ECO 601-001 Problem Set #7 KEY Fall 2004

1. $Q = 600K^2L^2 - K^3L^3$

(a) $MP_L = \frac{\partial Q}{\partial L} = 1200K^2L - 3K^3L^2$

$AP_L = \frac{Q}{L} = 600K^2L - K^3L^2$

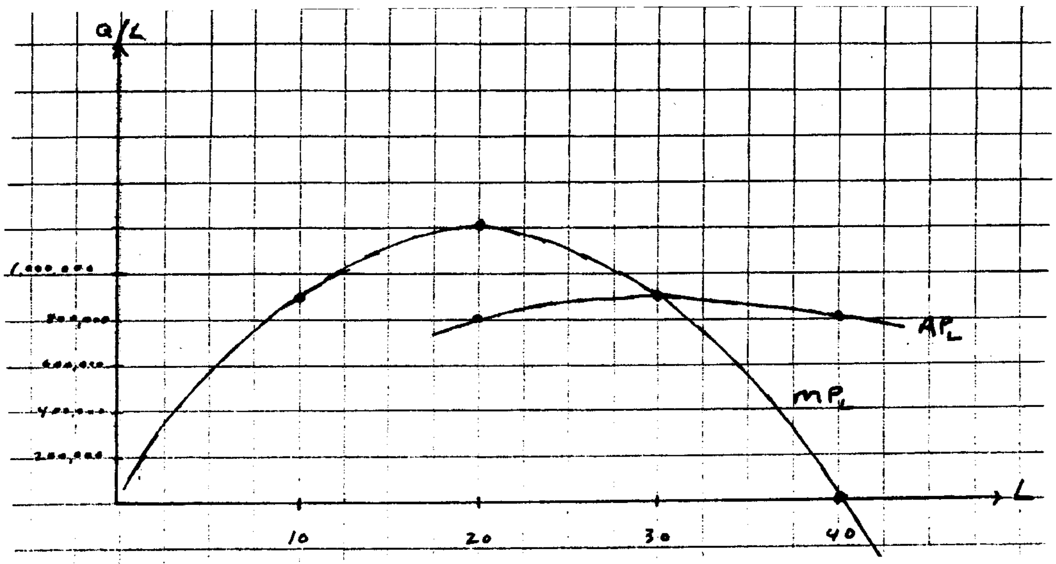
if $K = 10$, $MP_L = 120{,}000L - 3000L^2$

and $AP_L = 60{,}000L - 1000L^2$

critical points: max $MP_L \rightarrow \frac{\partial MP_L}{\partial L} = 120{,}000 - 6000L = 0$, $L = 20$

max $AP_L \rightarrow \frac{\partial AP_L}{\partial L} = 60{,}000 - 2000L = 0$, $L = 30$

$MP_L = 0 \rightarrow 120{,}000L - 3000L^2 = 0$, $L = 40$



(b) It is uneconomic to use the input combination $K = 15$, $L = 30$ if either MP_L or MP_K are negative:

$MP_L = 1200K^2L - 3K^3L^2$

$= (1200)(225)(30) - (3)(3375)(900) = -1{,}012{,}500$

$MP_K = 1200KL^2 - 3K^2L^3$

$= (1200)(15)(900) - (3)(27000)(225) = -2{,}025{,}000$

so both MP_L and MP_K are negative.

2. (a) $Q = 2K^{1/2}L^{1/2}$

$$MRTS_{L,K} = \frac{MP_L}{MP_K} = \frac{K^{1/2}L^{-1/2}}{K^{-1/2}L^{1/2}} = \frac{K}{L}$$

$Q = (K^{1/2} + L^{1/2})^2$

$$MRTS_{L,K} = \frac{MP_L}{MP_K} = \frac{2(K^{.5}+L^{.5})(.5)L^{-.5}}{2(K^{.5}+L^{.5})(.5)K^{-.5}} = \frac{K^{.5}}{L^{.5}}$$

