

Choosing a Licensee from Heterogeneous Rivals*

Anthony Creane[†] Chiu Yu Ko[‡] Hideo Konishi[§]

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Abstract

We examine a firm that can license its production technology to a rival when firms are heterogeneous in production costs. We show that a complete technology transfer from one firm to another always increases joint profit under weakly concave demand when at least three firms remain in the industry. A jointly profitable transfer may reduce social welfare, although a jointly profitable transfer from the most efficient firm always increases welfare. We also consider two auction games under complete information: a standard first-price auction and a menu auction by Bernheim and Whinston (1986). With natural refinement of equilibria, we show that the resulting licensees are ordered by degree of efficiency: menu auction, simple auction, and joint-profit maximizing licensees, in (weakly) descending order.

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[†]Department of Economics, University of Kentucky. (e-mail) acr233@email.uky.edu

[‡]Department of Economics, National University of Singapore. (e-mail) ecscyk@nus.edu.sg

[§]Corresponding author: Hideo Konishi, Department of Economics, Boston College, 140 Commonwealth Avenue, Chestnut Hill, MA 02467. (e-mail) hideo.konishi@bc.edu

1 Introduction

We examine the licensing of production technology to a rival firm in a product market while relaxing the standard assumption that the rivals are homogeneous in their production technologies. Specifically, we assume that firms engage in Cournot competition and differ in their constant marginal costs of production, and that a technology transfer reduces the licensee's marginal cost to the level of the licensor.¹ We are interested in the direct effects of licensing; so, to abstract from any possible effects of collusion, we adopt the standard assumption in the literature that the production decisions of firms remain independent after transfer. We focus on a setting of complete information where a single licensor chooses an exclusive licensing partner from heterogeneous rivals, resulting in negative externalities of licensing to third-party firms.² We allow for the possibility of some third-party firms' shutting down after the transfer. We analyze first the gains in joint profit for the licensor and a licensee from licensing, and then the social welfare gains. Then, we consider two auction games to determine the licensee.

We begin, following the seminal work by Katz and Shapiro (1985), by analyzing whether such a transfer is always jointly profitable in a Cournot model.³ Katz and Shapiro (1985) have shown that a complete technology transfer (where the licensee ends up with the same cost as the licensor) could reduce joint profit in a duopoly if the licensor has a near-monopoly position, because the transfer would reduce the licensor's near-monopoly profit. La Manna (1993) shows that when there are at least three firms in the market, a complete technology transfer to another firm is always joint profit improving if the demand is linear. We show that La Manna's result extends even if we allow for weakly concave demand. This is not a trivial exercise: we can neither explicitly calculate equilibrium nor use simple comparative static techniques, because a partial technology transfer can reduce joint profit.⁴ Nevertheless, by introducing artificial markets as a device, we can show that a complete technology transfer is always jointly profitable if the demand curve is weakly concave and there are at least three firms in the market after the transfer (Theorem 1). That is, a complete transfer is always jointly profitable independent of its

¹This implies that the size of the technology transfer varies with the licensee's efficiency (a less efficient rival receives a larger transfer).

²For example, Jehiel et al. (1996) and Jehiel and Moldovanu (2000) in examining a single transfer allow for the presence of private information at the auction stage. In this paper, we analyze auction methods in licensing, but we concentrate on the effects of (negative) externalities on auction outcomes.

³This is also equivalent to fixed fee licensing, examined by Kamien and Tauman (1986).

⁴See Creane and Konishi (2009b).

absolute size and the relative efficiency of the licensor.⁵

We then focus on which partner would maximize joint profit. At first glance, one might expect that this would be the most inefficient rival. We find that for weakly concave demand, neither the very inefficient nor the very efficient rival maximizes joint profit (Observation 1). With heterogeneous firms, the less efficient the licensee is, the greater the technology transfer will be. On the one hand, a technology transfer to a nearly equally efficient rival is very small and holds little benefit for the rival's profit, although such a transfer does not substantially reduce the market price and the licensor's profit. On the other hand, a technology transfer to a very inefficient firm benefits the licensee greatly but reduces the licensor's output and profit through a large reduction in the market price. Given that profit is convex in output, the licensor's profit reduction is large if the technology transfer is made to a very inefficient firm. Hence, the licensor is better off choosing a partner who is neither very efficient nor very inefficient.

