ECO 601 1^{st} Test Fall 2004

1. (10pts.) Draw this picture and briefly explain what is wrong with it. Mad Max is asked to reveal his preferences for several bundles. Bundle A consists of 5 Clothing and 2 Food. Bundle B consists of 4 Clothing and 4 Food. Bundle C consists of 2 Clothing and 5 Food. Max is indifferent between A and B, but prefers C to A.

2. (10 pts.) True, False, or Uncertain and Explain: Consider a lump-sum income transfer versus a per-unit subsidy for a good X. If a consumer is indifferent between the two, the lump-sum transfer will be more costly to provide than the per-unit subsidy.

3. (24 pts.) Graham and Alexander both consume cell phone conversations (X) and other goods (Y). Their current service provider sells them service at \$.50 per minute. They can purchase other goods at a price of \$1 per unit. Each of them has a monthly income of \$400. Graham is not very popular, and only talks on his phone 200 minutes per month. Alexander has more friends with whom he must stay in touch and talks on his phone 400 minutes per month.
   a) Illustrate each of their consumption choices in the attached diagram.
   b) A new cell phone provider comes to town and offers an alternative service plan. For a monthly fee of \$100 you get 400 "free" minutes. Additional minutes beyond the first 400 can be purchased at a per minute price of \$.25. Graham decides to stick with his original cell phone plan. Is Graham's decision rational? Explain using your diagram.
   c) Alexander decides to go with the new service plan. He increases his usage to 600 minutes per month. When asked to compare the marginal utility he gets from talking on his cell phone with the marginal utility he gets from consuming other goods, he says that $MU_X = 10$ and $MU_Y = 20$. Illustrate Alexander's new consumption choice using an indifference curve and budget constraint.

4. (24 pts.) Given that utility depends on consumption of X and Y: $U(X,Y) = 2*Y + 2*\ln X$.
   a) Derive the consumer's ordinary (Marshallian) demand functions for X and Y.
   b) Suppose $I = 12$ and $P_Y = 4$. Graph d_X on the attached diagram. Suppose I increases to 16. What happens to d_X? Show in your diagram.
   c) Using the Slutsky equation for X and your results from (b), solve for the compensated (Hicksian) demand curve for X and graph it on the same diagram as the ordinary demand curve.

5. (32 pts.) Hard Times Burt has a friend, Doug Cobb, who also consumes rotgut whiskey (X) and jelly doughnuts (Y). Doug's utility is given by $U(X,Y) = X^{.5}Y^{.5}$. His indirect utility function is given by $V = .5IP_X^{-.5}P_Y^{-.5}$. Doug's Marshallian demands for X and Y are given by $X^* = I/2P_X$ and $Y^* = I/2P_Y$. His Hicksian demand for X is given by $X^c = VP_Y^{.5}P_X^{-.5}$
   a) Is whiskey a gross substitute or a gross complement for doughnuts? (i.e. calculate $\partial X/\partial P_Y$). Illustrate in the attached diagram using $I = 8$, $P_X = 1$, $P_Y^0 = 4$, and $P_Y^1 = 2$.
   b) Calculate Doug's level of utility before and after the change in the price of Y.
   c) Write the general Slutsky equation for cross-price effects and use it to decompose $\partial X/\partial P_Y$ into its income and substitution components. Then reconcile your results with your answer to part (a).
   d) Illustrate the income and substitution effects of the change in the price of Y from 4 to 2 in your diagram.

