ECO 601 Final Exam Fall 2003

1. (10 pts.) Your sister owns the only coal mine in an isolated region in eastern Kentucky. As such, she faces a labor supply curve given by $L = 50w$, where L indicates the number of coal miners she employs and w indicates the hourly wage rate she pays her workers. She sells her coal in a competitive output market for \$10 per ton. Each worker produces two tons of coal per hour (which works out to be 16 tons per day, an output that Tennessee Ernie Ford made famous in a well-known song). How many workers should your sister employ and what wage rate should she pay in order to maximize profits? Compare this to the number of workers hired and the wage rate if your sister were a price taker in the labor market.

2. (15 pts.) Barnacle Bill Bennett's utility function is given by $U = W^2$, where U is Bill's utility and W is his level of wealth. Bill's initial wealth is \$100.
   a) Bill's economics professor offers to bet him \$50, where Bill wins \$50 if a fair coin turns up heads and loses \$50 if the coin turns up tails. Will Bill take the bet? Explain why or why not.
   b) Illustrate Bill's utility of wealth function and your answer to (a) in the attached diagram.
   c) What probability of winning would leave Bill indifferent between participating and not participating in the gamble?

3. (20 pts.) Suppose your uncle is athletics director at the University of Kentucky. For your birthday he gives you the monopoly rights to sell plastic wildcat hats at LadyCats basketball games. Your cost function is given by $C(Q) = 100 + Q^2$. The inverse demand function for plastic wildcat hats is given by $P(Q) = 40 - Q$. Q is the number of hats you sell at each game.
   a) What price should you charge in order to maximize profits? How many hats will you sell? What will your profits be?
   b) Illustrate the total cost, total revenue, and profit functions in the attached diagram. Explain how the MR = MC condition is illustrated in this diagram.

4. (30 pts.) Given that $Q = L^{2/3}K^{2/3}$, where Q represents the rate of output, and L and K represent labor and capital usages. Input prices for L and K are given by w and v.
   a) Derive the firm's total cost function, i.e., express total cost as a function of Q, w, and v.
   b) Suppose w = \$10 and v = \$10. Graph the firm's average and marginal cost curves in the attached diagram.
   c) Suppose the firm chooses to produce Q = 16. Assume v = \$10. Determine and graph two points on the firm's constant-output demand curve for labor when w = \$10 and w = \$5.

5. (25 pts.) Erma's utility function is given by $U = X^{1/2}Y^{1/2}$.
   a) If Erma's income is \$2 and she faces $P_x = \$.25$ and $P_y = \$1$. How much X and Y should Erma consume to Maximize utility? What will her level of utility be? Illustrate in the attached diagram.
   b) Suppose P_x increases to \$1. What happens to Erma's consumption of X? Illustrate in the above diagram and also in the attached demand curve diagram.
   c) Erma's Hicksian (compensated) demand function is given by $h_x(P_x, P_y, V) = VP_y^{.5}/P_x^{.5}$, where V represents the level of utility. Illustrate Erma's Hicksian demand curve for X in the above demand curve diagram, and then illustrate the income and substitution effects of the change in P_x from \$.25 to \$1 in your budget constraint-indifference curve diagram.

ECO 601 Final Exam KEY Fall 2003

1. $L = 50w \Rightarrow w = \frac{L}{50}$

$MP_L = 2$ tons / hr ; $P_{coal} = \$10$ / ton

$VMP_L = \$20$ per worker hour

<u>monopsony</u>: total labor cost $= w(L) \cdot L = \frac{L^2}{50}$

$MFC_L = \frac{d(TLC)}{dL} = \frac{L}{25}$

$VMP_L = MFC_L \Rightarrow \frac{L}{25} = 20 \Rightarrow L = 500$ workers

$w = \frac{L}{50} = \$10$ per hour for π max

<u>competition</u>: $VMP_L = w$, so $w = \$20$

$L = 50w$, so $L = 1000$ under competition

2. $u = w^2$, $w_0 = \$100$, $u(w_0) = 10{,}000$

(a) bet: $w = \$50$ w/ pr $1/2$, $w = \$150$ w/ pr $1/2$

expected utility of bet $= \frac{1}{2}u(50) + \frac{1}{2}u(150)$

$= \frac{1}{2}(2500) + \frac{1}{2}(22500) = 12{,}500$

Since the expected utility of the gamble exceeds the certain utility from not gambling, Bill will take the bet.

