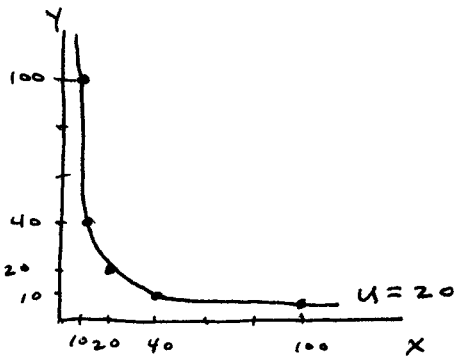
ECO 601 Problem Set #2 KEY Fall 2003

1. $U = X^{1/2} Y^{1/2}$

(a) $U = 20$ $\quad X^{1/2} Y^{1/2} = 20$, $\quad XY = 400$

X	4	100	10	40	20
Y	100	4	40	10	20



(b) $MRS_{X,Y} = MU_X / MU_Y$

$MU_X = \frac{1}{2} X^{-1/2} Y^{1/2}$

$MU_Y = \frac{1}{2} X^{1/2} Y^{-1/2}$

$MRS_{X,Y} = \dfrac{\frac{1}{2} X^{-1/2} Y^{1/2}}{\frac{1}{2} X^{1/2} Y^{-1/2}} = \dfrac{Y}{X}$

(c) $U = 20$, $X = 10$

$Y^{1/2} = U / X^{1/2}$

$Y = U^2 / X = 400/10 = 40$

$MRS_{X,Y} = Y/X = 40/10 = 4$

2. $U = X^{1/2} Y$

(a) $MU_X = \frac{dU}{dX} = \frac{1}{2} X^{-1/2} Y > 0$

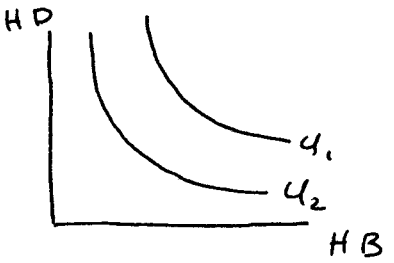
$MU_Y = \frac{dU}{dY} = X^{1/2} > 0$

$MU_X > 0$ and $MU_Y > 0$, so more is preferred to less for both X and Y.

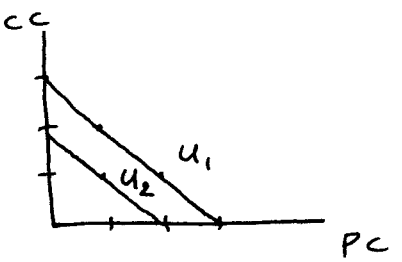
(b) $\dfrac{d MU_X}{dX} = -\frac{1}{4} X^{-3/2} Y < 0$, so X exhibits diminishing marginal utility

$\dfrac{d MU_Y}{dY} = 0$, so Y exhibits constant marginal utility

3. (a) hamburgers and hot dogs:

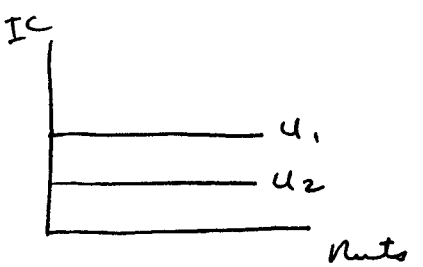


(b) Pepsi-Cola and Coca-Cola:



(c) Peanut butter and jelly:

(d) Nuts and ice cream:



(e) Apples and radishes:

4. (a) $u = 3x^{1/3}y^{2/3}$

$f_1 = x^{-2/3}y^{2/3}$

$f_2 = 2x^{1/3}y^{-2/3}$

$f_{11} = -2/3\, x^{-5/3}y^{2/3}$

$f_{22} = -4/3\, x^{1/3}y^{-5/3}$

$f_{12} = 2/3\, x^{-2/3}y^{-1/3}$

↳ $f_2^2 f_{11} - 2f_1 f_2 f_{12} + f_1^2 f_{22} < 0$?

↳ $(+)(-) - 2(+)(+)(+) + (+)(-) < 0$? yes ✓

(b) $u = -x^{-1} - y^{-1}$

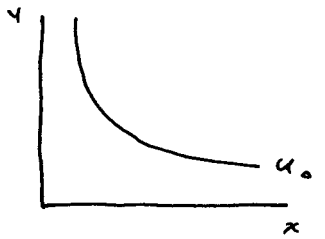
$f_1 = x^{-2}$

$f_2 = y^{-2}$

$f_{11} = -2x^{-3}$

$f_{22} = -2y^{-3}$

$f_{12} = 0$



↳ $f_2^2 f_{11} - 2f_1 f_2 f_{12} + f_1^2 f_{22} < 0$?

$(+)(-) - 2(+)(+)(0) + (+)(-) < 0$? yes ✓

(c) $u = x + \ln y$

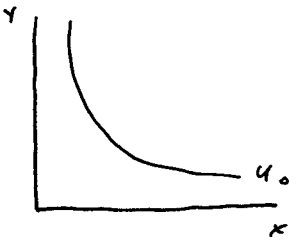
$f_1 = 1$

$f_2 = \frac{1}{y}$

$f_{11} = 0$

$f_{22} = -y^{-2}$

$f_{12} = 0$



↳ $f_2^2 f_{11} - 2f_1 f_2 f_{12} + f_1^2 f_{22} < 0$?

$(+)(0) - 2(+)(+)(0) + (+)(-) < 0$? yes ✓

(d) $u = 2x^2 + 3y^2$

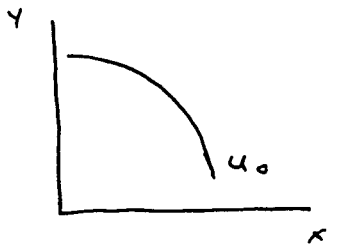
$f_1 = 4x$

$f_2 = 6y$

$f_{11} = 4$

$f_{22} = 6$

$f_{12} = 0$



↳ $f_2^2 f_{11} - 2f_1 f_2 f_{12} + f_1^2 f_{22} < 0$?

$(+)(+) - 2(+)(+)(0) + (+)(+) < 0$? NO!

5. Marsha: $u = x^{.5} y^{.5}$ Melinda: $u = 2x^{.75} y^{.75}$

Melinda's utility function is just a monotonic transformation of Marsha's utility function:

$$u = \left[x^{.5} y^{.5}\right]^{1.5} = x^{.75} y^{.75}$$

$$\left[x^{.75} y^{.75}\right] * 2 = 2x^{.75} y^{.75}$$

a graph of their indifference curves will confirm the identity.