Eco 601 Problem Set #8 Key Fall 2002

1. (a) $Q = L^{1/3} K^{1/3} E^{1/3}$ $TC = wL + vK + uE$

$$\mathcal{L} = wL + vK + uE + \lambda\left(Q_0 - L^{1/3} K^{1/3} E^{1/3}\right)$$

1st order conditions for cost minimum:

(1) $$\frac{\partial \mathcal{L}}{\partial L} = w - \frac{1}{3}\lambda L^{-2/3} K^{1/3} E^{1/3} = 0$$

(2) $$\frac{\partial \mathcal{L}}{\partial K} = v - \frac{1}{3}\lambda L^{1/3} K^{-2/3} E^{1/3} = 0$$

(3) $$\frac{\partial \mathcal{L}}{\partial E} = u - \frac{1}{3}\lambda L^{1/3} K^{1/3} E^{-2/3} = 0$$

combining (1) and (2)

$$\frac{w}{v} = \frac{L^{-2/3} K^{1/3} E^{1/3}}{L^{1/3} K^{-2/3} E^{1/3}} = \frac{K}{L} \Rightarrow K = \frac{w}{v} L$$

combining (1) and (3):

$$\frac{w}{u} = \frac{L^{-2/3} K^{1/3} E^{1/3}}{L^{1/3} K^{1/3} E^{-2/3}} = \frac{E}{L} \Rightarrow E = \frac{w}{u} L$$

plugging into production function constraint:

$$Q = L^{1/3}\left(\frac{w}{v}L\right)^{1/3}\left(\frac{w}{u}L\right)^{1/3}$$

$Q = w^{2/3} v^{-1/3} u^{-1/3} L$, so $L = Q v^{1/3} u^{1/3} w^{-2/3}$

and $K = \frac{w}{v} L = Q v^{-2/3} u^{1/3} w^{1/3}$

and $E = \frac{w}{u} L = Q u^{-2/3} v^{1/3} w^{1/3}$

thus $TC = wL + vK + uE = w\left(Q v^{1/3} u^{1/3} w^{-2/3}\right) + v\left(Q v^{-2/3} u^{1/3} w^{1/3}\right) + u\left(Q u^{-2/3} v^{1/3} w^{1/3}\right)$

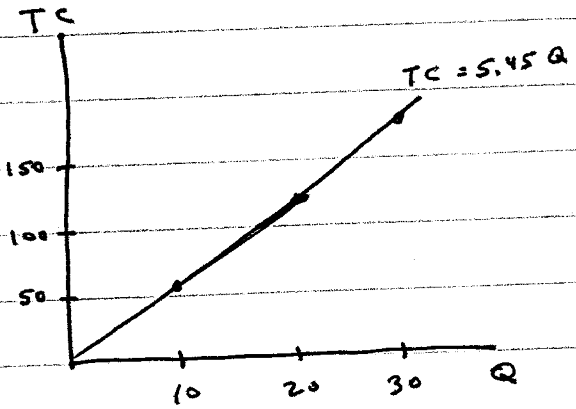
or $TC = 3Q(uvw)^{1/3}$

(b) $TC = 3Q(uvw)^{1/3}$

if $w = 1$, $v = 2$, and $u = 3$

then $TC = 3Q(6)^{1/3}$

or $TC = 5.45Q$



(c) $TC = 3Q(uvw)^{1/3}$

$$MC = \frac{dTC}{dQ} = 3(uvw)^{1/3}$$

$$AC = \frac{TC}{Q} = 3(uvw)^{1/3}$$

2.