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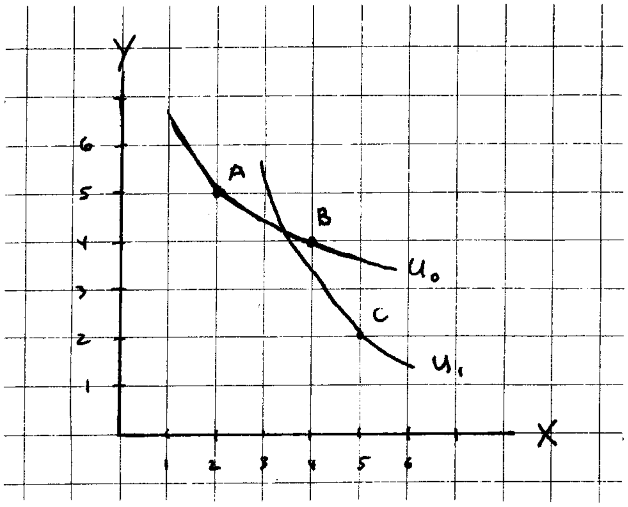
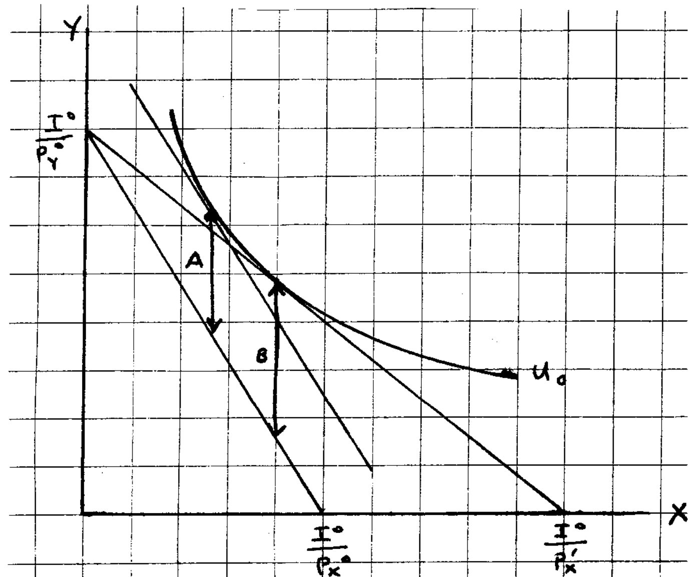
1\.



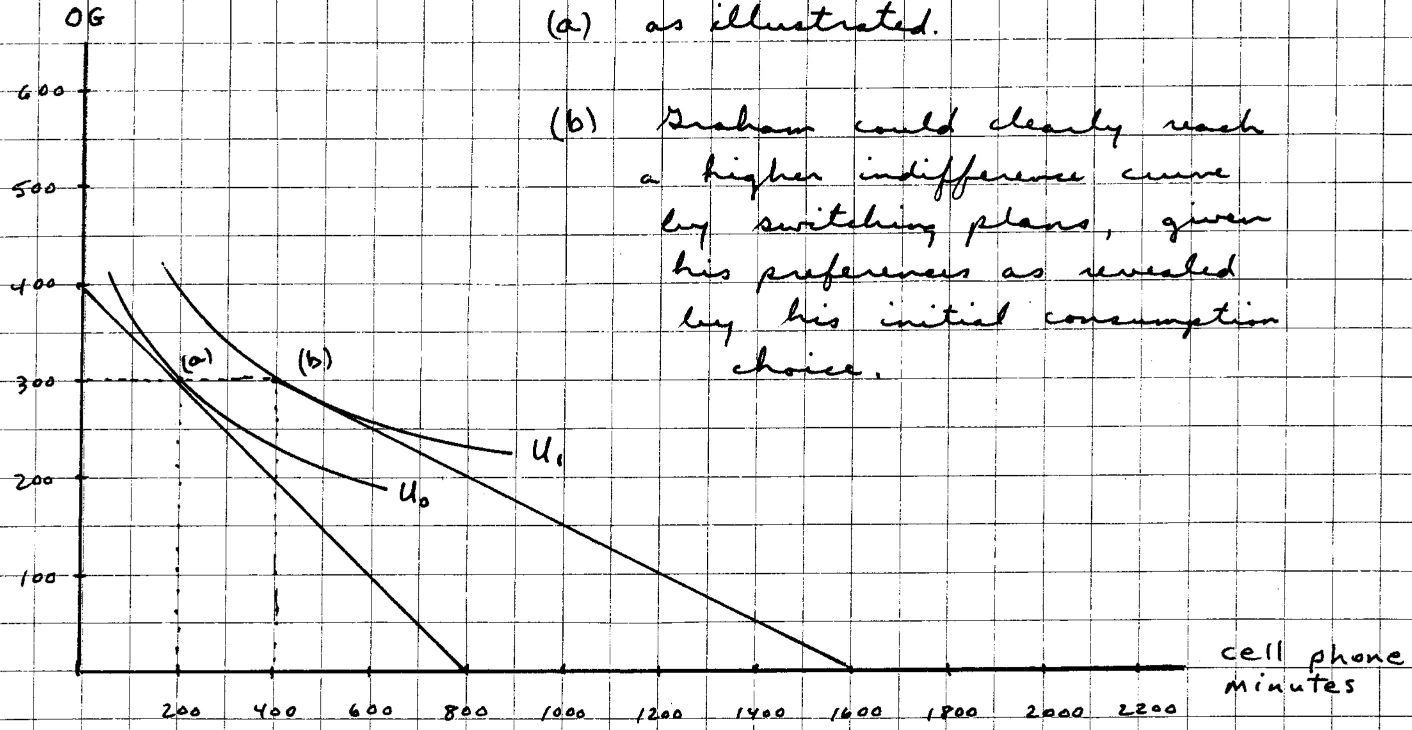
If the consumer is indifferent between A and B, they lie on the same indifference curve U_0. If C is preferred to A, C lies on a higher indifference curve than A. This is only possible if the indifference curves intersect or if the consumer has an increasing MRS.