Turning to the welfare effects of a technology transfer, it is known that making an inefficient firm slightly more efficient can actually reduce welfare (Lahiri and Ono 1988). This implies that, as a corollary of Theorem 1, a jointly profitable transfer can reduce social welfare if there are more than two firms and if both the licensor and licensee are sufficiently similar and inefficient (Observation 2). In contrast, Katz and Shapiro (1985) find that profitable transfers never reduce welfare in a duopoly, which underlies the importance of considering non-duopoly markets. Katz and Shapiro (1986) and Sen and Tauman (2007) find that with homogeneous firms, licensing always raises welfare, so heterogeneity is important in evaluating the welfare implications of licensing. Although a transfer from a sufficiently inefficient licensor can reduce welfare, we show that if the most efficient firm makes a complete transfer then social welfare always increases under general demand (Theorem 2). However, a joint-profit-maximizing licensee is not necessarily a social-welfare-maximizing licensee, because the joint-profit-maximizing selection does not take into account the negative externalities imposed on other firms. Since technology transfers affect the rival firms' production decisions, including those of efficient rivals, total costs can be lower with a *more* efficient licensee. The conclusion for the policy maker whose goal is to maximize social welfare is that the most efficient firm should not be discouraged from licensing its technology to rivals; but technology transfers between marginal firms should bear some scrutiny.

Analyzing the joint-profit-maximizing licensee is a natural benchmark because it allows comparisons to Katz and Shapiro (1985) as well as work that examines fixed-fee setting licensing (e.g., Kamien and Tauman 1986), and, as we will see, it is useful for

⁵The licensor does not have to be the most efficient firm for this result to hold.

later analysis.⁶ However, when there is more than one rival, licensing to a joint-profit-maximizing partner does not exploit the entire possible gains for the licensor if the licensor can credibly threaten to find a new partner during negotiations. That is, competition among potential licensees over the technology transfer should be more profitable in the presence of externalities. Katz and Shapiro (1986), and others since,⁷ have taken this into account when they examine an auction game in a homogeneous licensee environment by endogenizing the number of licenses. We follow their approach, but do so in a setting with heterogeneous firms, and ask which firm would win the right to use the technology and how much the licensor would collect from licensing. Specifically, we examine what happens when the most efficient firm (the natural analogy to homogeneous rivals) uses first-price auction mechanisms to sell the right to use its technology. In the first-price auction method (a simple auction game), which is a modification of the method used by Katz and Shapiro (1986) to take into account a heterogeneous firm environment, each potential licensee submits a bid and only the winner pays for the bid. Since there are many Nash equilibria and most of them are not very plausible, we refine the set of Nash equilibria by stipulating that non-licensees would not be worse off if the licensor happens to choose it: truthful Nash equilibrium (TNE in simple auction). Roughly speaking, this is akin to a “trembling-hand” refinement. In this refined set of Nash equilibria, the licensing fee can be pinned down, and the licensee is the partner that maximizes the joint profit of the licensee, the licensor, and any other potential rival.

Given the complex negative externalities created by technology transfer, even if a firm is not willing to bid enough to win the license it might find it profitable to bribe the licensor to influence which of its rivals does obtain the license.⁸ For this reason, we also consider a menu auction (Bernheim and Whinston 1986) in which each potential licensee submits a menu that specifies contingent payments to the licensor for each possible licensee the licensor might select. We again refine the set of Nash equilibria to the set of truthful Nash equilibria (TNEs in menu auction).⁹ We show that a simple auction licensee is at least

⁶This can also be justified by noting that often a licensee is selected and then the two parties negotiate the contract. Since negotiating a technology transfer is not trivial, it may be too costly for the licensor to credibly threaten to license to a different firm, and so the fee should be determined as a function of the increase in their joint profit. In this case, the joint-profit-maximizing licensee should be selected by the licensor as the recipient of the technology.

