Eco 601
Problem Set #1
Fall 2004

Note: This problem set is not to be turned for a grade. It is intended to review some mathematical concepts with which you should be familiar.

From Nicholson, Ch. 2, 8th edition:

Problems

2.1

For each of the following functions of one variable, determine all local maxima and minima and indicate points of inflection (where $f'' = 0$):

a. $f(x) = 4x^3 - 12x$
b. $f(x) = 4x - x^2$
c. $f(x) = x^3$

2.3

The height of a ball t seconds after it is thrown straight up is $-\frac{1}{2}gt^2 + 40t$ (where g is the acceleration due to gravity).

a. If $g = 32$ (as on the earth), when does the ball reach a maximum height? What is that height?
b. If $g = 5.5$ (as on the moon), when does the ball reach a maximum height and what is that height? Can you explain the reasons for the difference between this answer and the answer for part (a)?
c. In general, develop an expression for the change in maximum height for a unit change in g. Explain why this value depends implicitly on the value of g itself.

2.5

Suppose $U = (x,y) = 4x^2 + 3y^2$.

a. Calculate $\partial U/\partial x$, $\partial U/\partial y$.
b. Evaluate these partial derivatives at $x = 1$, $y = 2$.
c. Write the total differential for U.
d. Calculate dy/dx for $dU = 0$—that is, what is the implied trade-off between x and y holding U constant?
e. Show $U = 16$ when $x = 1$, $y = 2$.
f. In what ratio must x and y change to hold U constant at 16 for movements away from $x = 1$, $y = 2$?
g. More generally, what is the shape of the $U = 16$ contour line for this function? What is the slope of that line?

2.6

Suppose that $f(x,y) = xy$. Find the maximum value for f if x and y are constrained to sum to 1. Solve this problem in two ways: by substitution and by using the Langrangian multiplier method.

2.7

Suppose a firm's total revenues depend on the amount produced (q) according to the function

$$TR = 70q - q^2.$$

Total costs also depend on q:

$$TC = q^2 + 30q + 100$$

a. What level of output should the firm produce in order to maximize profits ($TR - TC$)? What will profits be?
b. Show that the second-order conditions for a maximum are satisfied at the output level found in part (a).
c. Does the solution calculated here obey the "marginal revenue equals marginal cost" rule? Explain.