2\.



FALSE. The lump sum transfer that leaves the household indifferent will be less costly than the per unit subsidy. This can be seen in the above diagram where the distance A represents the lump sum transfer and the distance B represents the per unit subsidy.

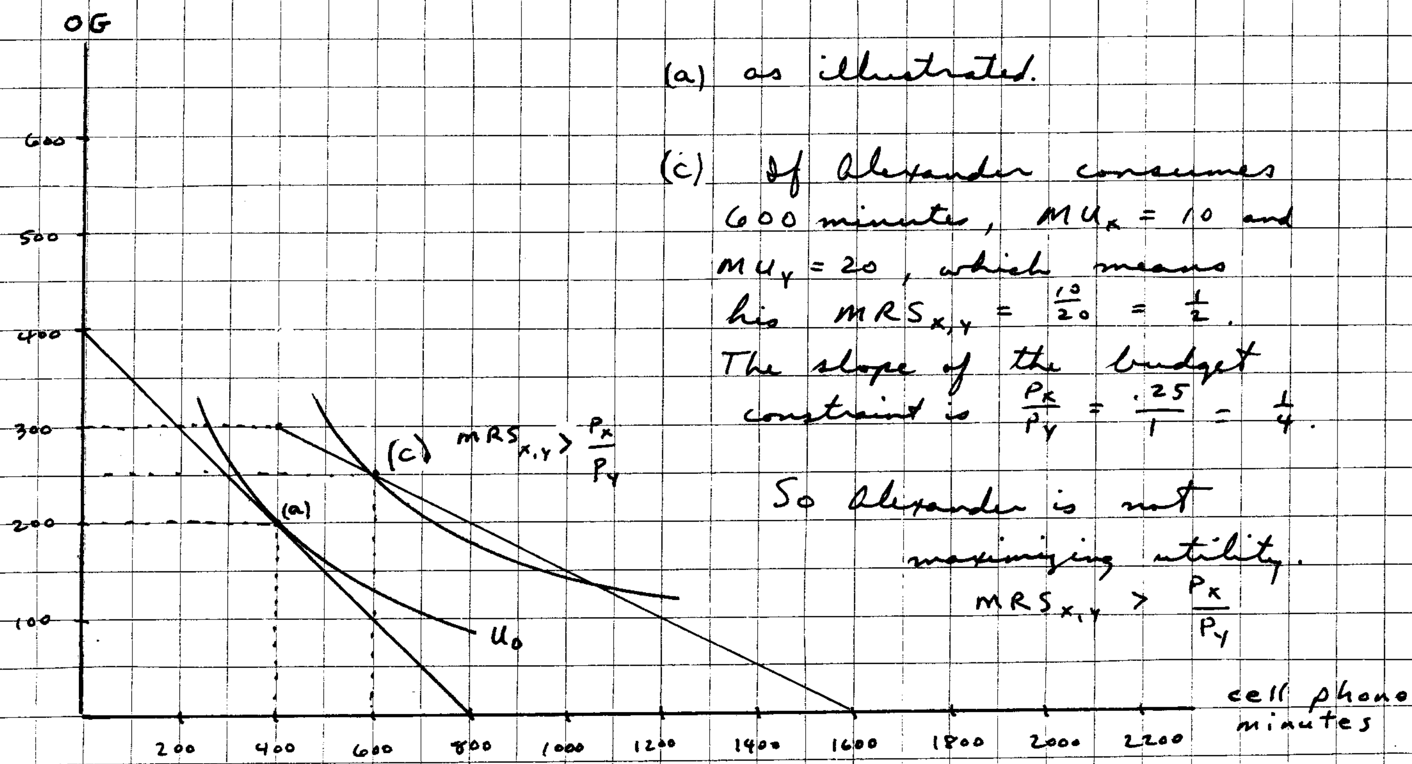
3.