(b) see diagram

(c) let x = probability of losing and $(1-x)$ = the probability of winning.

indifference implies

$(x)u(50) + (1-x)u(150) = u(100)$

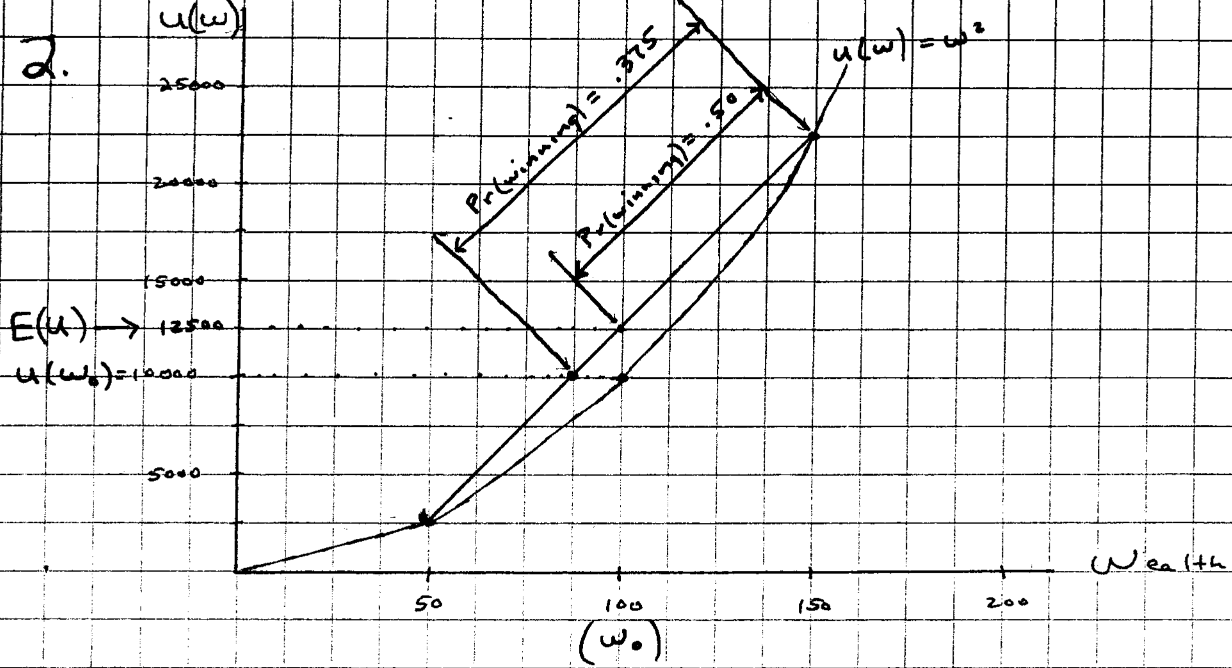
$(x)(2500) + (1-x)(22500) = 10000$

$20000(x) = 12{,}500$

$x = .625$, $1-x = .375$

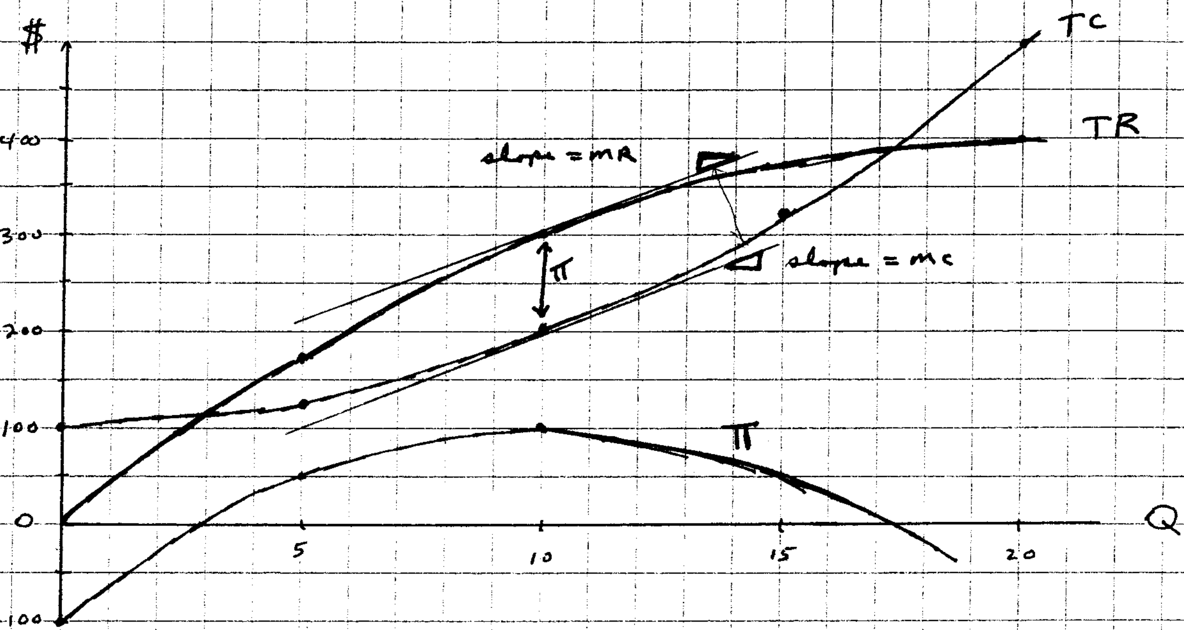
so a .375 chance of winning leaves Bill indifferent.