⁷For a review of auctions in licensing, see Giebe and Wolfstetter (2007).

⁸For example, recently, when it looked like Google would acquire bankrupt Nortel’s patents, a coalition of Apple, EMC, Ericsson, Microsoft, Research In Motion, and Sony outbid Google (Claburn, 2011).

⁹Truthful Nash equilibria in a simple auction and in a menu auction appear to be similar in their definitions, but their implications are somewhat different. In a simple auction, TNE is a rather innocuous refinement of Nash equilibrium, whereas in a menu auction, TNE has implications for communication-

as efficient as a joint-profit-maximizing partner, and a menu auction licensee is at least as efficient as a simple auction licensee (Theorem 3).

In the next section, we introduce the basic modeling assumptions. Section 3 examines the effect of the amount of technology transferred on profit and examines the implications of partner type. Section 4 contains the welfare analysis. Section 5 identifies which firm will get the right to use the technology in license auction games. Section 6 provides extensions of our analysis in the case of multiple licensees and license contracts with royalties. We provide a number of intriguing observations, but the results are sensitive to the setup of the license game. It seems difficult to obtain general results.

2 The Model

We consider a basic Cournot market structure. There is a commodity besides a numeraire good, and its (inverse) demand is a continuous function $P(Q)$ in $[0, \bar{Q}]$ that is twice continuously differentiable with $P'(Q) < 0$ for all $Q \in (0, \bar{Q})$ and $P(\bar{Q}) = 0$. There are K firms in the market with no fixed cost of production.

Firms are indexed as $i \in \{1, \dots, K\}$ and differ in their constant marginal costs c_i . We order firms by degree of efficiency: $c_1 \leq c_2 \leq \dots \leq c_K$. With a small abuse of notation, let the set $\{1, 2, \dots, K\}$ be denoted by K as well.

Each firm i 's production level is denoted by q_i . Firm i 's profit function is written as

$$\pi_i(q_i, q_{-i}) = (P(Q) - c_i) q_i,$$

where $Q = \sum_{i \in K} q_i$. The first-order condition for profit maximization (assuming an interior solution) is

$$P'(Q)q_i + P(Q) - c_i = 0. \tag{1}$$

This implies

$$q_i = \frac{(P(Q) - c_i)}{-P'(Q)},$$

and firm i 's profit is written as

$$\pi_i(q_i, q_{-i}) = \frac{(P(Q) - c_i)^2}{-P'(Q)}.$$

We assume the **strategic substitutability** condition throughout the paper: for all $i \in K$,

$$P''(Q)q_i + P'(Q) \leq 0.$$

based refinement (Bernheim and Whinston 1986).

Note that the second-order condition for profit maximization ($P''(Q)q_i + 2P'(Q) \leq 0$) is guaranteed by strategic substitutability. Strategic substitutability is weaker than the requirement that the demand is **weakly concave** $P''(Q) \leq 0$.¹⁰ In proving some of our main results, we strengthen strategic substitutability condition by the weak concavity of demand. The strategic substitutability condition guarantees the uniqueness of equilibrium. Let $C = \sum_{i \in K} c_i$ denote the aggregate marginal cost. We now present a standard result, whose derivation, which can be found in the appendix along with all subsequent proofs, will be useful for later analysis.

Lemma 1. Under the strategic substitutability condition, the equilibrium is unique. Moreover, keeping other firms' marginal costs intact, an increase in c_j decreases the equilibrium aggregate output Q if $c_j < P(Q)$, and has no effect otherwise.

3 Production Technologies and Transfers

Each firm i has its own technology for producing the commodity (the marginal cost of production is c_i), and it has the property right to its own technology (e.g., it holds a patent). We adopt the standard assumption in the literature that the output decisions remain independent after any transfer, a condition that is usually imposed by competition authorities. Firm i can license its technology with an exclusive usage agreement to another firm. As is also standard in the literature (Katz and Shapiro 1986, etc.), we assume **complete** technology transfer throughout the paper: the obtaining firm j (*licensee*) reduces its marginal cost to that of firm i . That is, if firms i and j have technologies with marginal costs c_i and c_j with $c_i < c_j$, respectively, then firm j can reduce its marginal cost of production to c_i by adopting firm i 's technology.

