ECO 601 Final Exam Fall 2004

Shorter questions, 10 points each:

1. Nominal income is unchanging along a Marshallian demand curve, and utility is unchanging along a Hicksian demand curve. If the Marshallian and Hicksian demand curves for good X coincide between P_0 and P_1 for an individual, then how is it that both nominal income and utility are both unchanging? Can you reconcile this seeming paradox?

2. The taxi business is inherently a constant-cost perfectly competitive industry. In a number of cities, the number of taxicabs is limited by city government via a medallion system, whereby you have to purchase a medallion from the city in order to operate a cab. In New York City, the medallions can be bought and sold, but the city has kept the number of medallions constant for a long time. Currently a taxi medallion sells for over $100,000. Explain how that market price is determined.

3. Homer consumes two goods, leisure and other goods. The price of other goods is $1, and of course Homer has 8760 hours each year to either work or consume as leisure. Homer can earn $20 per hour working and chooses to work 2000 hours per year. Then Congress passes a negative income tax system that changes Homer's incentives. Each person pays a tax to or receives a subsidy from the government according to the formula: $T = -20{,}000 + 0.5\ I$; where T is the tax paid to or subsidy received from the government, and I is income received from other sources, like working. Will Homer change his work effort?

4. Pete places an even-money $100,000 bet on the Cincinnati Reds to win the World Series. Pete's utility function is given by $U = \ln(W)$, where W is Pete's level of wealth. Pete's current level of wealth is $1,000,000. What does Pete believe the probability of the Reds winning the World Series is?

Longer questions, 20 points each:

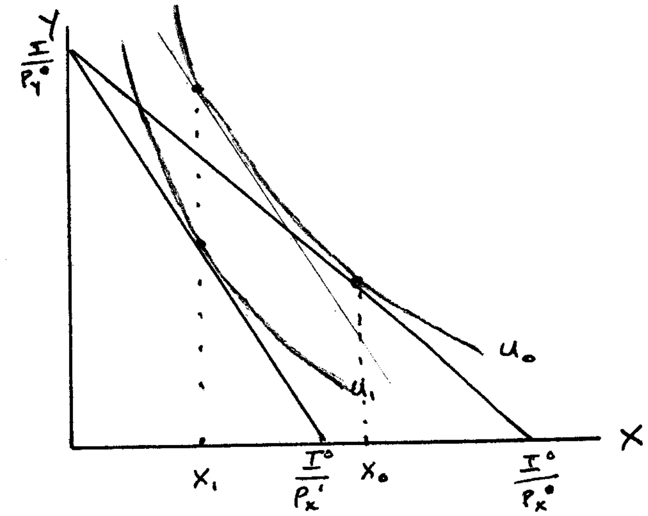
5. Marge consumes two goods, visits to the beauty parlor (X) and other goods (Y). Her utility is given by $U = X^{1/3}Y^{2/3}$. Trips to the beauty parlor cost $2, and the price of other goods is $P_Y = \$1$. Marge's income is $12.
   a) Derive Marge's ordinary demand curves for X and Y. How much of each will she choose to consume? How much utility does she get? Illustrate her initial situation in the attached diagram.
   b) The beauty parlor reduces the price of a visit to $1. How will Marge change her consumption behavior? Illustrate in your diagram. How will her utility change? Can you come up with a dollar estimate of this change in utils?

6. A monopolist faces a demand curve given by $P = 120 - 3Q$. The monopolist has two plants. The first has a marginal cost curve given by $MC_1 = 10 + 20Q_1$ and the second plant's marginal cost curve is given by $MC_2 = 60 + 5Q_2$.
   a) Explain how the monopolist should allocate production between the two plants and then illustrate the monopolist's profit-maximizing choice of total output and price in the attached diagram.
   b) Solve analytically for the profit-maximizing output and price and verify that they match your diagram.

