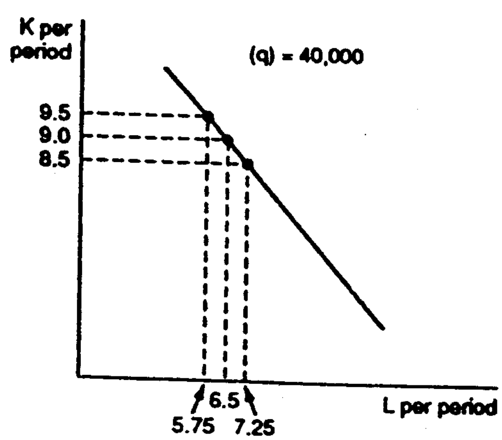
Function 2:

$$
\begin{aligned}
2(K + L) &= 10{,}000 \\
2K + 2L &= 10{,}000
\end{aligned}
$$

Thus, $9.5K$, $5.75L$ is on the 40,000 isoquant

Functions of K and L represent fractions of hours using whole units of capital and labor.

d.



4. TRUE. See problem 3 above.

5. (a) $MP_T = .35\ T^{-.3} B^{.4}$

$MP_B = .2\ T^{.7} B^{-.6}$

(b) $\frac{\partial MP_T}{\partial T} = -.105\ T^{-1.3} B^{.4} < 0$

$\frac{\partial MP_B}{\partial B} = -.12\ T^{.7} B^{-1.6} < 0$

so yes, diminishing marginal productivity

(c) $MRTS = MP_T / MP_B = \frac{.35 T^{-.3} B^{.4}}{.2 T^{.7} B^{-.6}} = 1.75\ B/T$

yes, it is homothetic since MRTS depends only on B/T and not on scale.

(d) diminishing MRTS if $f_B^2 f_{TT} - 2 f_B f_T f_{BT} + f_T^2 f_{BB} < 0$

$f_B^2 f_{TT} \Rightarrow (.04\ T^{1.4} B^{-1.2})(-.105\ T^{-1.3} B^{.4})$

$-2 f_B f_T f_{BT} \Rightarrow -2(.2T^{.7} B^{-.6})(.35\ T^{-.3} B^{.4})(.14\ T^{-.3} B^{-.6})$

$f_T^2 f_{BB} \Rightarrow (.1225\ T^{-.6} B^{.8})(-.12\ T^{.7} B^{-1.6})$

since all three terms are negative, the entire expression will be negative, so MRTS is diminishing.

(e) $\ln E = \ln .5 + .7 \ln T + .4 \ln B$

$\frac{\partial \ln E}{\partial \ln T} = .7 \qquad \frac{\partial \ln E}{\partial \ln B} = .4$

(f) $.7 + .4 = 1.1$, so increasing returns to scale. Perhaps there are some mass production techniques that can be used as the size (scale) of the school increases.