The following lemma also plays a key role in the subsequent analysis.

Lemma 2. Suppose that there are initially K firms engaging in production. Pick three firms i , j , and j' with $c_i < c_j < c_{j'}$, and consider two scenarios: (i) firm i transfers its technology to firm j , and (ii) firm i transfers its technology to firm j' . Then, equilibrium aggregate output \hat{Q} in scenario (i) is not more than equilibrium aggregate output \tilde{Q} in scenario (ii), resulting in $P(\hat{Q}) \geq P(\tilde{Q})$.

¹⁰That is, the weak concavity of demand implies the second-order condition for profit maximization.

3.1 Jointly Profitable Transfers

Katz and Shapiro (1985) show that complete transfers could reduce joint profits in a duopoly; here, we examine whether this result can extend to markets with more than two firms. We can show that under weakly concave demand (which includes linear demand), a complete technology transfer is always profitable if there is at least one other firm in the market. This somewhat surprising result has been known under a linear demand assumption by La Manna (1993). However, with more general demand, Katz and Shapiro (1985) found such transfers could be unprofitable when there are only two firms in the market, and Creane and Konishi (2009b) show that partial transfers (i.e., the licensee's cost is not completely reduced to the licensor's cost) when firms are heterogeneous could reduce joint profit. Since we have to compare profit levels of two discrete cases and a small transfer may reduce joint profit, we cannot simply rely on comparative statics on technology transfers: we need to utilize an artificial economy to prove the theorem.

Theorem 1. Consider firms $i, j \in K$ with $c_i < c_j$. Assume that firm i is in operation originally, and that even after firm i transfers technology to firm j , another firm k is still in operation ($q_k > 0$) with $c_k \neq c_i$. If demand is weakly concave ($P''(Q) \leq 0$), then a complete technology transfer from firm i to firm j is joint profit improving.

Notice that we assume that at least three firms remain in the market after the technology transfer. Although Katz and Shapiro (1985) obtain conditions for a complete transfer to reduce joint profit, they examine a duopoly case. The existence of a third firm drives the theorem, as part of the gain to the licensee comes from the lost profits of the other firm(s). However, since a partial transfer could reduce joint profit, one may wonder how a complete transfer always increases joint profit. To intuitively see the reason for this, consider what happens when a partial technology transfer would reduce joint profit. In this case, consider what happens if, instead, the licensee's cost is *increased* (thereby raising joint profit) until the licensee is driven out of the market. Joint profit has now increased. At this point we note from the divisionalization literature (Baye et al. 1996) that if the licensee could create a second, identical division, then its profits would increase.

3.2 Joint-Profit-Maximizing Partner

Whereas in the previous subsection we considered the profitability of technology transfers, in this subsection we consider which partner would maximize joint profit. That is, for firm i , which firm j would create the greatest increase in joint profit from a technology

transfer? Recall that when the licensor chooses a less efficient partner, this leads to a larger technology transfer.

Since we need to compare profits when different partners have been chosen, for heuristic reasons it is more convenient to use linear demand with explicit solutions and assume that all firms are in operation: $q_k > 0$ for all $k \in K$. Let $P(Q) = 1 - Q$. With this demand curve we have $Q = \frac{K-C}{K+1}$, $P = \frac{1+C}{K+1}$, and $q_i = \frac{1+C}{K+1} - c_i$. Then the change in the joint profit by firms i and j from the technology transfer is

