ECO 601
Fall 2003
Problem Set #7

Due: Wednesday, October 29

1. Crude oil is carried by pipelines from oil fields and storage areas over hundreds of miles to urban and industrial centers. The output of such pipelines is the amount of oil carried per day, and the two principal inputs are the diameter of the pipeline and the horsepower applied to the oil carried. It has been estimated that the production function for a pipeline with a 10-inch diameter is: $Q = 286H^{.37}$, where Q is the amount of crude oil carried per day and H is horsepower.
    a) Derive a formula for the marginal product of horsepower.
    b) Do increases in horsepower result in diminishing marginal returns?
    c) Derive a formula for the average product of horsepower.
    d) Graph TP_L, MP_L, and AP_L.

2. Arby's produces roast beef sandwiches according to the production function $q=10K^{.5}L^{.5}$, where q is the number of roast beef sandwiches per hour and K and L refer to inputs of capital and labor per hour.
    a) On a sheet of graph paper, plot isoquants for q=40 and q=80.
    b) To produce 20 roast beef sandwiches per hour Arby's could use either (K=4, L=1) or (K=2, L=2). What is the elasticity of substitution between capital and labor over this range of the q=20 isoquant? (Hint: a complete answer will define elasticity of substitution and either compute it between the two points on the isoquant or solve for it algebraically.)

3. Suppose the production function is given by $Q = LK^{.5}$. Graph the isoquants corresponding to Q = 10, Q = 20, and Q = 50.
    a) Do these isoquants exhibit diminishing marginal rate of technical substitution?
    b) What can you say about returns to scale for this production function? Illustrate in your diagram.

4. Consider the production function $Q = (K^{.5} + L^{.5})^2$.
    a) What is the elasticity of substitution for this production function? Does it change as the capital-labor ratio changes? (Hint: it is an example of a CES production function.
    b) Does this production function exhibit constant, increasing, or decreasing returns to scale?