ECO 601 2^{nd} Test Fall 2004

1. (15 pts.) Velma is getting ready to graduate and has a job offer from a small company that will pay her a token salary of \$4,000, plus a bonus of \$100,000 if the company manages to become profitable during the upcoming year. Velma assesses the company's chances of success as 50-50. Suppose Velma's utility of income function is given by: $U = I^{.5}$.
    a) What is her expected level of income? What is her expected utility if she takes the job?
    b) This company really wants to hire her. What salary with certainty would they have to offer Velma that she would consider equivalent to the above bonus plan?
    c) In some industries worker pay is tied to the profitability of the employer, while in other industries pay is guaranteed. Other things the same, would you expect average pay to be higher in one industry than the other? Briefly explain your reasoning.

2. (10 pts.) Blue Nile is an online diamond retailer. It competes with brick-and-mortar jewelry stores that also sell diamond jewelry. Recently a stock analyst wrote the following:

    > "Wholesale prices of diamonds and precious metals have been on the rise in recent months. That has been bad news for Blue Nile, because it does not carry an inventory of loose diamonds. Instead, it purchases diamonds from its suppliers after customers have ordered them from the company. This means the company is much more at the mercy of changing costs than traditional retailers who often hold inventory for a year or longer. In turn, Blue Nile has passed those rising costs on to consumers. Even if diamond prices stabilize in the near term, Blue Nile's pricing problem likely won't go away anytime soon. That's because the company's offline competitors likely won't need to pass on rising diamond costs for another year or so, when they turn over their current inventory."

    What do you think of this person's analysis of the economic costs of diamond retailers?

3. (25 pts.) As VP for Operations of Fallen Arch Coal Co., you commission a study of all the coal mines that your company operates. Data are collected on quarterly output of coal in tons, seam thickness (S) in inches in each mine, and the number of labor hours (L) worked in each mine per quarter. Past experience indicates that a Cobb-Douglas production function ($Q=AS^{\alpha}L^{\beta}$) represents the production technology well. Your research assistants regress the natural log of output on the natural logs of seam thickness and labor. The results are displayed below.

    Ln Q = 0.138 + 0.463 Ln(Seam Thickness) + 0.856 Ln (Labor Hours)

    a) Write the equation for a Cobb-Douglas production function, i.e. express Q as a function of S and L. (Remember, if ln $A = x$, then $A = e^x$.) Derive the expressions for the marginal product of labor and the marginal product of seam thickness.

b) For a typical Fallen Arch coal mine, seam thickness equals 60 inches, and 45 workers each work 40 hours for 13 weeks each quarter. Compute the marginal products of both seam thickness and labor.

c) Suppose the wage rate for additional labor hours is \$25 per hour and it costs \$3000 per quarter to lease coal deposits that are one inch thicker than the ones your company is currently mining. What do you think about your company's current input mix? Are you minimizing cost?

4. (20 pts.) Suppose that $Q = L^{1/3}K^{2/3}$.
   a) Given input prices w and v, solve for the firm's total cost function.
   b) Graph the firm's total cost function for w = \$2 and v = \$2.
   c) Pick a point on the firm's total cost function in (b) and illustrate it in an isoquant-isocost diagram, carefully labeling all relevant information.

5. (20 pts.) John and Bill operate a lawn mowing service in the highly competitive lawn care industry. The going market price for lawn mowing is \$20 per acre. Their costs are given by $TC = 0.1q^2 + 10q + 50$, where q = the number of acres John and Bill choose to cut each day.
   a) How many acres should they choose to cut in order to maximize profit?
   b) Calculate their maximum daily profit.
   c) Graph these results and label their supply curve.

6. (10 pts.) McLazarus, a local department store, is attempting to determine the profit-maximizing price for one of its best-selling perfumes. The marginal cost to the store of each bottle of perfume is \$15. Based on past price experiments, the manager of the perfume department has found that:

At a price of:	A 10% change in price is associated with a change in quantity of:
\$20	30%
\$25	25%
\$30	22%

Given this information, and if you are only going to choose between a price of \$20, \$25, or \$30, what should you charge? Briefly explain your reasoning.

Eco 601 2nd Test KEY Fall 2004

1. (a) $E(\text{income}) = \frac{1}{2}(4000) + \frac{1}{2}(104\,000) = \$54{,}000$

$E(\text{utility}) = \frac{1}{2}U(4000) + \frac{1}{2}U(104000)$; $U = I^{1/2}$

$= \frac{1}{2}(63.25) + \frac{1}{2}(322.49) = 192.87$

(b) $U = I^{1/2}$

$192.87 = I^{1/2}$, $I = \$37{,}198$

(c) If workers are risk averse, they will require a premium to bear the risk of the uncertain compensation scheme. So we would expect average income/pay to be higher among those workers.