$$\begin{aligned}
& (\text{joint profit after transfer}) - (\text{joint profit before transfer}) \\
&= 2 \left(\frac{1+C - (c_j - c_i)}{K+1} - c_i \right)^2 - \left(\frac{1+C}{K+1} - c_i \right)^2 - \left(\frac{1+C}{K+1} - c_j \right)^2 \\
&= \underbrace{\left(\frac{1+C}{K+1} - c_i - \frac{c_j - c_i}{K+1} \right)^2 - \left(\frac{1+C}{K+1} - c_i - (c_j - c_i) \right)^2}_{\text{increase in firm } j\text{'s profit}} \\
&\quad + \underbrace{\left(\frac{1+C}{K+1} - c_i - \frac{c_j - c_i}{K+1} \right)^2 - \left(\frac{1+C}{K+1} - c_i \right)^2}_{\text{decrease in firm } i\text{'s profit}} \\
&= 2 \left(\frac{1+C}{K+1} - c_i \right) (c_j - c_i) - \frac{(K(K+2) - 1)(c_j - c_i)^2}{(K+1)^2}.
\end{aligned}$$

This is a quadratic function in the difference in marginal costs $c_j - c_i$. The first positive term increases if firm j is a less efficient partner, while the second negative term gains in magnitude as firm j is a less efficient partner. Hence, the gain is highest when c_j is neither too big nor too small. Firm i should choose some firm in the middle. Although the above analysis is based on an assumption of linear demand, a quantitatively similar result applies for general demand (see Creane and Konishi 2009b).

Observation 1. With a complete transfer, a joint-profit-maximizing partner for a firm is neither too efficient nor too inefficient relative to the firm under weakly concave demand.

This condition is intuitive: you cannot make a rival who is already efficient that much more efficient. Thus, there is some benefit to picking a less efficient rival, as there is a greater transfer and so an expected increase in profit of the licensee from the transfer. However, you do not want to pick too inefficient of a rival. The reason is that as you pick a more inefficient rival the technology transfer causes the price to fall more, harming you as well as the rival. At the same time, when considering sufficiently inefficient firms, a slightly more inefficient firm will not yield that much less profit (since its output is approaching zero, i.e., marginal cost is approaching the price) and the gain from selecting a slightly more inefficient rival approaches zero.

4 Welfare Effects

We now investigate the effects of technology transfers on social welfare, which is the sum of the firms' profits and consumer surplus. Since technology transfers reduce production cost, social welfare tends to increase with the amount of technology transferred. Indeed, Katz and Shapiro (1985) show that in a duopoly, licensing that increases joint profit always increases welfare (and welfare-decreasing licensing always decreases joint profit). Likewise, Sen and Tauman (2007) find licensing to be welfare improving under general licensing schemes when firms are homogeneous.

However, a profitable licensing could reduce welfare when firms are heterogeneous. This possibility arises if a very inefficient firm obtains a technology transfer that reduces its cost only slightly; then its resulting increase in production will displace the production of more efficient firms, thereby reducing social welfare. This result has already been observed by Lahiri and Ono (1988). The question, then, is whether this implies that jointly profitable licensing can reduce welfare, contrary to previous results. Combining Theorem 1 with Lahiri and Ono's result, we are able to state that the previous results do not generalize to the case where there are more than two firms and firms are heterogeneous: profitable licensing can be welfare reducing.

Given this result, one may wonder if there are conditions that guarantee that a technology transfer raises welfare. We show that if the most efficient firm makes a complete technology transfer, then welfare increases. The policy implications of these results appear straightforward. On the one hand, competition authorities should pay close attention to technology transfers (through licensing, joint venture, or merger) between marginal firms (in the technological efficiency sense) in an industry. On the other hand, the most efficient firm within an industry should not be discouraged from making a technology transfer to a rival.

4.1 Welfare-reducing Profitable Licensing

We begin by presenting Lahiri and Ono's (1988) condition that a reduction in the marginal cost of an inefficient firm reduces social welfare.

Observation 2. (Lahiri and Ono 1988): When firm j 's marginal cost (c_j) decreases, social welfare decreases if c_j is sufficiently high, though consumer welfare (surplus) increases.

This observation leads to an immediate corollary to Theorem 1 that yields a result contrary to previous ones in the literature: there are profitable technology transfers that reduce total welfare while benefiting consumers.