12.8 a. $q = 2\sqrt{\overline{K}L}$ so $q^2 = 4\overline{K}L \qquad L = q^2/4\overline{K}$

$TC = v\overline{K} + wL = v\overline{K} + wq^2/4\overline{K}$

b. $\dfrac{\partial TC}{\partial K} = v - wq^2/4\overline{K}^2 = 0$ so $\overline{K} = \dfrac{q}{2}w^{½}v^{-½}$

c. $TC = vK + wL = \dfrac{q}{2}w^{½}v^{½} + \dfrac{q}{2}w^{½}v^{½} = qw^{½}v^{½}$ (a special case of Problem 12.5)

d. If $w = 4 \quad v = 1, \quad TC = 2q$

$STC = (\overline{K} = 100) = 100 + q^2/100$

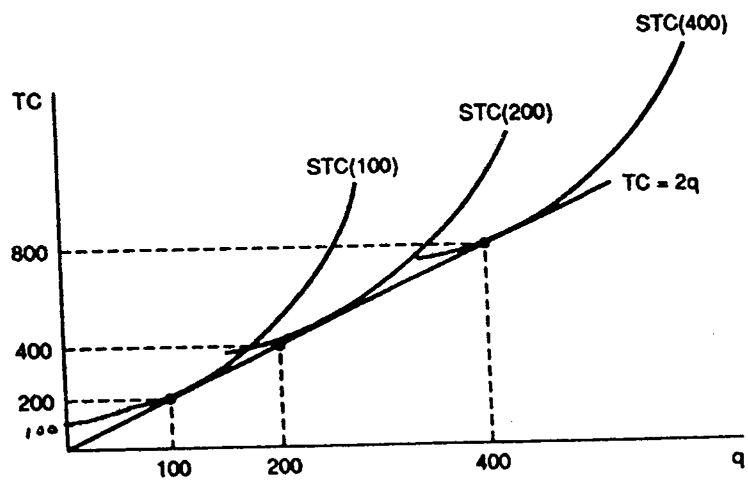
$STC = TC$ for $q = 100$

$STC = (\overline{K} = 200) = 200 + q^2/200$

$STC = TC$ for $q = 200$

$STC = (\overline{K} = 400) = 400 + q^2/400$

$STC = TC$ for $q = 400$



3. Uncertain. It depends on whether or not the firm's desired level of output changes, and on whether its production function is homothetic. If the optimal output remains the same then the firm will use the same input mix and total cost will be 30% higher. If Q changes and the production function in homothetic, then K/L will stay the same since w/v has not changed. If the production is not homothetic then K/L will change as the firm adjusts its output.

4. False. All cost functions are homogeneous of degree one in input prices. Only constant-returns-to-scale production functions are homogeneous of degree one in input employments.

5. True. Isoquants are radial blow-ups of one another if the production function is homothetic, i.e. the MRTS along a ray from the origin is the same at any point along the ray. Therefore the firm's expansion path cannot have backward-bending portions.

6. False. In a random lottery draft, some individuals who are very productive in the private sector but are unproductive (i.e., lousy soldiers) in the military sector would still end up serving in the army. If such individuals were able, they would likely hire other persons whose talents made them relatively productive in the military but unproductive in the private sector. Total output (the sum of military plus private sector) would be greater if hiring substitutes were permitted.

7. 13.6 Total cost $= .25q^2 = .25(q_A + q_L)^2$

$q_A = 100 - 2P_A \qquad q_L = 100 - 4P_L$

$P_A = 50 - q_A/2 \qquad P_L = 25 - q_L/4$

$R_A = P_A q_A = 50q_A - q_A^2/2$

$R_L = P_L q_L = 25q_L - q_L^2/4$

$MR_A = 50 - q_A \qquad MR_L = 25 - q_L/2$

$MC_A = .5(q_A + q_L) \qquad MC_L = .5(q_A + q_L)$

$MR_A = MC_A \qquad MR_L = MC_L$

$50 - q_A = .5q_A + .5q_L \qquad 25 - \frac{q_L}{2} = .5q_A + .5q_L$

$100 - 2q_A = q_A + q_L \qquad 50 - q_L = q_A + q_L$

$100 - 3q_A = q_L$

$50 - (100 - 3q_A) = q_A + 100 - 3q_A \text{ ; } -50 + 3q_A = 100 - 2q_A$

$q_A = 30 \qquad P_A = 35$

$q_L = 10 \qquad P_L = 22.5 \qquad \pi = 1050 + 225 - 400 = 875$