(a) as illustrated.

(b) Graham could clearly reach a higher indifference curve by switching plans, given his preferences as revealed by his initial consumption choice.

GRAHAM



(a) as illustrated.

(c) If Alexander consumes 600 minutes, $MU_x = 10$ and $MU_y = 20$, which means his $MRS_{x,y} = \frac{10}{20} = \frac{1}{2}$. The slope of the budget constraint is $\frac{P_x}{P_y} = \frac{.25}{1} = \frac{1}{4}$.

So Alexander is not maximizing utility.

$MRS_{x,y} > \frac{P_x}{P_y}$

ALEXANDER

4. $u(X,Y) = 2Y + 2\ln X$

(a) $\mathcal{L} = 2Y + 2\ln X + \lambda(I - P_x X - P_Y Y)$

$$\frac{\partial \mathcal{L}}{\partial X} = \frac{2}{X} - \lambda P_x = 0$$
$$\frac{\partial \mathcal{L}}{\partial Y} = 2 - \lambda P_Y = 0$$
$$\frac{\partial \mathcal{L}}{\partial \lambda} = I - P_x X - P_Y Y = 0$$

$$\left.\begin{array}{l}\frac{\partial \mathcal{L}}{\partial X} = 0\\ \frac{\partial \mathcal{L}}{\partial Y} = 0\end{array}\right\} \quad \frac{1}{X} = \frac{P_x}{P_Y} \Rightarrow P_x X = P_Y$$

using above condition: $P_x X = P_Y$

$$X^* = \frac{P_Y}{P_x}$$

$$I - P_Y - P_Y Y = 0 \quad , \quad Y^* = \frac{I - P_Y}{P_Y}$$

(b) $I = 12$ and $P_Y = 4$

P_x	4	3	2	1
X^*	1	$\frac{4}{3}$	2	4

If $I = 16$ and $P_Y = 4$, then the Marshallian demand curve for X does not change!

(c) $$\frac{\partial X}{\partial P_x} = \left.\frac{\partial X}{\partial P_x}\right|_{\bar{u}} - X\frac{\partial X}{\partial I}$$

but $\frac{\partial X}{\partial I} = 0$, so $\left.\frac{\partial X}{\partial P_x}\right|_{\bar{u}} = \frac{\partial X}{\partial P_x}$

and $X^c = P_Y / P_x$, i.e. the Marshallian and the Hicksian demand curves for X coincide.

