ECO 601
Fall 2002
Test #2

1. (15 pts.) Suppose that Joanna Morganstern has a utility (U) function that can be written as $U = W^{1/2}$. She has a house which may or may not burn down. If there is a fire, her wealth (W) equals \$2,500 and if there is no fire her wealth equals \$40,000. The probability of a fire is 1/10. (a) Find the expected utility of her uncertain wealth. (b) What level of wealth with certainty would yield the same utility as the above gamble? (c) If fire insurance is available for \$1000 plus the actuarially fair premium, would Joanna purchase it? Explain and illustrate your answer with a diagram.

2. (10 pts.) The production function for small plastic gears that go into computer printers is given by $Q = (K^{1/2} + L^{1/2})^2$, where Q denotes units of output, L denotes person-hours, and K denotes machine hours. Does this production function exhibit decreasing, constant, or increasing returns to scale? Provide a brief proof of your answer.

3. (15 pts.) Production of widgets requires the usage of both capital and labor. While a copy of the only known picture of a widget factory can be seen on Professor Garen's door, the mathematical relationship that describes the production process is given by $Q = KL^2 - L^3$. Suppose K is fixed in the short run at K=600. Sketch the total product of labor function, and then in a separate diagram sketch the average product and marginal product curves that go along with it. Be careful to identify the three different regions of production in your diagram, specifying the output levels that define each region.

4. (20 pts.) Going back to the production of plastic gears in printers in question #2 above.
   (a) Write expressions for the marginal products of labor and for capital.
   (b) Suppose w, the wage rate, is \$10 per person-hour and v, the rental rate on machines, is \$1 per machine hour. Find the cost-minimizing combination of labor and capital for a manufacturer who wants to produce 12,100 plastic gears.

5. (25 pts.) Suppose that after completing your graduate studies you decide to go back to work in the family business, which involves making plastic parts for laptop computers. After a couple of weeks on the job, you realize that in the real world, it is impossible to make plastic parts using just capital and labor. Raw materials are also a necessary ingredient. The production process in your factory is given by the following function: $Q = K^{.5}L^{.25}M^{.25}$, where K and L represent capital and labor inputs, and M represents the raw material input.
   (a) The per unit prices of K, L, and M are represented by v, w, and u. Solve for the firm's total cost function TC = C(Q, v, w, u).
   (b) Given that v=2, w=16, and u=1. Graph the firm's total, average, and marginal cost curves.

6. (15 pts.) City council grants your sister-in-law a franchise monopoly to be the only pet crematory in Lexington. The total cost function for cremating animals is given by $TC = 1200 + 0.5Q^2$. The market demand curve for pet cremation services is given by $P(Q) = 300 - Q$.
   a) Your sister-in-law asks you to help her determine the profit-maximizing output and price for this service. What will her economic profits be?
   b) She does know how to use the inverse elasticity rule to determine whether the price you suggest maximizes profit. Calculate marginal cost and elasticity of demand at the above price and quantity and verify for her that the inverse elasticity pricing rule holds.