Corollary 1. Suppose that demand is weakly concave and that there are more than two firms. Then, if firm j has a sufficiently high marginal cost (c_j) and firm i 's marginal cost is sufficiently close to firm j 's, then firm i licensing its technology to firm j is jointly profitable and welfare reducing even though consumer welfare (surplus) increases.

Previous results in the literature may at first glance appear to be similar even though they are quite distinct. First, Katz and Shapiro (1985) have shown that in a duopoly a technology transfer can reduce welfare, but only when it reduces joint profit. Hence, such transfers would never actually occur. In contrast, here there can be technology transfers that reduce welfare, but increase joint profit. Second, Faulí-Oller and Sandonís (2002) have shown that in a duopoly profitable licensing can reduce welfare, but this requires the use of a royalty (raising the recipient's marginal cost) and only occurs in price competition. As they note, "the royalty works as a collusive device" and so reduces welfare. More generally, licensing contracts can reduce welfare if these contracts have collusive effects (e.g., Shapiro 1985). However, such effects are absent in our model.

4.2 Welfare-improving Profitable Licensing

Since technology transfers between inefficient firms can reduce welfare, the next question is whether there are conditions that guarantee that transfers increase welfare. Since the social welfare is reduced only because relatively inefficient firms' production crowds out more efficient firms' production, we can naturally propose that if the licensor is the most efficient firm then the social welfare should improve. Indeed, we can show that this is the case. For this result, we need no condition on demand function.

Theorem 2. Suppose that the most efficient firm (firm 1) makes a complete transfer to any firm j ($c_1 \leq c_2 \leq \dots \leq c_j \leq \dots \leq c_K$ and $c_1 < c_j$). Then, the social welfare improves.

Interestingly, a social-welfare-maximizing partner is not necessarily the least efficient firm. Although aggregate output and consumer surplus are maximized by choosing the least efficient firm as partner, industry profit, which is also part of social welfare, may not be maximized. The following example illustrates how the harm to industry profit shows that welfare is not maximized by licensing to the least efficient firm.

Example 1. Consider a market with five firms with marginal costs $c_1 = 0$, $c_2 = 0.05$, $c_3 = 0.1$, $c_4 = 0.14$, and $c_5 = 0.2$. The demand function is linear $P(Q) = 1 - Q$. Firm 1 is the unique licensor ($i = 1$). The following table shows the resulting profits for all possible

transfers:

$i = 1$	P	π_1	π_2	π_3	π_4	π_5	industry	CS	SW
$j = 1$	0.24833	0.06167	0.03934	0.022	0.01174	0.00234	0.13708	0.28250	0.41958
$j = 2$	0.24	0.0576	0.0576	0.0196	0.01	0.0016	0.1464	0.2888	0.4352
$j = 3$	0.23167	0.05367	0.033	0.05367	0.0084	0.001	0.14975	0.29517	0.44492
$j = 4$	0.225	0.05063	0.03063	0.01563	0.05063	0.00063	0.14813	0.30031	0.44844
$j = 5$	0.215	0.04623	0.02723	0.01323	0.00563	0.04623	0.13853	0.30811	0.44664

The social-welfare-maximizing partner is firm 4, the consumer-surplus-maximizing partner is firm 5, and the industry-profit-maximizing partner is firm 3. A different way to see why firm 4 maximizes social welfare is to note that social welfare equals consumers' total benefit less the cost of production (which appears in industry profit). Licensing to the least efficient firm (firm 5) does result in the greatest cost reduction; however, two countervailing effects result in lower total costs when firm 4 is licensed. First, total production by the lowest-cost firms is greater when firm 4 is licensed (the price is higher). Second, when firm 4 is licensed, the least efficient firm still produces (firm 5), but its production is quite small so its contribution to total cost is negligible, whereas when firm 5 is licensed firm 4's production is several times larger. As a result, in the above example, although licensing firm 5 increases output by 0.01, total cost increases by 0.004, yielding a "marginal cost" of 0.4 well above the price. \square

5 Choosing a Licensee through Auctions

In this section, we study (i) which firm wins the right to use the technology and (ii) how much is paid when the technology is licensed by auction. In a duopoly, there is only one potential licensee, and so a fixed-price licensing fee (calculated as the difference between the licensee's post-transfer profit minus its original profit) is optimal. However, if there are multiple potential licensees, then they compete over the exclusive license. As noted by Katz and Shapiro (1986), if one firm obtains the license, its rivals suffer from the market price reduction caused by the licensing. A firm's auction bid, then, must take this externality into consideration. However, unlike in Katz and Shapiro (1986), here the potential licensees are heterogeneous, and so the non-licensees differ in the harm from a given firm winning the license and so in their willingness-to-pay.