7. Suppose you operate a small coal mine in an isolated area of eastern Kentucky. You use capital and labor to produce coal, and your production function is given by $Q = K^{.5}L^{.5}$, where Q is daily output measured in tons of coal, and K and L are capital and labor inputs. Assume that you have locked into a level of capital equal to 16, and are stuck with that K=16 throughout the production interval involved in answering this question. Labor is measured in person-days, where a person-day is an eight-hour shift worked by one person. You sell your coal in a world market, where the market price of coal is $20 per ton.
   a) Suppose there are other employment opportunities for workers in your small corner of the world and that you are a price taker in the market for labor. The going wage rate is $10 per hour ($80 per day). How many workers will you want to employ to maximize profits? Illustrate.
   b) Now suppose that you are the only employer within reasonable travel distance, and so you have some monopsony power. The labor supply curve is given by $L = w/10$, where w is the wage payment for an eight-hour shift (i.e. at $10 per hour w = $80, and eight workers would show up wanting to work for you). How many workers would you want to employ? Illustrate.

ECO 601 Final Exam Key Fall 2004

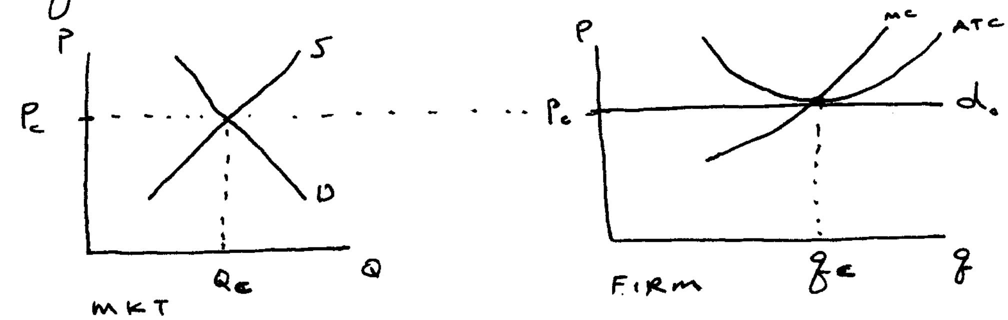
1. If the Marshallian and Hicksian demand curves coincide, the income effect of a price change must be zero. This is not inconsistent with nominal income being held constant in the Marshallian experiment and utility being held constant in the Hicksian experiment:



Marshallian: holding nominal income constant at I_0, quantity demanded declines from X_0 to X_1 when the price of X rises from P_X^0 to P_X^1.

Hicksian: holding utility constant at U_0, quantity demanded declines from X_0 to X_1 when the price of X rises from P_X^0 to P_X^1

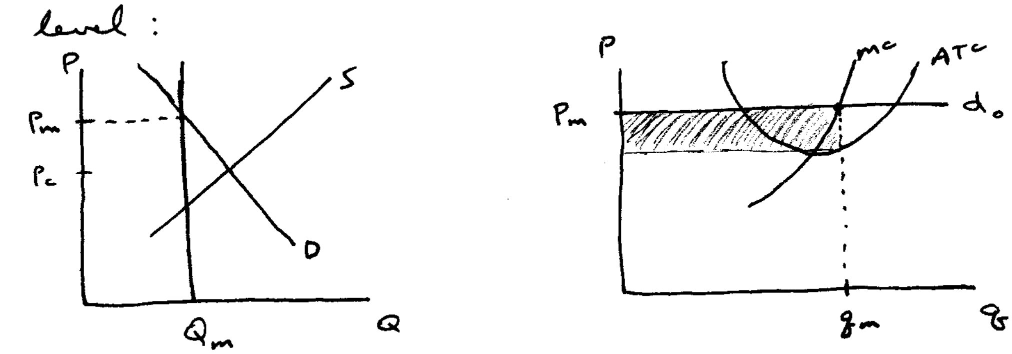
2. If there were no restrictions on the number of taxicabs, the market would look like this:



Taxi owners would earn a normal return and the number of taxis would be determined by

