ECO 601 Problem Set # 8 KEY Fall 2003

1. (12/6) a. $q = 2\sqrt{K \cdot L}$ $K = 100,\ q = 2\sqrt{100 \cdot L}$ $q = 20\sqrt{L}$

$$\sqrt{L} = \frac{q}{20} \qquad L = \frac{q^2}{400}$$

$$STC = vK + wL = 1(100) + 4\left(\frac{q^2}{400}\right) = 100 + \frac{q^2}{100}$$

$$SAC = \frac{STC}{q} = \frac{100}{q} + \frac{q}{100}$$

b. $SMC = \frac{q}{50}$. If $q = 25$, $STC = 100 + \left(\frac{25^2}{100}\right) = 106.25$

$$SAC = \frac{100}{25} + \frac{25}{100} = 4.25 \qquad SMC = \frac{25}{50} = .50$$

If $q = 50$, $STC = 100 + \left(\frac{50^2}{100}\right) = 125$

$$SAC = \frac{100}{50} + \frac{50}{100} = 2.50 \qquad SMC = \frac{50}{50} = 1$$

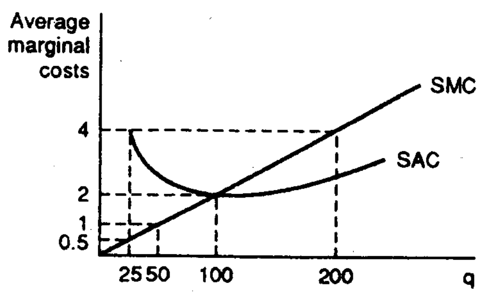
If $q = 100$, $STC = 100 + \left(\frac{100^2}{100}\right) = 200$

$$SAC = \frac{100}{100} + \frac{100}{100} = 2 \qquad SMC = \frac{100}{50} = 2.$$

If $q = 200$, $STC = 100 + \left(\frac{200^2}{100}\right) = 500$

$$SAC = \frac{100}{200} + \frac{200}{100} = 2.50 \qquad SMC = \frac{200}{50} = 4.$$

c.



d. As long as the marginal cost of producing one more unit is below the average-cost curve, average costs will be falling. Similarly, if the marginal cost of producing one more unit is higher than the average cost, then average costs will be rising. Therefore, the *SMC* curve must intesect the *SAC* curve at its lowest point.

2. $Q = \left(K^{1/2} + L^{1/2}\right)^2 = K + 2K^{1/2}L^{1/2} + L$

(a) $mP_L = \frac{dQ}{dL} = K^{1/2}L^{-1/2} + 1 = L^{-1/2}\left(K^{1/2} + L^{1/2}\right)$

$mP_K = \frac{dQ}{dK} = K^{-1/2}L^{1/2} + 1 = K^{-1/2}\left(K^{1/2} + L^{1/2}\right)$

(b) $\min TC = wL + vK$ S.T. $Q = 12{,}100$; $w = 10$, $v = 1$

$$\mathcal{L} = 10L + K + \lambda\left[12{,}100 - \left(K^{1/2} + L^{1/2}\right)^2\right]$$

$$\frac{d\mathcal{L}}{dL} = 10 - \lambda\left[L^{-1/2}\left(K^{1/2} + L^{1/2}\right)\right] = 0$$

$$\frac{d\mathcal{L}}{dK} = 1 - \lambda\left[K^{-1/2}\left(K^{1/2} + L^{1/2}\right)\right] = 0$$

$$\frac{d\mathcal{L}}{d\lambda} = 12{,}100 - \left(K^{1/2} + L^{1/2}\right)^2 = 0$$

combining first two 1st order conditions:

$$\frac{10}{1} = \frac{L^{-1/2}\left(K^{1/2} + L^{1/2}\right)}{K^{-1/2}\left(K^{1/2} + L^{1/2}\right)}$$

or $10 = \sqrt{\frac{K}{L}}$, $\frac{K}{L} = 100$, $K = 100L$

plug back into production ftn. constraint:

$$\left[(100L)^{1/2} + L^{1/2}\right]^2 = 12{,}100$$

$\left(10L^{1/2} + L^{1/2}\right)^2 = 12{,}100$; $\left(11L^{1/2}\right)^2 = 12{,}100$

$11L^{1/2} = 110$, $L^{1/2} = 10$, $L = 100$ and $K = 10{,}000$

5. Burton has not considered the opportunity cost of his time (approximately $5000) per month, nor has he factored in the interest income foregone on $350,000 that is tied up in his business. In addition, presumably his truck is depreciating with use and with time. All of these things together suggest that he is probably suffering economic losses. So your spouse should continue working for JBHunt and you should keep the mutual fund.