2.



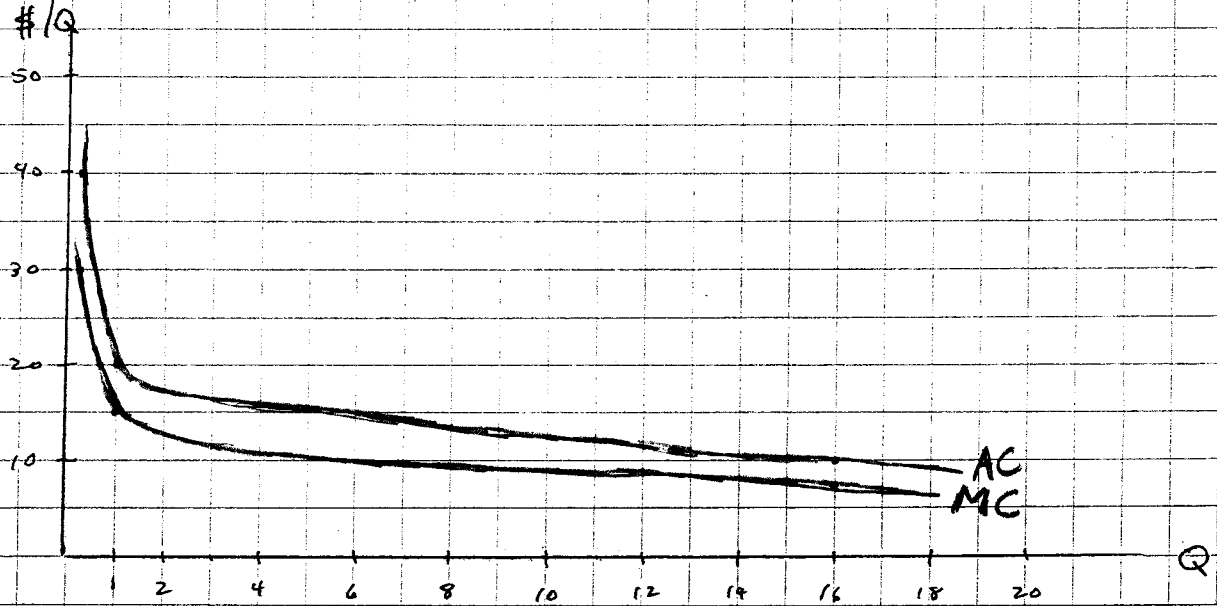
u(w)
25000
20000
15000
E(u) → 12500
u(w₀) = 10000
5000
Pr(winning) = .375
Pr(winning) = .50
u(w) = w²
Wealth
50
100
150
200
(w₀)

3.



$
TC
TR
slope = MR
slope = MC
π
π
400
300
200
100
0
-100
5
10
15
20
Q

4.



$/Q
50
40
30
20
10
AC
MC
Q
2
4
6
8
10
12
14
16
18
20

3. $C(Q) = 100 + Q^2$, $MC = \frac{dC}{dQ} = 2Q$

$P(Q) = 40 - Q$, $TR = 40Q - Q^2$, $MR = 40 - 2Q$

(a) $MR = MC \Rightarrow 40 - 2Q = 2Q$, $Q = 10$

$P = 40 - Q = \$30$

$\pi = TR - TC = 40Q - Q^2 - 100 - Q^2 = \$100$

(b) see diagram. The MR = MC necessary condition for maximum profit is illustrated by the slopes of the TR and TC functions. They are equal where profits are maximized.