5. $u(X,Y) = X^{1/2} Y^{1/2}$

$V = .5\, I\, P_X^{-1/2} P_Y^{-1/2}$

$X^* = I/2P_X \qquad Y^* = I/2P_Y$

$X^c = V P_Y^{1/2} P_X^{-1/2}$

(a) $\frac{dX}{dP_Y} = 0$, so whiskey is neither a gross substitute nor a gross complement for doughnuts

$I = 8,\ P_X = 1,\ P_Y = 4:\quad X^* = 4 \ \&\ Y^* = 1$

$I = 8,\ P_X = 1,\ P_Y = 2:\quad X^* = 4 \ \&\ Y^* = 2$

(b) $u(4,1) = 4^{1/2}\, 1^{1/2} = 2$

$u(4,2) = 4^{1/2}\, 2^{1/2} = 2\sqrt{2} = 2.83$

(c) $\frac{dX}{dP_Y} = \frac{dX}{dP_Y}\Big|_{\bar{u}} - Y\frac{dX}{dI}$

- substitution component: $\frac{dX}{dP_Y}\Big|_{\bar{u}}$

given $X^c = V P_Y^{1/2} P_X^{-1/2}$

$\frac{dX}{dP_Y}\Big|_{\bar{u}} = \frac{1}{2} V P_Y^{-1/2} P_X^{-1/2}$

but $V = u(X,Y) = \left(\frac{I}{2P_X}\right)^{1/2}\left(\frac{I}{2P_Y}\right)^{1/2} = .5\, I\, P_X^{-1/2} P_Y^{-1/2}$

so $\frac{dX}{dP_Y}\Big|_{\bar{u}} = \frac{1}{2}\left(\frac{1}{2} I P_X^{-1/2} P_Y^{-1/2}\right) P_X^{-1/2} P_Y^{-1/2} = \frac{I}{4P_X P_Y}$

- income component: $-Y\frac{dX}{dI}$

$-Y\frac{dX}{dI} = -\left(\frac{I}{2P_Y}\right)\left(\frac{1}{2P_X}\right) = -\frac{I}{4P_X P_Y}$

So the income effect exactly offsets the substitution effect, and the total effect is zero.

(d) $I = 8,\ P_X = 1,\ P_Y = 4,\ u = 2$

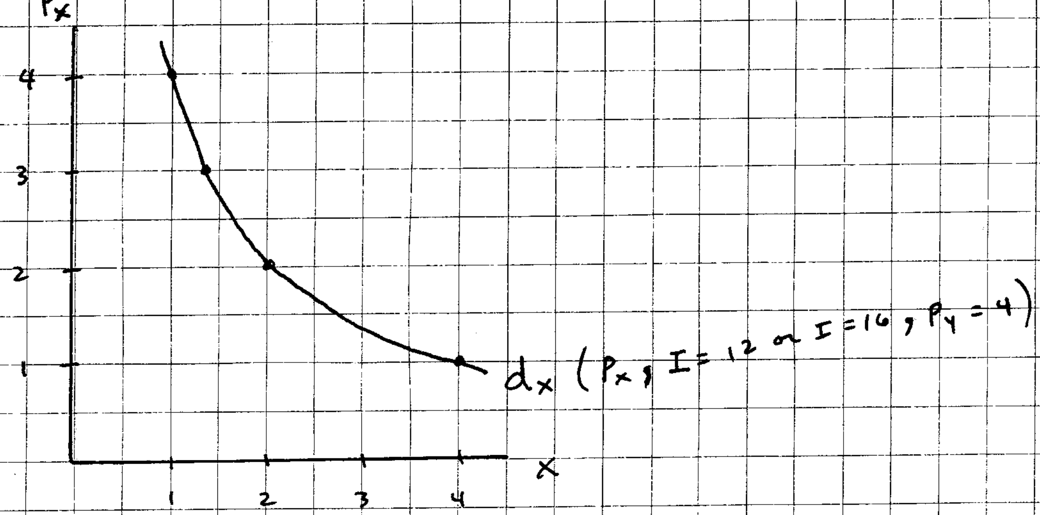
$MRS_{X,Y} = \frac{Y}{X} = \frac{P_X}{P_Y} = \frac{1}{4}$ initially