2. The "cost of goods sold" is the same for Blue Nile as it is for bricks-and-mortar jewelry stores — it is the current opportunity cost of a diamond. In placing a value on one's inventory, historical costs are not relevant. Instead one should use the current replacement cost of the item. So it does not matter whether one bought a one-carat diamond a year ago for \$1000 or six months ago for \$750 or yesterday for \$1250. When one sells that diamond today and has to buy a new one for the display case, the cost of the diamond that was sold is what one has to pay for a replacement. So this stock analyst does not understand economic costs and hence economic profit.

3. (a)

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$$Q = A S^{\alpha} L^{\beta}$$

$$MP_S = \frac{\partial Q}{\partial S} = \alpha A S^{\alpha-1} L^{\beta}$$

$$MP_L = \frac{\partial Q}{\partial L} = \beta A S^{\alpha} L^{\beta-1}$$

$\alpha = .463$

$\beta = .856$

$\ln A = .138$, so $A = e^{.138} = 1.148$, and $\underline{Q = 1.148 S^{.463} L^{.856}}$

so $MP_S = (.463)(1.148) S^{-.537} L^{.856}$

$MP_L = (.856)(1.148) S^{.463} L^{-.144}$

(b)

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$\bar{S} = 60$ inches

$\bar{L} = (45 \text{ workers})(40 \text{ hrs/week})(13 \text{ weeks}) = 23{,}400$ hours

$MP_S = (.463)(1.148)(60^{-.537})(23400^{.856})$

$MP_S = 324.12$ tons / inch of seam thickness

$MP_L = (.856)(1.148)(60^{.463})(23{,}400^{-.144})$

$MP_L = 1.54$ tons / hour of labor

(c) cost minimization requires that

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$$\frac{MP_L}{P_L} = \frac{MP_S}{P_S}$$

$P_L = \$25$ and $P_S = \$3000$

$$\frac{1.54}{25} \stackrel{?}{=} \frac{324.12}{3000}$$

$.0616 \neq .1080$, so we are not minimizing cost. We should use less labor relative to seam thickness.

$\frac{MP_L}{MP_S} = .00475$ $\frac{P_L}{P_S} =$ [illegible]

4. (a) 10

$$Q = L^{1/3} K^{2/3}$$

$$MP_L = \frac{dQ}{dL} = \frac{1}{3} L^{-2/3} K^{2/3}$$

$$MP_K = \frac{dQ}{dK} = \frac{2}{3} L^{1/3} K^{-1/3}$$

cost minimization requires $\dfrac{\frac{1}{3} L^{-2/3} K^{2/3}}{\frac{2}{3} L^{1/3} K^{-1/3}} = \dfrac{w}{v}$

or $\dfrac{K}{2L} = \dfrac{w}{v}$

or $vK = 2wL$, so $K = \dfrac{2wL}{v}$ and $L = \dfrac{vK}{2w}$

substitute into production ftn:

$$Q = L^{1/3}\left(\frac{2wL}{v}\right)^{2/3} = \left(\frac{2w}{v}\right)^{2/3} L$$

so $L^* = \dfrac{Q}{\left(\frac{2w}{v}\right)^{2/3}}$

$$Q = \left(\frac{vK}{2w}\right)^{1/3} K^{2/3} = \left(\frac{v}{2w}\right)^{1/3} K$$

so $K^* = \dfrac{Q}{\left(\frac{v}{2w}\right)^{1/3}}$

substitute into definition of TC:

$$TC = wL^* + vK^* = \frac{wQ}{\left(\frac{2w}{v}\right)^{2/3}} + \frac{vQ}{\left(\frac{v}{2w}\right)^{1/3}}$$