3. (12.8) (a) $q_{total} = q_1 + q_2$

$q_1 = \sqrt{25L_1} = 5\sqrt{L_1} \qquad L_1 = q_1^2/25$

$q_2 = \sqrt{100L_2} = 10\sqrt{L_2} \qquad L_2 = q_2^2/100$

if $w = v = 1$ then

$C_1 = 25 + L_1 = 25 + q_1^2/25$

$C_2 = 100 + L_2 = 100 + q_2^2/100$

$C_{TOTAL} = C_1 + C_2 = 125 + \frac{q_1^2}{25} + \frac{q_2^2}{100}$

Minimize TC s.t. $q_1 + q_2 = q_{TOTAL}$

$\mathcal{L} = 125 + \frac{q_1^2}{25} + \frac{q_2^2}{100} + \lambda(q_T - q_1 - q_2)$

$$\left.\begin{aligned} (1)\quad \frac{\partial \mathcal{L}}{\partial q_1} &= \frac{2q_1}{25} - \lambda = 0 \\ (2)\quad \frac{\partial \mathcal{L}}{\partial q_2} &= \frac{2q_2}{100} - \lambda = 0 \end{aligned}\right\} \quad 4q_1 = q_2$$

Note: if $4q_1 = q_2$, then $\frac{dC_1}{dq_1} = \frac{dC_2}{dq_2}$, i.e. $MC_1 = MC_2$

(b) $4q_1 = q_2$, $q_1 + q_2 = q_T$, so $q_1 = \frac{1}{5}q_T$ and $q_2 = \frac{4}{5}q_T$

$C = 125 + \frac{\left(\frac{1}{5}q\right)^2}{25} + \frac{\left(\frac{4}{5}q\right)^2}{100} = 125 + \frac{q^2}{125}$

$MC = \frac{2q}{125} \qquad AC = \frac{125}{q} + \frac{q}{125}$

$MC(100) = \frac{200}{125} = \$1.60 \qquad MC(125) = \$2.00 \qquad MC(200) = \$3.20$

(c) $q = L^{1/2}K^{1/2} \Rightarrow$ constant returns to scale, so it doesn't really matter how production is allocated.

$q = L^{1/2}K^{1/2}$; cost min requires that $\frac{w}{v} = \frac{K}{L}$

so $q = \left(\frac{v}{w}K\right)^{1/2}K^{1/2} \Rightarrow K = \left(\frac{w}{v}\right)^{1/2}q = q$

similarly $L = \left(\frac{v}{w}\right)^{1/2}q = q$, so $LTC = wL + vK = 2q$

$LMC = 2 = LAC$

(d) decreasing returns to scale, so divide production equally between two plants because the smaller the output in each plant, the lower is avg. cost

4. (a) $Q = K^{1/2} L^{1/4} M^{1/4}$

minimize $TC = wL + vK + uM$ s.t. $Q \geq Q_0$

$$\mathcal{L} = wL + vK + uM + \lambda(Q_0 - K^{1/2} L^{1/4} M^{1/4})$$

1st order conditions:

(1) $\frac{\partial \mathcal{L}}{\partial L} = w - \frac{1}{4}\lambda K^{1/2} L^{-3/4} M^{1/4} = 0$

(2) $\frac{\partial \mathcal{L}}{\partial K} = v - \frac{1}{2}\lambda K^{-1/2} L^{1/4} M^{1/4} = 0$

(3) $\frac{\partial \mathcal{L}}{\partial M} = u - \frac{1}{4}\lambda K^{1/2} L^{1/4} M^{-3/4} = 0$

(4) $\frac{\partial \mathcal{L}}{\partial \lambda} = Q_0 - K^{1/2} L^{1/4} M^{1/4} = 0$

combining equations and simplifying:

(1) and (2) $\Rightarrow \frac{w}{v} = \frac{K}{2L} \Rightarrow vK = 2wL$

(1) and (3) $\Rightarrow \frac{w}{u} = \frac{M}{L} \Rightarrow wL = uM$

(2) and (3) $\Rightarrow \frac{v}{u} = \frac{2M}{K} \Rightarrow vK = 2uM$

now solve for L, K, and M in terms of Q, w, v, and u:

$$Q = \left(\frac{2wL}{v}\right)^{1/2} L^{1/4} \left(\frac{wL}{u}\right)^{1/4}$$

$$Q = L\left(\frac{2w}{v}\right)^{1/2} \left(\frac{w}{u}\right)^{1/4}$$

$$L = (Q v^{1/2} u^{1/4} w^{-3/4}) / \sqrt{2}$$

$$wL = \frac{1}{\sqrt{2}} Q v^{1/2} w^{1/4} u^{1/4}$$

$$vK = 2wL = \frac{2}{\sqrt{2}} Q v^{1/2} w^{1/4} u^{1/4}$$

$$uM = wL = \frac{1}{\sqrt{2}} Q v^{1/2} w^{1/4} u^{1/4}$$

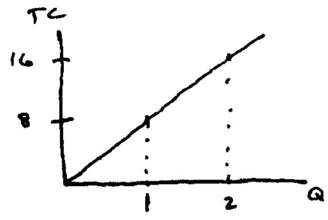
$$TC = \frac{1}{\sqrt{2}} Q v^{1/2} w^{1/4} u^{1/4} + \frac{2}{\sqrt{2}} Q v^{1/2} w^{1/4} u^{1/4} + \frac{1}{\sqrt{2}} Q v^{1/2} w^{1/4} u^{1/4}$$

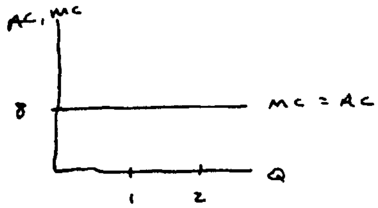
$$TC = \frac{4}{\sqrt{2}} Q v^{1/2} w^{1/4} u^{1/4} = 2.828\, Q v^{1/2} w^{1/4} u^{1/4}$$

(b) if $v = 2$, $w = 16$, $u = 1$, then

$$TC = 2.828\, Q (2)^{1/2} (16)^{1/4} (1)^{1/4} = 8Q$$

$$AC = TC/Q = 8 \qquad MC = \frac{dTC}{dQ} = 8$$





(c) $Q = K^{1/2} L^{1/4} m^{1/4}$, $\bar{K} = 10$, so $Q = \sqrt{10}\, L^{1/4} m^{1/4}$

short run cost minimization requires that

$$\frac{MP_L}{MP_m} = \frac{w}{u} \text{ , or } \frac{\frac{1}{4}\sqrt{10}\, L^{-3/4} m^{1/4}}{\frac{1}{4}\sqrt{10}\, L^{1/4} m^{-3/4}} = \frac{w}{u}$$

or $\frac{m}{L} = \frac{w}{u} \Rightarrow wL = uM$

so if $w = 16$ and $u = 1$, then $M = 16L$

substitute back into production ftn:

$$Q = \bar{K}^{1/2} L^{1/4} (16L)^{1/4} = 2L^{1/2} \bar{K}^{1/2}$$

and $L = Q^2 / 4\bar{K}$, $m = 16L = 4Q^2/\bar{K}$

$$STC = wL + v\bar{K} + uM$$

$$STC = w\frac{Q^2}{4\bar{K}} + v\bar{K} + u\frac{4Q^2}{\bar{K}}$$

if $w = 16$, $v = 2$, and $u = 1$, $STC = 8Q^2/\bar{K} + 2\bar{K}$

$\bar{K} = 10 \Rightarrow STC = 20 + .8Q^2$ $\quad SAC = \frac{20}{Q} + .8Q$

$\bar{K} = 20 \Rightarrow STC = 40 + .4Q^2$ $\quad SAC = \frac{40}{Q} + .4Q$

$\bar{K} = 40 \Rightarrow STC = 80 + .2Q^2$ $\quad SAC = \frac{80}{Q} + .2Q$

Q	5	10	20	40
SAC(K=10)	8	10	17	32.5
SAC(K=20)	10	8	10	17
SAC(K=40)	17	10	8	10