(b) $Q = 2K^{1/2}L^{1/2}$

if $Q=4$: $4 = 2K^{1/2}L^{1/2}$

$L = \frac{4}{K}$

K	1	4	2
L	4	1	2

if $Q=8$: $8 = 2K^{1/2}L^{1/2}$

$L = \frac{16}{K}$

K	1	16	4	2	8
L	16	1	4	8	2

$Q = (K^{1/2} + L^{1/2})^2$

if $Q=4$: $4 = (K^{1/2}+L^{1/2})^2$

$K^{1/2} + L^{1/2} = 2$

$L = 4 - 4K^{1/2} + K$

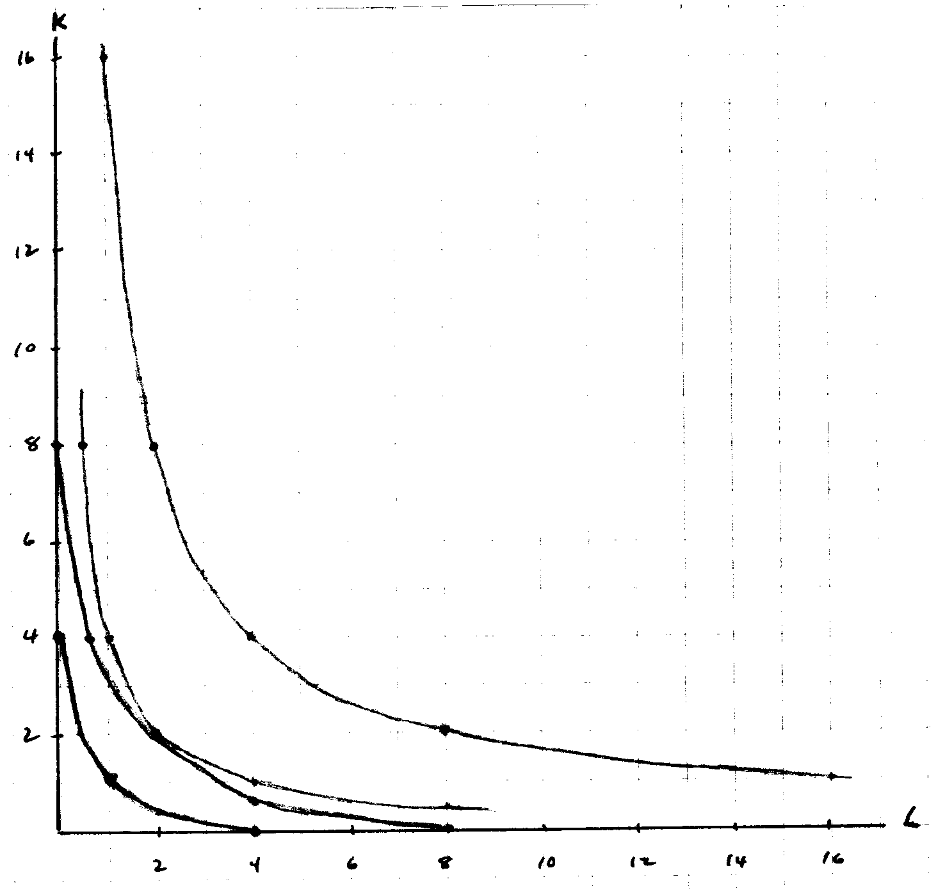
K	4	0	1	2	.34
L	0	4	1	.34	2

if $Q=8$: $8 = (K^{1/2}+L^{1/2})^2$

$K^{1/2}+L^{1/2} = \sqrt{8}$

$L = 8 - 5.66K^{1/2} + K$

K	8	0	4	.68	2
L	0	8	.68	4	2



3. (a) $MP_T = .35\,T^{-.3}B^{.4}$

$MP_B = .2\,T^{.7}B^{-.6}$

(b) $\frac{\partial MP_T}{\partial T} = -.105\,T^{-1.3}B^{.4} < 0$

$\frac{\partial MP_B}{\partial B} = -.12\,T^{.7}B^{-1.6} < 0$

so yes, diminishing marginal productivity

(c) $MRTS = MP_T/MP_B = \frac{.35T^{-.3}B^{.4}}{.2T^{.7}B^{-.6}} = 1.75\,B/T$

yes, it is homothetic since MRTS depends only on B/T and not on scale.