$$TC = wQ\left(\frac{2w}{v}\right)^{-2/3} + vQ\left(\frac{v}{2w}\right)^{-1/3}$$

$$= Q\, w^{1/3} v^{2/3}\, 2^{-2/3} + Q\, w^{1/3} v^{2/3}\, 2^{1/3}$$

$$= Q\, w^{1/3} v^{2/3}\left(2^{-2/3} + 2^{1/3}\right)$$

$$TC = 1.87\, Q\, w^{1/3} v^{2/3}$$

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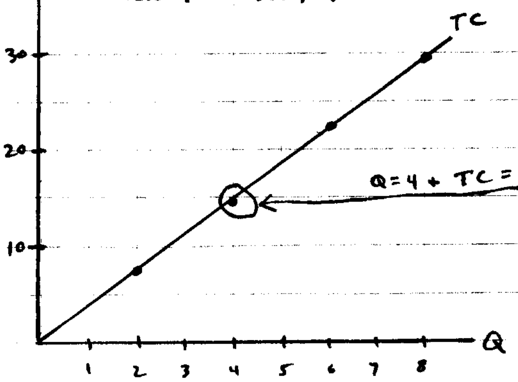
(b) if $w = 2$ and $v = 2$

then $TC = 3.74Q$



Q	2	4	6	8
TC	7.48	14.96	22.44	29.92

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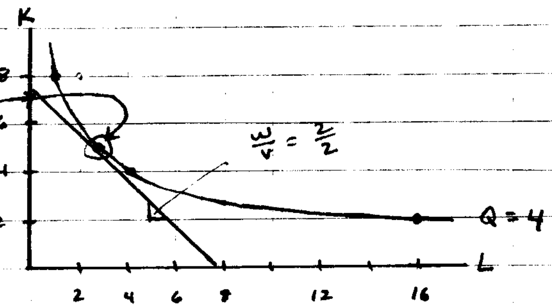
(c) let $Q = 4$: $4 = L^{1/3}K^{2/3}$, or $64 = LK^2$, so

for cost minimization: $\dfrac{K}{2L} = \dfrac{w}{v}$

if $w = v = 2$: cost min $\Rightarrow$ $K = 2L$

L	K
4	4
1	8
16	2
2.51	5.03



5. (a) $P = mr = \$20$

$TC = .1 q^2 + 10q + 50$

$mc = .2q + 10$

$P = mc \Rightarrow .2q + 10 = 20, \quad q = 50$ acres per day

(b) $\pi = TR - TC$

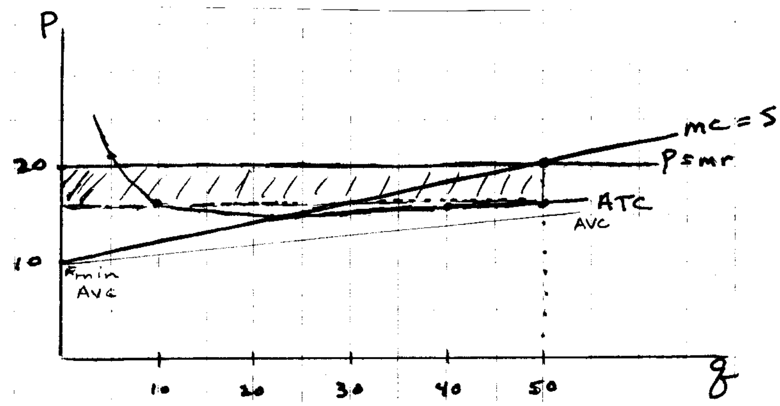
$\pi = (50)(20) - (.1)(50)^2 - (10)(50) - 50$

$\pi = \$200$

(c) $P = mc$ for supply curve (above min AVC)

$P = .2q + 10 \Rightarrow q = 5p - 50$

at $q = 50$, $ATC = .1q + 10 + \frac{50}{q} = 16$



$TC = .1q^2 + 10q + 50$

$TVC = .1q^2 + 10q$

$ATC = .1q + 10 + \frac{50}{q}$

$AVC = .1q + 10$

$mc = ATC$:

$.2q + 10 = .1q + 10 + \frac{50}{q}$

$.1q = \frac{50}{q}, \quad q = 22.36$

if $q = 10$ $ATC = 16$

if $q = 30$ $ATC = 14.66$

6. Inverse elasticity rule: $\frac{P - mc}{P} = \frac{1}{\varepsilon}$ for π max

$mc = 15$

at $P = 20$, $\varepsilon = 3$

$\frac{P - mc}{P} = \frac{5}{20} \Leftrightarrow \frac{1}{3}$ No.

at $P = 25$, $\varepsilon = 2.5$

$\frac{P - mc}{P} = \frac{10}{25} \Leftrightarrow \frac{1}{2.5}$ YES

at $P = 30$, $\varepsilon = 2.2$

$\frac{P - mc}{P} = \frac{15}{30} \Leftrightarrow \frac{1}{2.2}$ No

so $P = \$25$ maximizes profit.