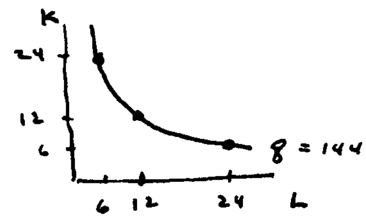
ECO 601-001 Problem Set #8 KEY Fall 2004

1. $q = 10K^{1/2}L^{1/2}$

(a) $q = 120$: $120 = 10\,K^{1/2}L^{1/2}$

$K^{1/2}L^{1/2} = 12$, so $K \cdot L = 144$

K	12	16	9	18	8	24	6
L	12	9	16	8	18	6	24



(b) for cost minimization : $\dfrac{MP_L}{MP_K} = \dfrac{w}{v}$; $w = 6$, $v = 6$

$MP_L = 5\,K^{1/2}L^{-1/2}$

$MP_K = 5\,K^{-1/2}L^{1/2}$

$MRTS_{L,K} = \dfrac{MP_L}{MP_K} = \dfrac{5K^{1/2}L^{-1/2}}{5K^{-1/2}L^{1/2}} = \dfrac{K}{L}$

cost minimum $\Rightarrow$ $\dfrac{K}{L} = \dfrac{6}{6}$ $\Rightarrow$ $K = L$

for $q = 120$, $K \cdot L = 144$, and since $K = L$,

$K^* = 12$ and $L^* = 12$

(c) $q = 10\,K^{1/2}L^{1/2}$; For cost minimization, $K = L$;

so $q = 10\,K^{1/2}K^{1/2}$ and $q = 10\,L^{1/2}L^{1/2}$

or $q = 10K$, $K = \frac{q}{10}$ and $q = 10L$, $L = \frac{q}{10}$

substitute into total cost definition :

$TC = vK + wL = 6\left(\frac{q}{10}\right) + 6\left(\frac{q}{10}\right)$

$TC = 1.2q$

$MC = \dfrac{dTC}{dq} = 1.2$

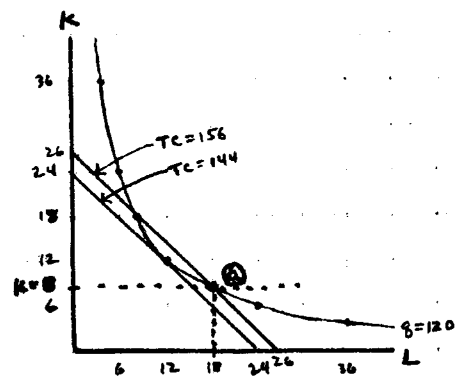
Alternatively, an additional unit of labor can produce 5 additional sandwiches at an additional cost of \$6, which works out to \$1.20 each.

2. $q = 10K^{1/2}L^{1/2}$, and $\bar{K} = 8$; $w = 6$, $v = 6$

(a) $KL = 144$ for $q = 120$

if $K = 8$, then $L = 18$

$TC = 6(8) + 6(18) = 156$



(b) slope of isoquant $= MRTS_{L,K} = \frac{MP_L}{MP_K} = \frac{K}{L} = \frac{8}{18}$

slope of isocost $= \frac{w}{v} = \frac{6}{6} = 1$

$\frac{8}{18} < 1$, so $\frac{MP_L}{MP_K} < \frac{w}{v}$

we are underutilizing capital and overutilizing labor.

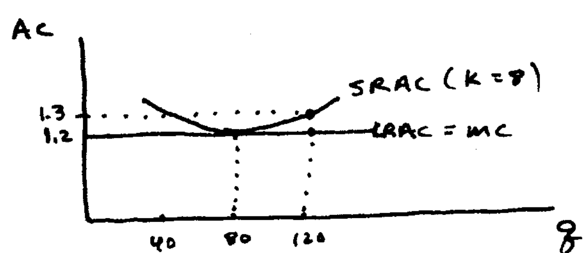
(c) From 2(c) we know that $TC = 1.2q$

Hence the LRAC is flat at $AC = 1.2 = MC$

But to produce $q = 120$ with $K = 8$ and $L = 18$,

we know that $TC = 156$ and $AC = \frac{156}{120} = 1.3$

So,



3.

a) Starting with the tangency condition we have

$$\frac{MP_L}{MP_K} = \frac{w}{r}$$

$$\frac{[L^{1/2} + K^{1/2}]L^{-(1/2)}}{[L^{1/2} + K^{1/2}]K^{-(1/2)}} = \frac{2}{1}$$

$$\frac{K}{L} = 4$$

$$K = 4L.$$

Plugging this into the total cost function yields

$$Q = [L^{1/2} + (4L)^{1/2}]^2$$

$$Q = [3L^{1/2}]^2$$

$$Q = 9L$$

$$L = \frac{Q}{9}.$$

Inserting this back into the solution for K above gives

$$K = \frac{4Q}{9}$$

b)

$$TC = 2\left(\frac{Q}{9}\right) + \frac{4Q}{9}$$

$$TC = \frac{2Q}{3},$$

$$AC = \frac{TC}{Q} = \left(\frac{2Q}{3}\right)\Big/Q$$

$$AC = \frac{2}{3}.$$

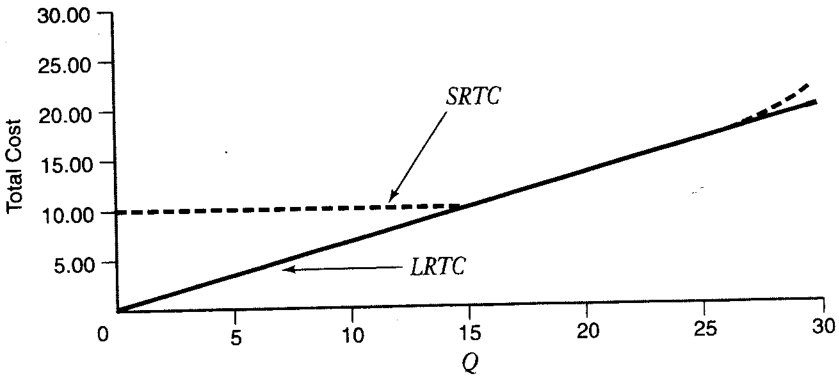


Figure for Problem 8.3.e.

c) When $Q < 9$, the firm needs no labor. If $Q > 9$, the firm does hire labor. Setting $\overline{K} = 9$ and plugging in for capital in the production function yields

$$Q = [L^{1/2} + 9^{1/2}]^2$$

$$Q^{1/2} = L^{1/2} + 3$$

$$L^{1/2} = Q^{1/2} - 3$$

$$L = [Q^{1/2} - 3]^2,$$

Thus,

$$L = \begin{cases} (Q^{1/2} - 3)^2 + 9 & \text{when } Q > 9. \\ 0 & \text{when } Q \leq 9 \end{cases}$$

d)

$$TC = \begin{cases} 2(Q^{1/2} - 3)^2 + 9 & \text{when } Q > 9. \\ 9 & \text{when } Q \leq 9 \end{cases}$$

Graphically, short-run and long-run total cost are shown in the figure.

4. $C = 3Q^{.5} w^{.5} v^{.5} u^{.5}$ is not a well-behaved cost function because it is not homogeneous of degree one in input prices. so FALSE.