Eco 601 Problem Set #8 Key Fall 2003

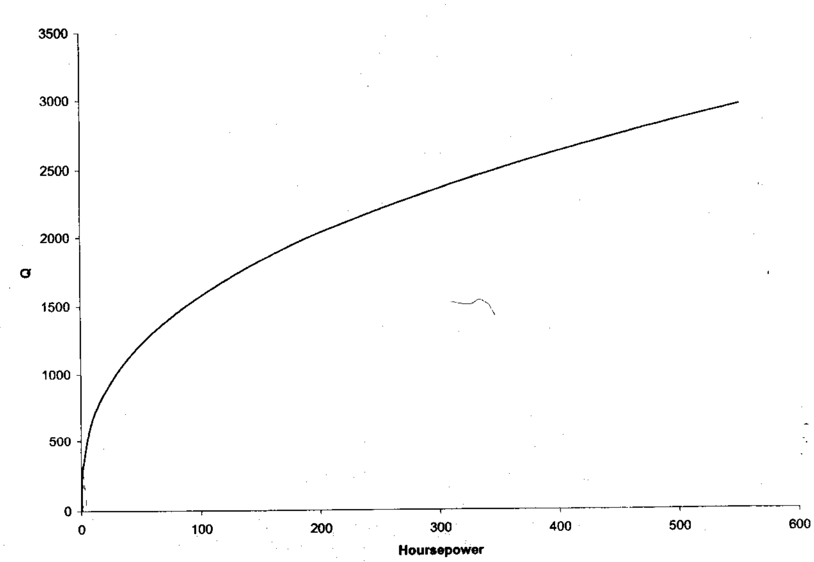
1. $Q = 286 H^{.37}$

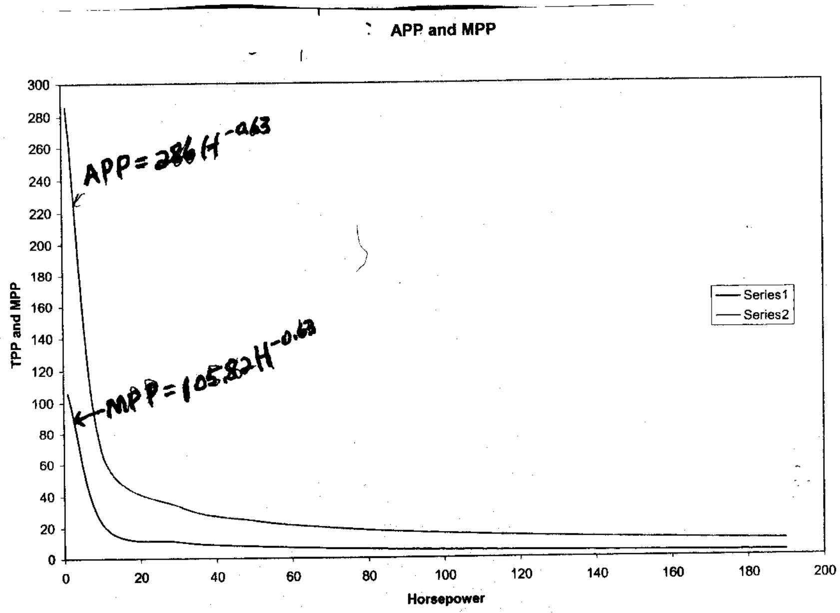
(a) $MP_H = \frac{dQ}{dH} = .37(286)H^{-.63} = 105.82 H^{-.63}$

(b) Yes, since MP_H decreases as horsepower increases

(c) $AP_H = Q/H = 286 H^{-.63}$

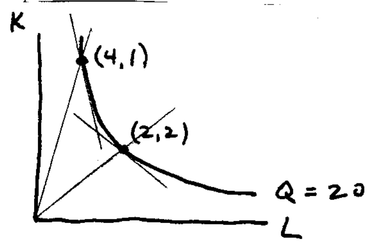
(d)





2. (a) see diagram.