4. $Q = L^{2/3} K^{2/3}$, $TC = wL + vK$

(a) $\mathcal{L} = wL + vK + \lambda(Q_0 - L^{2/3}K^{2/3})$

$$\left.\begin{aligned}\frac{\partial \mathcal{L}}{\partial L} &= w - \lambda \tfrac{2}{3} L^{-1/3} K^{2/3} = 0 \\ \frac{\partial \mathcal{L}}{\partial K} &= v - \lambda \tfrac{2}{3} L^{2/3} K^{-1/3} = 0\end{aligned}\right\} \quad \frac{K}{L} = \frac{w}{v}$$

$$\frac{\partial \mathcal{L}}{\partial \lambda} = Q_0 - L^{2/3} K^{2/3} = 0$$

substituting $L = \frac{vK}{w}$ and $K = \frac{wL}{v}$:

$$Q = \left(\frac{v}{w} K\right)^{2/3} K^{2/3} = \left(\frac{v}{w}\right)^{2/3} K^{4/3}$$

so $K = Q^{3/4} \left(\frac{w}{v}\right)^{1/2}$

similarly $L = Q^{3/4} \left(\frac{v}{w}\right)^{1/2}$

$$TC = wL + vK = wQ^{3/4}\left(\frac{v}{w}\right)^{1/2} + vQ^{3/4}\left(\frac{w}{v}\right)^{1/2}$$

or $TC = 2Q^{3/4} w^{1/2} v^{1/2}$

(b) suppose $w = 10$ and $v = 10$

$TC = 20Q^{3/4}$

$AC = \frac{20}{Q^{1/4}}$

$MC = \frac{15}{Q^{1/4}}$

Q	AC	MC
1/16	40	30
1	20	15
16	10	7.5

see diagram for graphs

4. (c) $w = 10$, $v = 10$

cost minimization requires $\frac{MP_L}{MP_K} = \frac{w}{v}$,

so $\frac{2/3\ L^{-1/3}\ K^{2/3}}{2/3\ L^{2/3}\ K^{-1/3}} = \frac{10}{10} = \frac{K}{L} = 1$ or $K = L$

$Q = L^{2/3} K^{2/3} \Rightarrow Q = L^{4/3}$

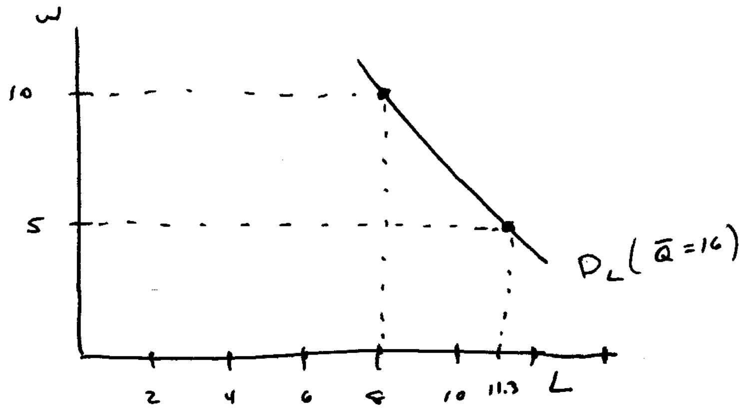
if $Q = 16$, $L = 8$

$w = 5$, $v = 10$

$\frac{K}{L} = \frac{5}{10}$, or $L = 2K$

$Q = L^{2/3} (\frac{1}{2}L)^{2/3} = (\frac{1}{2})^{2/3} L^{4/3} = .63 L^{4/3}$

$L^{4/3} = 1.59\ Q$, so $L = 11.3$



5. $U = X^{1/2} Y^{1/2}$

(a) $\frac{mu_X}{mu_Y} = \frac{P_X}{P_Y} \Rightarrow \frac{\frac{1}{2} X^{-1/2} Y^{1/2}}{\frac{1}{2} X^{1/2} Y^{-1/2}} = \frac{P_X}{P_Y} \Rightarrow P_X X = P_Y Y$