P_Y falls to 2, but keep U constant for subst. effect:

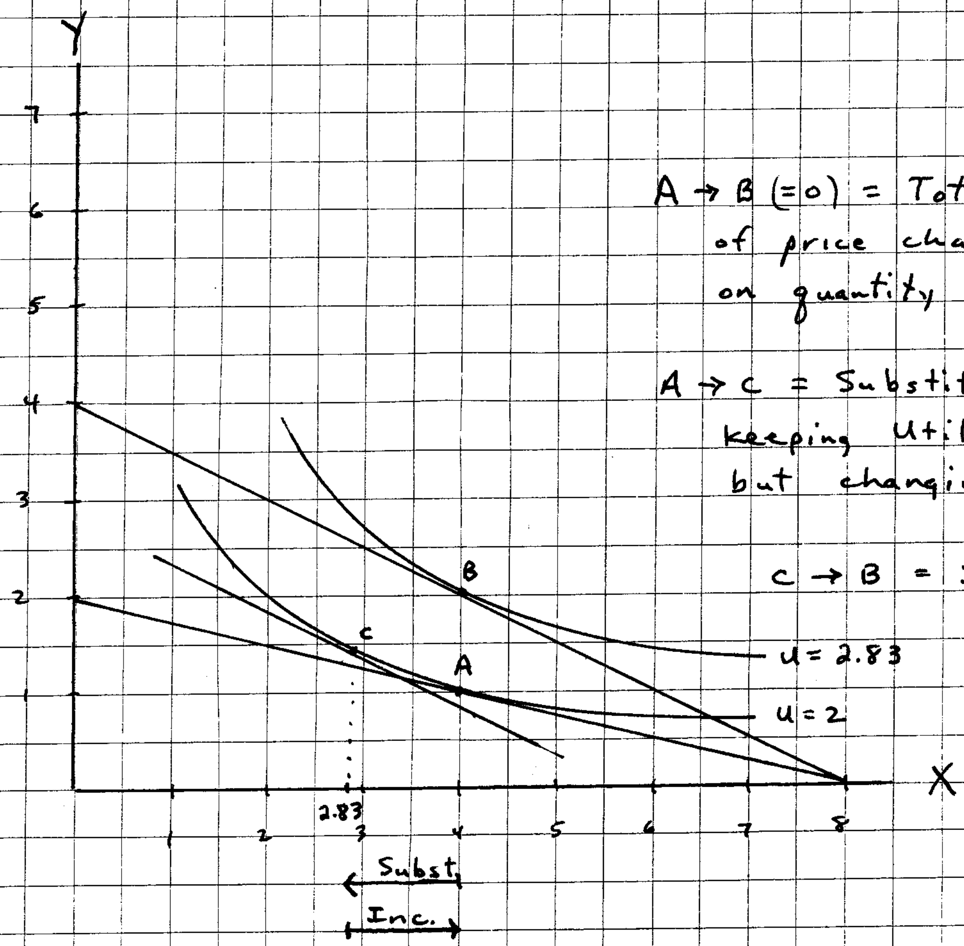
$\frac{Y}{X} = \frac{1}{2}$, so $X = 2Y$; $(2Y)^{1/2}(Y)^{1/2} = 2$, $Y = 1.41$

and $X = 2.83$ (alternatively, $X^c = u P_Y^{1/2} P_X^{-1/2} = 2.83$)

4.



5.



A → B (= 0) = Total effect
of price change in Y
on quantity demanded of X

A → C = Substitution effect,
keeping Utility constant,
but changing P_x / P_y

C → B = Income effect