(d) diminishing MRTS if $f_B^2 f_{TT} - 2f_B f_T f_{BT} + f_T^2 f_{BB} < 0$

$f_B^2 f_{TT} \Rightarrow (.04\,T^{1.4}B^{-1.2})(-.105\,T^{-1.3}B^{.4})$

$-2f_B f_T f_{BT} \Rightarrow -2(.2T^{.7}B^{-.6})(.35\,T^{-.3}B^{.4})(.14\,T^{-.3}B^{-.6})$

$f_T^2 f_{BB} \Rightarrow (.1225\,T^{-.6}B^{.8})(-.12\,T^{.7}B^{-1.6})$

since all three terms are negative, the entire expression will be negative, so MRTS is diminishing.

(e) $\ln E = \ln .5 + .7\ln T + .4\ln B$

$\frac{\partial \ln E}{\partial \ln T} = .7 \qquad \frac{\partial \ln E}{\partial \ln B} = .4$

(f) $.7 + .4 = 1.1$, so increasing returns to scale. Perhaps there are some mass production techniques that can be used as the size (scale) of the school increases.

4. $Q = f(L, K)$

translog production function:

$$\ln Q = \beta_0 + \beta_L \ln L + \beta_K \ln K + \tfrac{1}{2}\beta_{LL}(\ln L)^2$$
$$+ \tfrac{1}{2}\beta_{LK} \ln L \ln K + \tfrac{1}{2}\beta_{KL} \ln K \ln L + \tfrac{1}{2}\beta_{KK}(\ln K)^2$$

or
$$\ln Q = \beta_0 + \beta_L \ln L + \beta_K \ln K + \beta_{LK} \ln K \ln L + \tfrac{1}{2}\beta_{LL}(\ln L)^2 + \tfrac{1}{2}\beta_{KK}(\ln K)^2$$

(a) Cobb-Douglas: $Q = A L^{\alpha} K^{\beta}$

$$\ln Q = \ln A + \alpha \ln L + \beta \ln K$$

translog reduces to this form if $\beta_{LL} = \beta_{LK} = \beta_{KK} = 0$:

$$\ln Q = \beta_0 + \beta_L \ln L + \beta_K \ln K$$

(b) increase all inputs by proportion $\underline{t}$:

does $tQ = f(tL, tK)$?

$$\Rightarrow \beta_0 + \beta_L \ln tL + \beta_K \ln tK + \beta_{LK} \ln tL \ln tK + \tfrac{1}{2}\beta_{LL} \ln tL \ln tL + \tfrac{1}{2}\beta_{KK} \ln tK \ln tK$$

$$= \beta_0 + \beta_L \ln L + \beta_L \ln t + \beta_K \ln K + \beta_K \ln t + \beta_{LK} \ln K \ln L + \beta_{LK}(\ln t)^2 + \tfrac{1}{2}\beta_{LL} \ln L \ln L + \tfrac{1}{2}\beta_{LL}(\ln t)^2 + \tfrac{1}{2}\beta_{KK} \ln K \ln K + \tfrac{1}{2}\beta_{KK}(\ln t)^2$$

$$= \beta_0 + \beta_L \ln L + \beta_K \ln K + \beta_{LK} \ln K \ln L + \tfrac{1}{2}\beta_{LL} \ln L \ln L + \tfrac{1}{2}\beta_{KK} \ln K \ln K + (\beta_L + \beta_K)\ln t + (\beta_{LK} + \tfrac{1}{2}\beta_{LL} + \tfrac{1}{2}\beta_{KK})(\ln t)^2$$

if $\beta_L + \beta_K = 1$ and $\beta_{LK} + \tfrac{1}{2}\beta_{LL} + \tfrac{1}{2}\beta_{KK} = 0$

then this equals to $\ln Q + \ln t = \ln tQ$,

which is what is necessary for constant returns to scale.