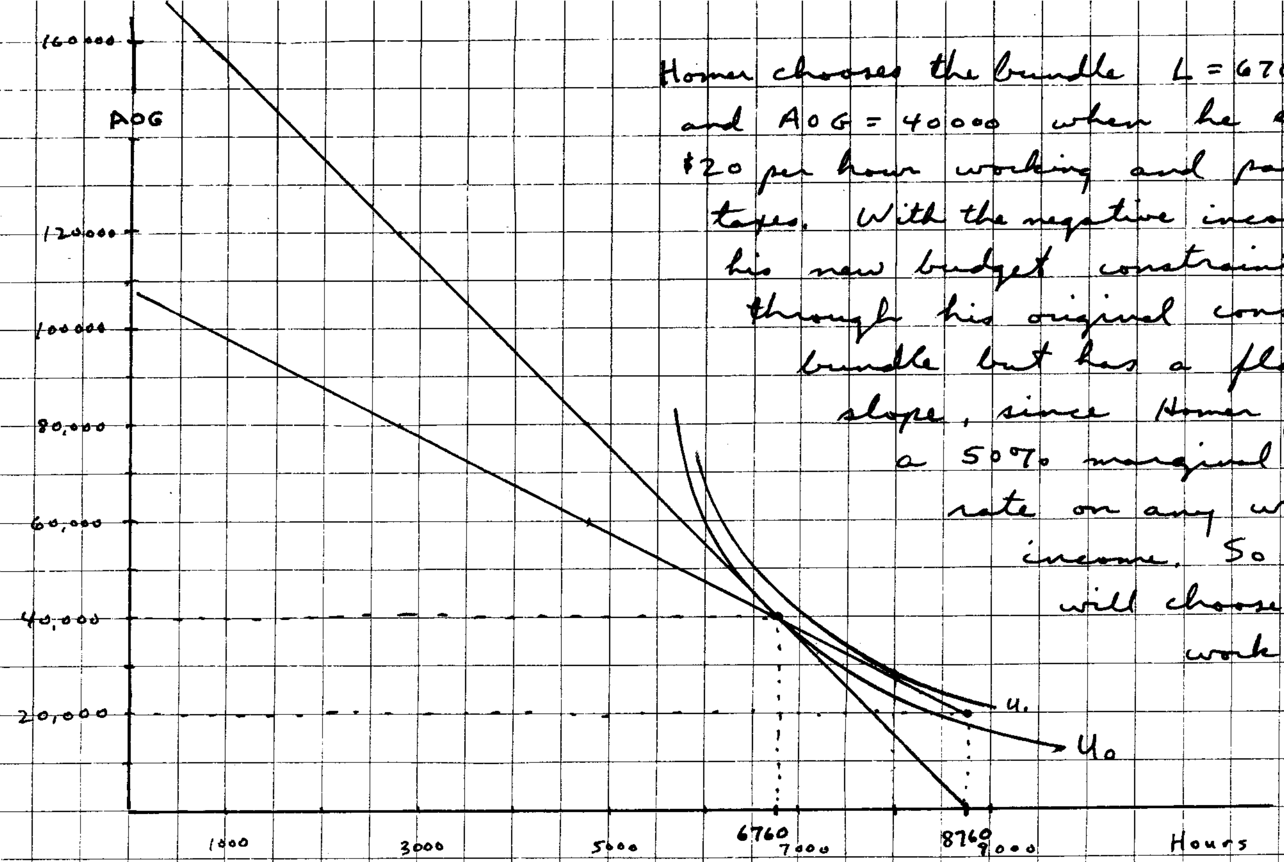
$$\sum_{i=1}^{n} q_c = Q_c$$

With a medallion system that limits the number of taxicabs, price will rise above the competitive level:



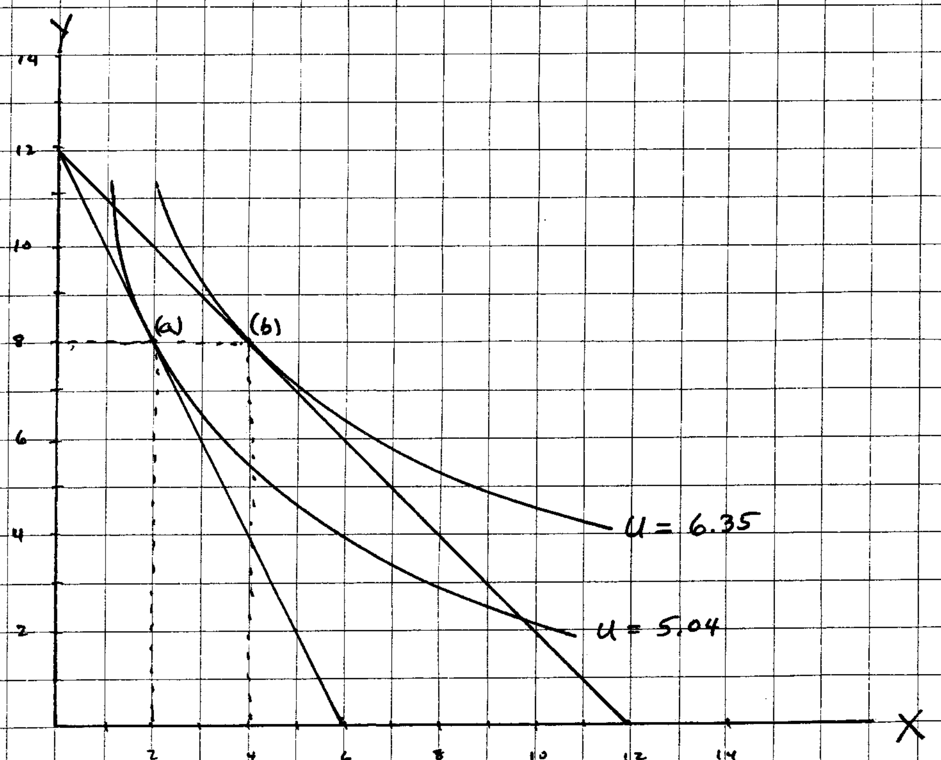
$\sum_{i=1}^{m} q_m = Q_m$. Each taxi owner will drive their cab more intensively when the price they can charge rises to P_m. Taxi owners will earn an above-normal return that will <u>not</u> be competed away by the entry of new firms. This above-normal return will be capitalized in the market value of a medallion.

3.



Homer chooses the bundle $L = 6760$ and $AOG = 40000$ when he earns \$20 per hour working and pays no taxes. With the negative income tax his new budget constraint passes through his original consumption bundle but has a flatter slope, since Homer pays a 50% marginal tax rate on any wage income. So Homer will choose to work less.

5.



3. see diagram.

4. current wealth = \$1,000,000

current utility = $\ln(1{,}000{,}000) = 13.816$

expected utility from the gamble:

$$E(u) = p \cdot u(1{,}100{,}000) + (1-p)\, u(900{,}000)$$

$$= p \ln(1{,}100{,}000) + (1-p) \ln(900{,}000)$$

$$= p \cdot 13.911 + (1-p) \cdot 13.710$$

He is indifferent between gambling and not gambling if

$$13.816 = p \cdot 13.911 + (1-p) \cdot 13.710$$

$$0.106 = .201\, p$$

$$p = .527$$

So since he is risk averse, Pete must think that the probability of the Reds winning is at least 0.527 if he places the bet.

5. (a) $U = X^{1/3} Y^{2/3}$

$$\mathcal{L} = X^{1/3} Y^{2/3} - \lambda (I - X P_x - Y P_Y)$$

$$\left.\begin{aligned} (1)\quad \frac{\partial \mathcal{L}}{\partial X} &= \frac{1}{3} X^{-2/3} Y^{2/3} - \lambda P_x = 0 \\ \frac{\partial \mathcal{L}}{\partial Y} &= \frac{2}{3} X^{1/3} Y^{-1/3} - \lambda P_Y = 0 \end{aligned}\right\} \quad 2 X P_x = Y P_Y$$

