ECO 601 Problem Set # 4 KEY Fall 2003

1. 5.6 a. Lagrangian is $\mathcal{L} = -\frac{1}{X} - \frac{1}{Y} + \lambda(I - P_X X - P_Y Y)$

yielding the first-order condition

$$\frac{\partial \mathcal{L}}{\partial X} = \frac{1}{X^2} - \lambda P_X = 0 \qquad \frac{\partial \mathcal{L}}{\partial Y} = \frac{1}{Y^2} - \lambda P_Y = 0$$

$$\frac{\partial \mathcal{L}}{\partial \lambda} = I - P_X X - P_Y Y = 0.$$

Hence $\frac{P_X}{P_Y} = \frac{Y^2}{X^2}$ or $Y = X\sqrt{P_X/P_Y}$.

Substitution into the budget constraint yields

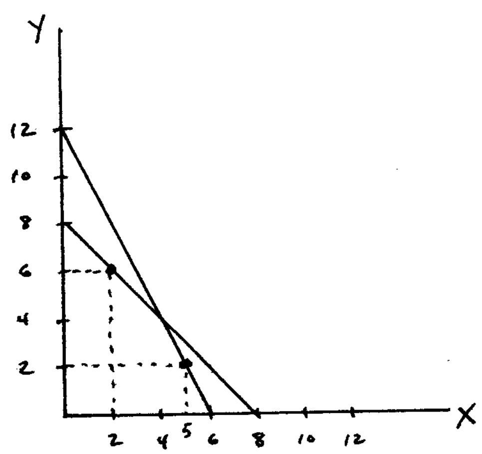
$$X = I/[P_X + P_Y(\sqrt{P_X/P_Y})]$$

$$Y = I/[P_Y + P_X(\sqrt{P_Y/P_X})].$$

b. Clearly a doubling of P_X, P_Y, and I leaves these demand functions unchanged.

c. Increases in I increase X and Y proportionally here. An increase in P_Y reduces the demand for X and *vice versa*.

2.



$I_0 = 24 \quad P_X^0 = 4 \quad P_Y^0 = 2 \quad X_0 = 5 \quad Y_0 = 2$

$I_1 = 32 \quad P_X' = 4 \quad P_Y' = 4 \quad X_1 = 2 \quad Y_1 = 6$

initially (X_0, Y_0) is revealed preferred to (X_1, Y_1):

$$P_X^0 X_0 + P_Y^0 Y_0 \geq P_X^0 X_1 + P_Y^0 Y_1$$

$$24 \geq 20$$

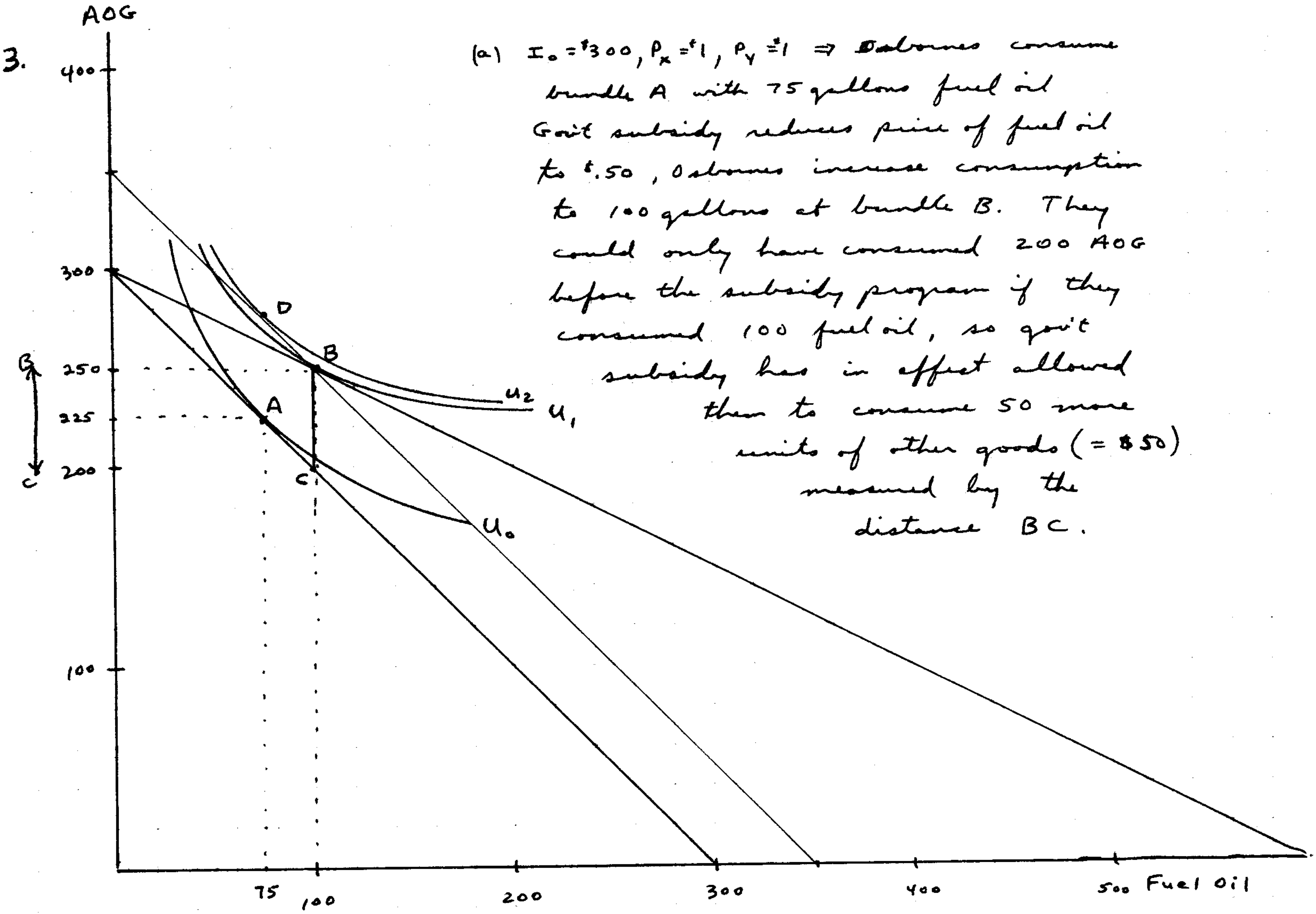
later (X_1, Y_1) is revealed preferred to (X_0, Y_0):