(b) $\sigma = \dfrac{\%\ \Delta\ (K/L)}{\%\ \Delta\ MRTS_{L,K}} \Rightarrow$



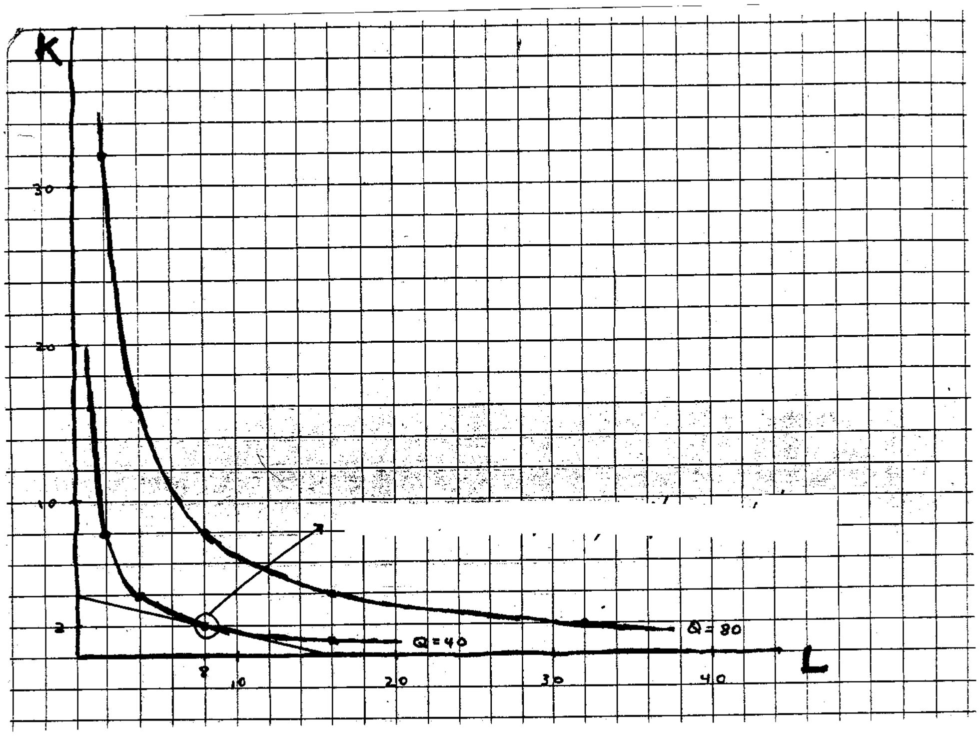
$\%\ \Delta\ (K/L) \Rightarrow \frac{4}{1} \rightarrow \frac{2}{2}$

$\%\ \Delta\ MRTS_{L,K} = \%\ \Delta \dfrac{MP_L}{MP_K} = \%\ \Delta\ \dfrac{5K^{1/2}L^{-1/2}}{5K^{-1/2}L^{1/2}} = \%\ \Delta\ (K/L)$