$I - P_X X - P_Y Y = 0$

if $P_X = .25$, $P_Y = 1$, and $I = 2$

then $I - 2 P_X X = 0 \Rightarrow X = 4$ and $Y = 1$

$u = 4^{1/2} 1^{1/2} = 2$

(b) $I - 2 P_X X = 0$, $P_X = 1$, $X = 1$ and $Y = 1$

$u = 1^{1/2} 1^{1/2} = 1$

(c) $h_X = V P_Y^{1/2} / P_X^{1/2}$

if $V = 2$ and $P_X = .25$, $h_X = 2(1)^{1/2} / (\frac{1}{4})^{1/2}$

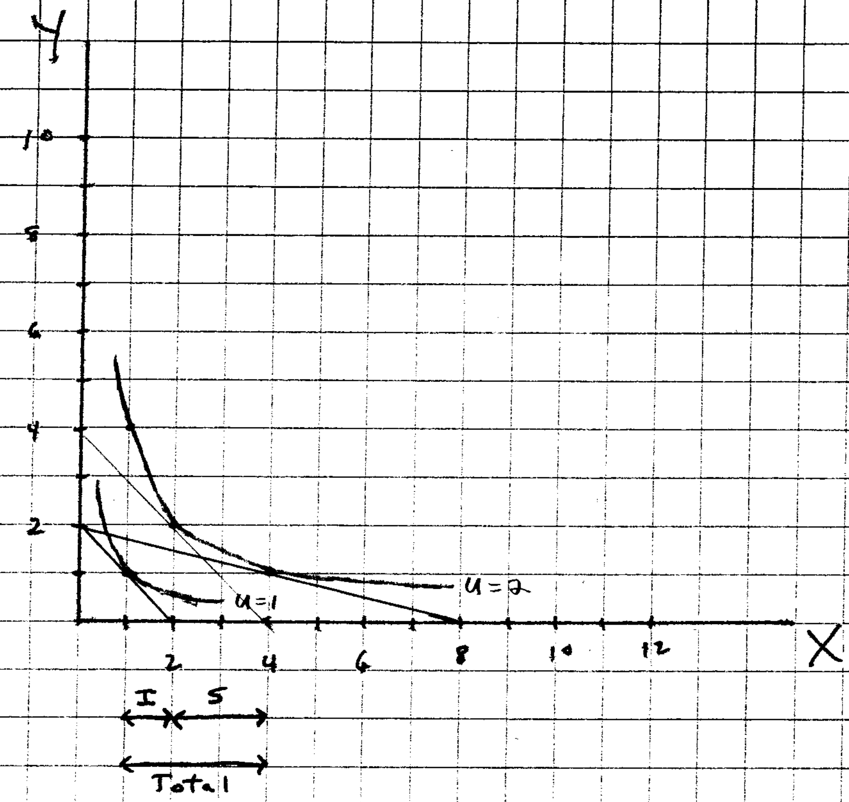
$h_X = 4$

if $V = 2$ and $P_X = 1$, $h_X = 2(1)^{1/2} / (1)^{1/2}$

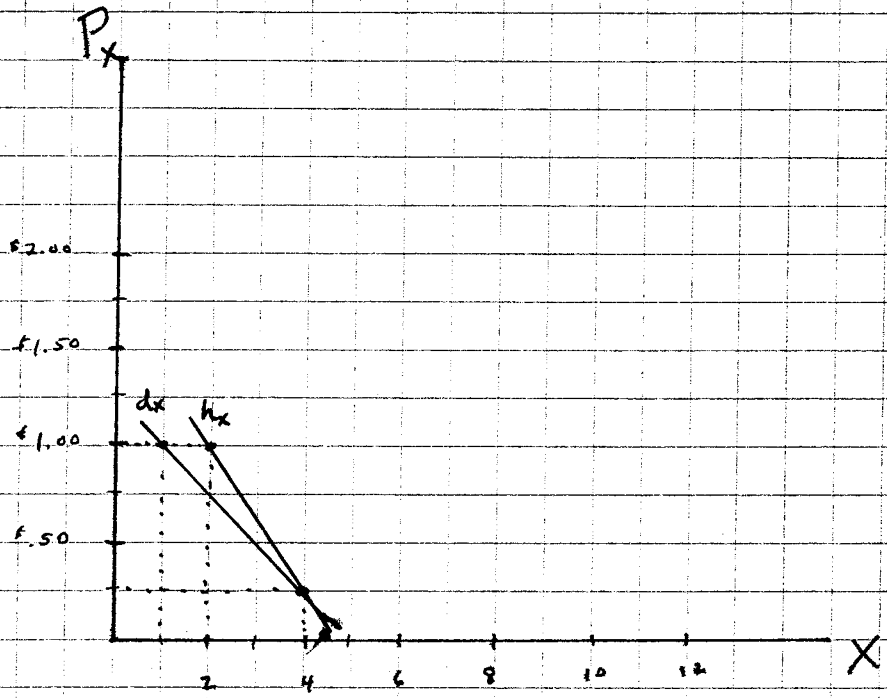
$h_X = 2$

so, the total effect of price change from $P_X = .25$ to $P_X = 1$ is that quantity demanded of X falls from $X = 4$ to $X = 1$. The change from $X = 4$ to $X = 2$ is a substitution effect, and from $X = 2$ to $X = 1$ is an income effect.

5.



Y
10
8
6
4
2
u=2
u=1
X
2
4
6
8
10
12
I
S
Total



P_x
$2.00
$1.50
$1.00
$.50
dx
hx
X
2
4
6
8
10
12