$$\frac{\partial \mathcal{L}}{\partial \lambda} = I - X P_x - Y P_Y = 0$$

substituting: $I - X P_x - 2 X P_x = 0$, so $X^* = \frac{I}{3 P_x}$

similarly for Y, $Y^* = \frac{2 I}{3 P_Y}$

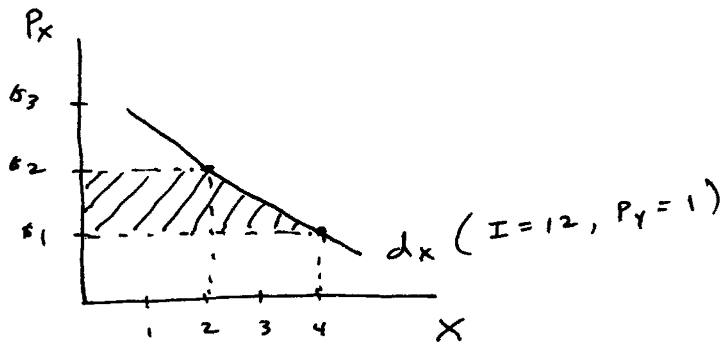
if $I = 12$, $P_x = 2$, and $P_Y = 1$, then $X^* = 2$ and $Y^* = 8$

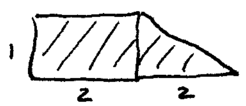
$U = 2^{1/3} 8^{2/3} = 5.04$

(b) if $I = 12$, $P_x = 1$, and $P_Y = 1$, then $X^* = 4$ and $Y^* = 8$

$U = 4^{1/3} 8^{2/3} = 6.35$

so Marge gains $6.35 - 5.04 = 1.31$ utils when the price of a trip to the beauty parlor falls from \$2 to \$1. A dollar estimate of of the gain to Marge is the gain in consumer's surplus, which can be gotten from her ordinary demand curve for good X:





= \$3 gain in consumer's surplus

6. (a) The monopolist's overall marginal cost is the horizontal sum of the marginal cost curves for each of the two plants. To maximize profit the firm should equate marginal revenue with overall marginal cost. Output should be allocated between the two plants so that the marginal cost at each plant is equal. See the diagram.

(b) $P = 120 - 3Q$, $TR = 120Q - 3Q^2$, $MR = 120 - 6Q$

The overall marginal cost curve is the horizontal sum of the individual plant MC curves:

$MC_1 = 10 + 20Q_1$ $\qquad MC_2 = 60 + 5Q_2$

so $Q_1 = -\frac{1}{2} + \frac{1}{20} MC_1$ and $Q_2 = -12 + \frac{1}{5} MC_2$

and $Q_1 + Q_2 = -12.5 + 0.25 MC_T$,

which is the horizontal sum of MC_1 and MC_2

since $Q_1 + Q_2 = Q_T$: $Q_T = -12.5 + .25 MC_T$

or $MC_T = 50 + 4Q$

$MR = MC \Rightarrow 120 - 6Q = 50 + 4Q$

or $Q = 7$ for π max

and $P = 120 - 3(7) = \$99$

The 7 units should be allocated between the two plants as follows: $MC_1 = MC_2$

$MC_T = 50 + 4Q = 78$

$MC_1 = 10 + 20Q_1 = 78 \Rightarrow Q_1 = 3.4$

$MC_2 = 60 + 5Q_2 = 78 \Rightarrow Q_2 = 3.6$

7. (a) $Q = K^{1/2} L^{1/2}$

$\bar{K} = 16$, so in short run $Q = 4L^{1/2}$

$MP_L = 2L^{-1/2} = \frac{2}{\sqrt{L}}$

L	4	2	1	1/2	1/4	1/16
MP_L	1	1.41	2	2.83	4	8

$MRP_L = P_{coal} \cdot MP_L = 20 \cdot 2L^{1/2} = \frac{40}{\sqrt{L}}$

so if the mkt. wage rate = \$80 and the firm is a price taker in the labor market, equate w and MRP_L: $80 = 40/\sqrt{L}$, $L = 1/4$