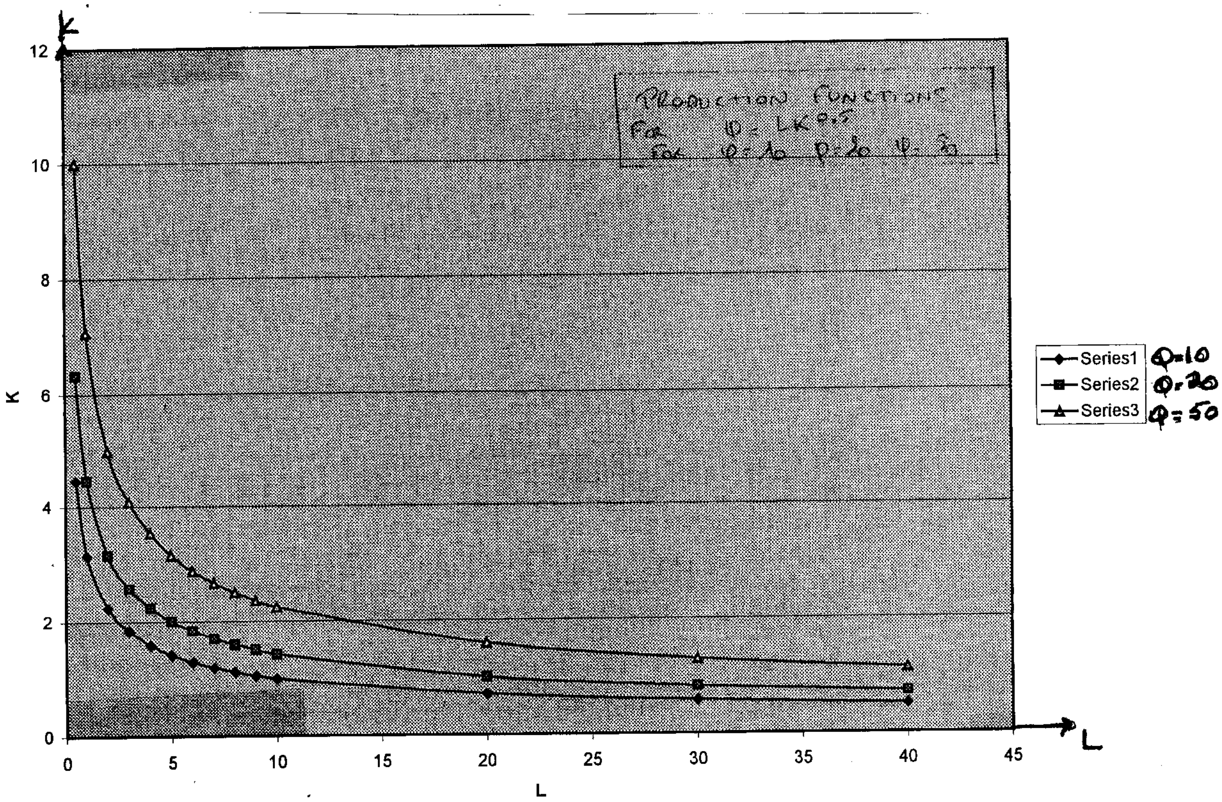
so $\%\ \Delta\ MRTS_{L,K} = \%\ \Delta\ (K/L) \Rightarrow \frac{4}{1} \rightarrow \frac{2}{2}$ and

is equal to the numerator of σ.

Hence $\sigma = 1$



3.



(a) $Q = LK^{1/2}$

diminishing $MRTS_{L,K}$ if $f_K^2 f_{LL} - 2 f_K f_L f_{KL} + f_L^2 f_{KK} < 0$

$f_L = \frac{dQ}{dL} = K^{1/2} \qquad f_{LL} = \frac{df_L}{dL} = 0$

$f_K = \frac{dQ}{dK} = \frac{1}{2} L K^{-1/2} \qquad f_{KK} = \frac{df_K}{dK} = -\frac{1}{4} L K^{-3/2}$

$f_{LK} = f_{KL} = \frac{1}{2} K^{-1/2}$

$\left(\frac{1}{2} L K^{-1/2}\right)^2 (0) - 2\left(\frac{1}{2} L K^{-1/2}\right)\left(K^{1/2}\right)\left(\frac{1}{2} K^{-1/2}\right) + \left(K^{1/2}\right)^2\left(-\frac{1}{4} L K^{-3/2}\right)$

$(+)(0) - (2)(+)(+)(+) + (+)(-) < 0$?? YES

since the above expression is negative, diminishing $MRTS_{L,K}$

(b) $f(mL, mK) \gtreqless m f(L,K)$?

$(mL)(mK)^{1/2} = m^1 m^{.5} L K^{.5}$

$= m^{1.5} f(L,K) > m f(L,K)$

so increasing returns to scale.

In the diagram, isoquants are getting closer together for equi-proportionate change in output

4\. (a) $Q = (K^{1/2} + L^{1/2})^2$

$MRTS_{L,K} = MP_L / MP_K$

$MP_L = 2(K^{1/2} + L^{1/2})(\frac{1}{2}L^{-1/2})$

$MP_K = 2(K^{1/2} + L^{1/2})(\frac{1}{2}K^{-1/2})$

so $MRTS_{L,K} = \left(\frac{K}{L}\right)^{1/2}$

$$\sigma = \frac{d \ln (K/L)}{d \ln MRTS_{L,K}} = \frac{d \ln(\frac{K}{L})}{d \ln (\frac{K}{L})^{1/2}}$$

$$\sigma = \frac{d \ln K/L}{\frac{1}{2} d \ln K/L} = 2$$

(b) $f(mL, mK) = \left[(mK)^{1/2} + (mL)^{1/2}\right]^2$

$= (m^{1/2})^2 (K^{1/2} + L^{1/2})^2$

$= m\, f(L, K)$

so constant returns to scale.