$$P_X' X_1 + P_Y' Y_1 \geq P_X' X_0 + P_Y' Y_0$$

$$32 \geq 28$$

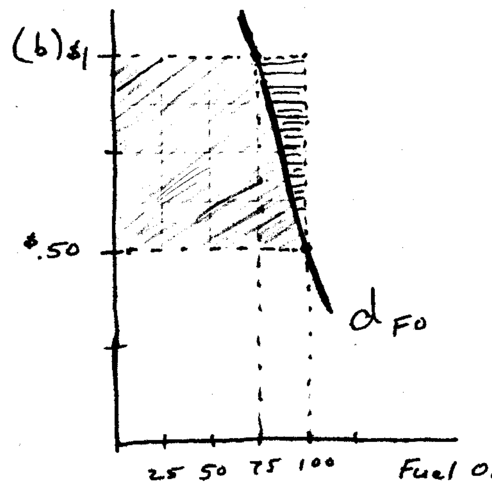
This behavior is not rational.

3.



(a) $I_0 = \$300$, $P_x = \$1$, $P_Y = \$1$ ⇒ Osbornes consume bundle A with 75 gallons fuel oil. Gov't subsidy reduces price of fuel oil to \$.50, Osbornes increase consumption to 100 gallons at bundle B. They could only have consumed 200 AOG before the subsidy program if they consumed 100 fuel oil, so gov't subsidy has in effect allowed them to consume 50 more units of other goods (= \$50) measured by the distance BC.

(b)



(c) light shaded area under demand curve = consumer's surplus = gain to Osbornes from the subsidy program (≈ \$43.75)

Total cost to gov't includes darker shaded area as well (≈ \$50.00)

(d) \$50 per month lump-sum transfer shifts Osborne's budget line parallel to original budget constraint but passing through new consumption bundle B. They can reach higher IC by substituting toward AOG away from fuel oil, bundle D in the diagram.

4. (a) $u(X,Y) = 3X^{1/3}Y^{2/3}$; $I = 48$, $P_X = 2$, $P_Y = 4$

$\mathcal{L} = 3X^{1/3}Y^{2/3} + \lambda(I - P_X X - P_Y Y)$

1st o.c.:

(1) $\frac{\partial \mathcal{L}}{\partial X} = X^{-2/3}Y^{2/3} - \lambda P_X = 0$

(2) $\frac{\partial \mathcal{L}}{\partial Y} = 2X^{1/3}Y^{-1/3} - \lambda P_Y = 0$