(b) Total Factor Cost if monopsonist:

$TFC_L = w(L) \cdot L$, where $L = \frac{w}{10}$ or $w = 10L$

$TFC_L = 10L^2$

$MFC_L = \frac{dTFC_L}{dL} = 20L$

equate MRP_L and MFC_L:

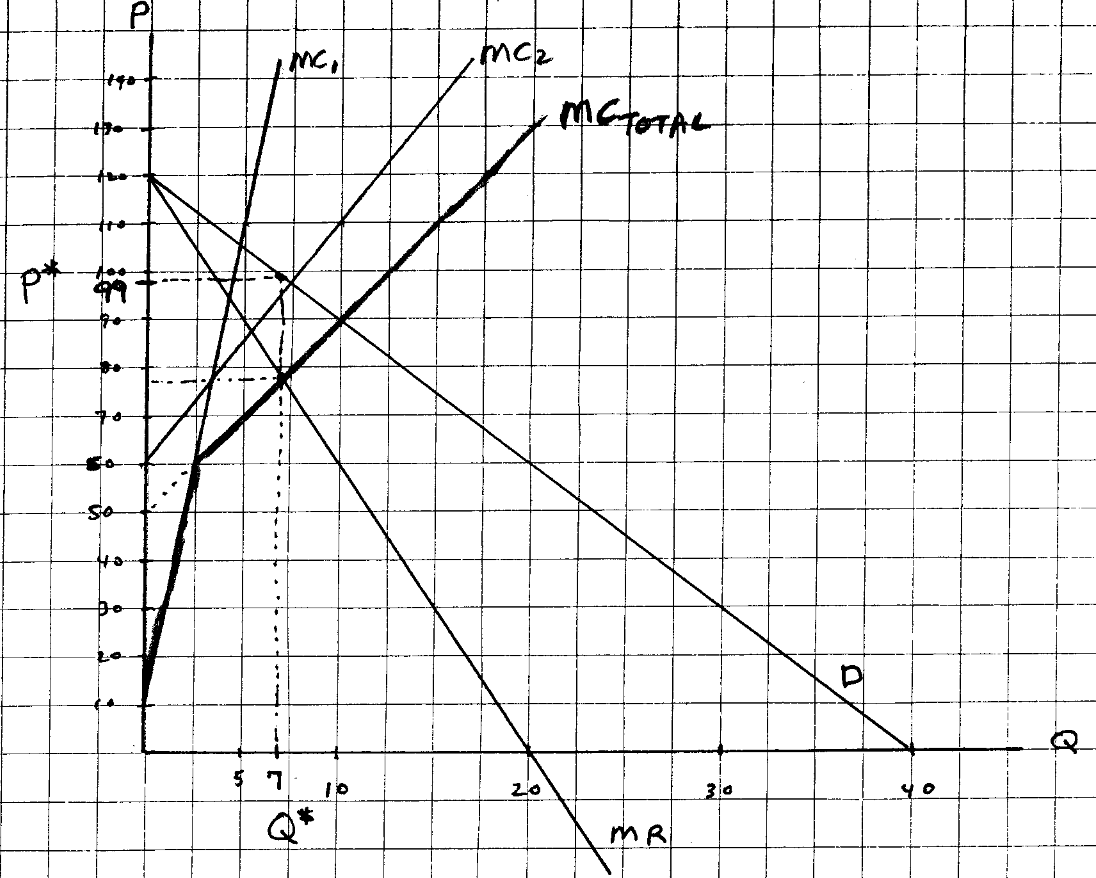
$$\frac{40}{\sqrt{L}} = 20L$$

$$L^{3/2} = 2$$

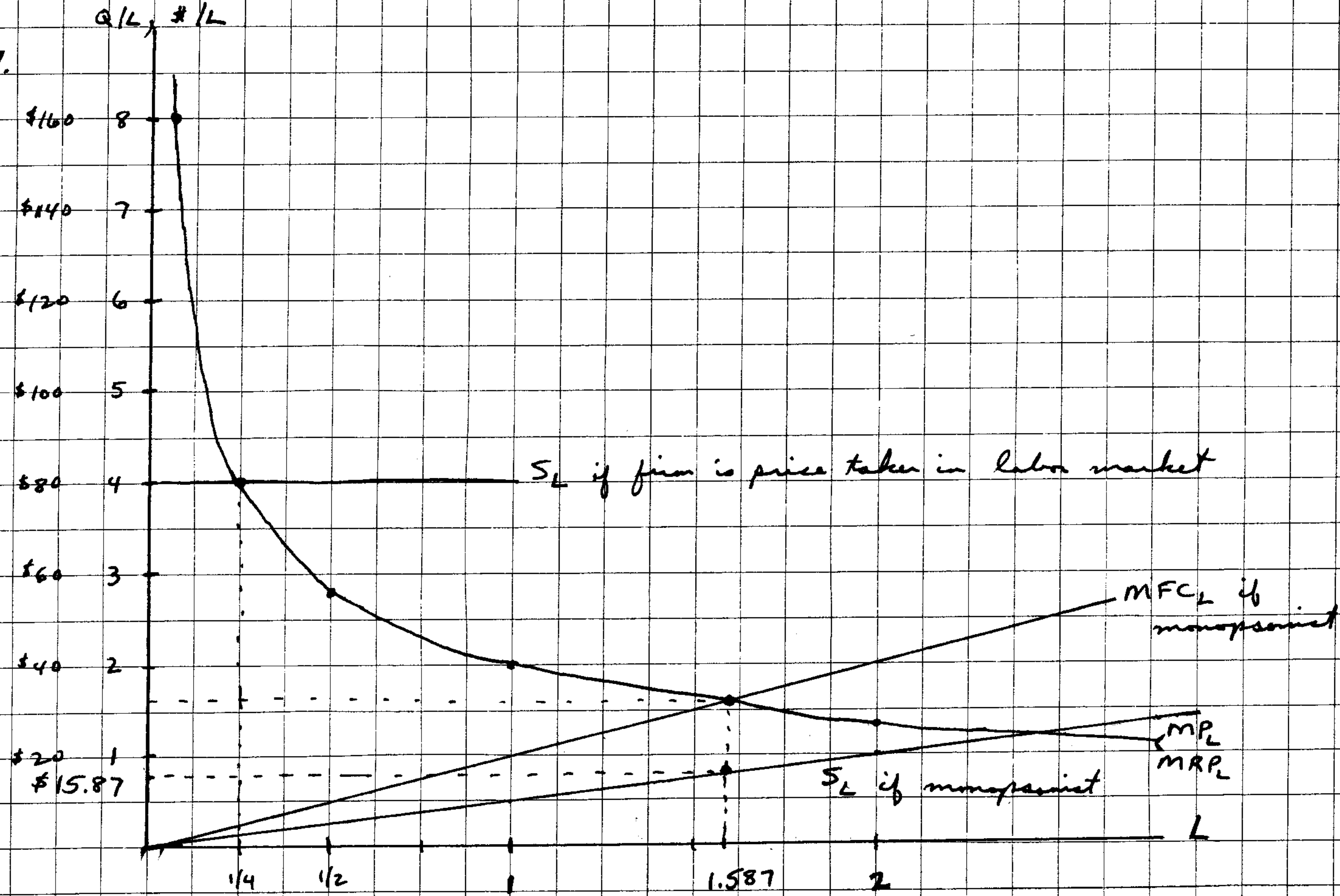
$$L = 1.587$$

$$w = 10L = \$15.87$$

6.


P
MC1
MC2
MCTOTAL
P*
99
D
Q
MR
Q*
5
7
10
20
30
40

7.


Q/L, $/L
$160 8
$140 7
$120 6
$100 5
$80 4
$60 3
$40 2
$20 1
$15.87
S_L if firm is price taker in labor market
MFC_L if monopsonist
MP_L
MRP_L
S_L if monopsonist
L
1/4
1/2
1
1.587
2