ECO 601 Problem Set #2 KEY Fall 2004

1. $u(x,y) = x^{1/2} y^{1/2}$

$mu_x = \frac{1}{2} x^{-1/2} y^{1/2}$

$mu_y = \frac{1}{2} x^{1/2} y^{-1/2}$

$\Big\} \; MRS_{x,y} = \frac{mu_x}{mu_y} = \frac{Y}{X}$

(a) $u(x,y) = 10 x^{1/2} y^{1/2}$; $MRS_{x,y} = \frac{Y}{X}$

(b) $u(x,y) = X \cdot Y$

$mu_x = Y$, $mu_y = X$, $MRS_{x,y} = \frac{Y}{X}$

(c) $u(x,y) = \ln X + \ln Y$

$mu_x = \frac{1}{X}$, $mu_y = \frac{1}{Y}$, $MRS_{x,y} = \frac{Y}{X}$

So, each of these utility functions have the same $MRS_{x,y}$, and will lead to the same consumption choice for any given P_x, P_y, I.

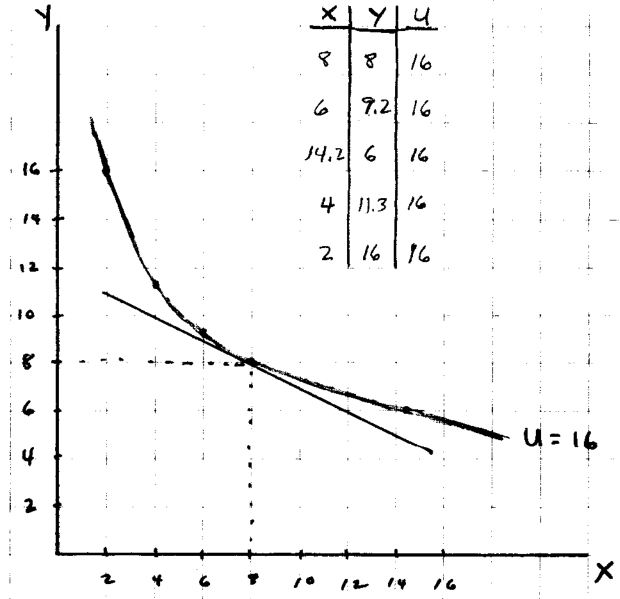
2. $u = 2x^{1/3} y^{2/3}$; current consumption : $x = 8$, $y = 8$

(a) $u = 2(8)^{1/3}(8)^{2/3} = 16$

(b) $MRS_{x,y} = \frac{mu_x}{mu_y} = \frac{\frac{2}{3} X^{-2/3} Y^{2/3}}{\frac{4}{3} X^{1/3} Y^{-1/3}} = \frac{Y}{2X}$

(c)

X	Y	u
8	8	16
6	9.2	16
14.2	6	16
4	11.3	16
2	16	16

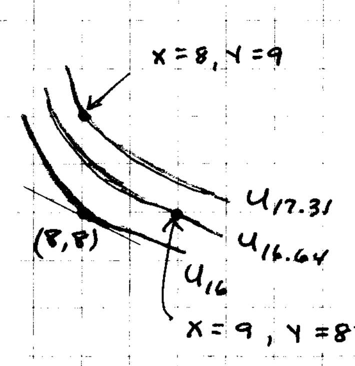


(d) $u(8,8) = 16$

$u(9,8) = 2(9)^{1/3}(8)^{2/3} = 16.64$

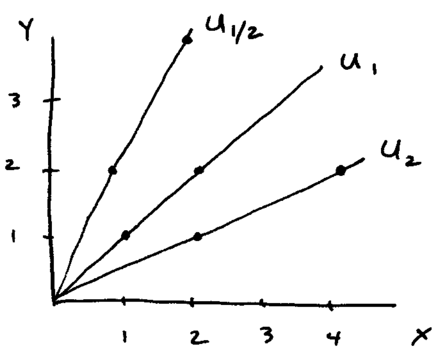
$u(8,9) = 2(8)^{1/3}(9)^{2/3} = 17.31$

$\Rightarrow$



3. $u = \min(.5H, S)$ where H = bites of hamburger and S = sips of milkshake

4. $u = X/Y$ $\quad mu_X = \frac{1}{Y} \quad mu_Y = -\frac{X}{Y^2}$



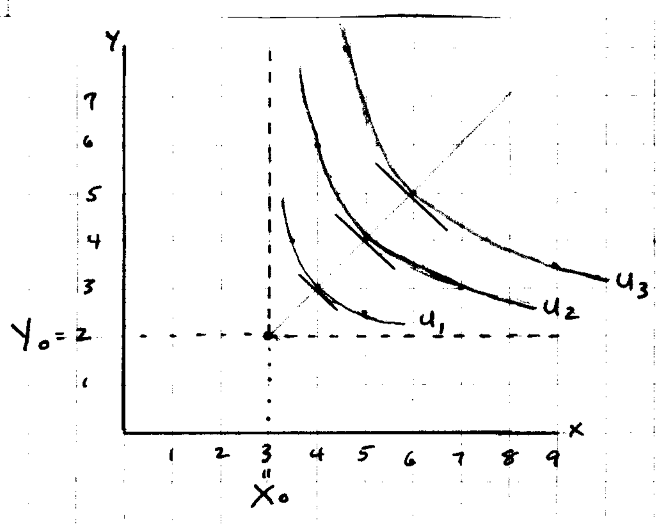
utility is increasing in X and decreasing in Y, so X is a good and Y is a bad.

5. 3.8 a. $MRS = \frac{\partial U/\partial x}{\partial U/\partial y} = \frac{\alpha x^{\alpha-1} y^{\beta}}{\beta x^{\alpha} y^{\beta-1}} = \frac{\alpha}{\beta}(y/x)$

This result does not depend on the sum α + β which, contrary to production theory, has no significance in consumer theory because utility is unique only up to a monotonic transformation.

b. Mathematics follows directly from part a. If α > β the individual values *x* relatively more highly; hence, $dy/dx > 1$ for $x = y$.

c. The function is homothetic in $(x - x_0)$ and $(y - y_0)$, but not in *x* and *y*.



Let $u = (X - X_0)^{\alpha}(Y - Y_0)^{\beta}$. Choose $X_0 = 3$ and $Y_0 = 2$. Also let $\alpha = \beta = 1/2$

X	4	5	7	4	6	9	4.5
Y	3	4	3	6	5	3.5	8
u	1	2	2	2	3	3	3