(3) $\frac{\partial \mathcal{L}}{\partial \lambda} = I - P_X X - P_Y Y = 0$

(1) + (2) imply $\frac{X^{-2/3}Y^{2/3}}{2X^{1/3}Y^{-1/3}} = \frac{P_X}{P_Y}$

or $2XP_X = YP_Y$

plugging into (3): $I - P_X X - 2P_X X = 0$

so $X^* = \frac{I}{3P_X}$

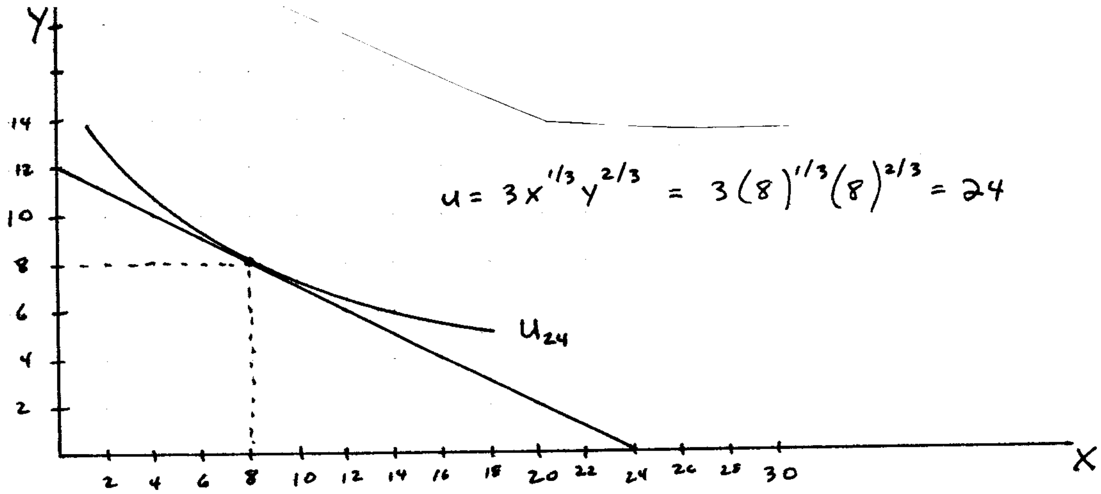
similarly for Y: $I - \frac{1}{2}P_Y Y - P_Y Y = 0$

so $Y^* = \frac{2I}{3P_Y}$

if $I = 48$ + $P_X = 2$ then $X^* = 8$

if $I = 48$ + $P_Y = 4$ then $Y^* = 8$

(b)



(c) $u = 3x^{1/3}y^{2/3} = 3\left(\frac{I}{3P_x}\right)^{1/3}\left(\frac{2I}{3P_y}\right)^{2/3}$

$u = 2^{2/3} I \,/\, (P_x)^{1/3} (P_y)^{2/3}$

or $I = u (P_x)^{1/3} (P_y)^{2/3} / 2^{2/3}$

Hicksian demand is constant real income demand, so substitute for I in Marshallian demand:

$x^* = \frac{I}{3P_x} = \frac{u P_x^{1/3} P_y^{2/3} / 2^{2/3}}{3 P_x}$

$x^* = \frac{u P_y^{2/3}}{3 \cdot 2^{2/3} \cdot P_x^{2/3}} = .21\, u\, P_y^{2/3} / P_x^{2/3}$

similarly for Y^*:

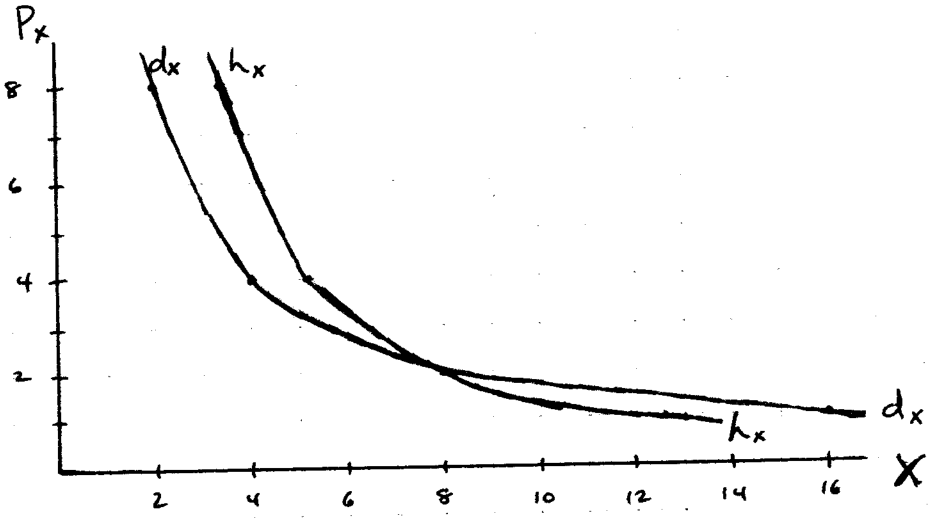
$Y^* = .42\, u\, P_x^{1/3} / P_y^{1/3}$

(d) $d_x = \frac{I}{3P_x}$; $h_x = \frac{.21\, u\, P_y^{2/3}}{P_x^{2/3}}$; $I = 48$, $P_y = 4$, $u = 24$

so

$d_x = 16/P_x$ and $h_x = \frac{(.21)(24)(4)^{2/3}}{P_x^{2/3}} = 12.7 / P_x^{2/3}$

$h_x = d_x \Rightarrow 12.7/P_x^{2/3} = 16/P_x \Rightarrow P_x = 2$ ✓



P_x	d_x	h_x
1	16	12.7
2	8	8
4	4	5.04
8	2	3.175