We consider two types of auctions: In the first, each potential licensee bids for the right to use the technology, and when a winner is selected, only the winner pays the license fee according to its bid (simple auction). In the second, each potential licensee

offers a menu that describes how much it will pay the licensor depending on which of the potential licensees gets the technology; when a winner is selected all potential licensees pay the licensor according to their bids for that particular winner (menu auction). These two license auctions have both advantages and disadvantages. A simple auction can be considered as a “natural” auction, since only the winner of the license auction pays for the license. However, the externalities created by potential licensees are not identical. If a firm is harmed more by firm j getting the license than firm k , it obviously would prefer that firm k instead of firm j obtain the license and so might be willing to pay to bring this about. Hence, a menu auction captures a firm’s willingness to help one firm obtain the license in order to lessen the negative externality from another firm in a licensing market, although it seems less “natural” at first glance (and for this reason may be viewed unfavorably by competition authorities).

For the rest of the paper we assume that the licensor is the most efficient firm, firm 1: i.e., $c_1 < c_2 \leq c_3 \leq \dots \leq c_K$. This is a natural setup for the licensing problem as it is (trivially) the structure discussed in the literature when the licensees are homogeneous, and as Theorem 2 assures, such a licensing will certainly improve the welfare. With a small abuse of notation, we denote $\pi_k(j)$ as the profit of firm k when firm j obtains the license from firm 1 for k for $j = 1, \dots, K$.

5.1 Simple Auction

A **simple auction** is a version of the first-price auction played by firms $2, 3, \dots, K$, in which each firm $k \in \{2, \dots, K\}$ simultaneously offers $T_k \geq 0$ to be the unique licensee to the licensor who chooses as a licensee firm (say, firm j) that maximizes the sum of firm 1’s profit and T_j : i.e., $j \in M(T) \equiv \arg \max_{k \in \{1, \dots, K\}} (\pi_1(k) + T_k)$, where $T = (T_1, T_2, \dots, T_K)$. (Recall that when $j = 1$, firm 1’s technology is not transferred to any firm.) Knowing this, firm $k \in \{2, \dots, K\}$ chooses its bid T_k . In a simple auction, an outcome (j^*, T^*) is a **Nash equilibrium** if $j^* \in M(T^*)$ and there is no $k \in \{2, \dots, K\}$ and T_k such that $k \in M(T_k, T_{-k}^*)$ and $U_k(k, T) > U_k(j^*, T)$, where $U_k(k, T) = \pi_k(k) - T_k$ and $U_k(j, T) = \pi_k(j)$ if $j \neq k$. Although Nash equilibrium is the most basic equilibrium concept, there are too many equilibria including unnatural ones in which the winner of license may pay a high license fee only to match another firm’s offer that is an empty threat because the latter firm would not want its offer accepted). In the light of this, we consider a reasonable refinement of Nash equilibrium that is a version of truthful equilibrium. The idea is loosely related to the trembling-hand argument for the licensor. The licensor may make a slight mistake in choosing a potential licensee. Hence, each

firm would be better off by making a weakly dominant offer relative to the equilibrium outcome. For firm $j \in K \setminus \{1\}$, a strategy T_j is **truthful relative to \bar{j}** if and only if either (i) $U_j(j, T) = U_j(\bar{j}, T)$ or (ii) $U_j(j, T) < U_j(\bar{j}, T)$ and $T_j(j) = 0$. A **truthful Nash equilibrium (TNE)** is a Nash equilibrium (j^*, T^*) such that each firm chooses a truthful strategy relative to j^* . With this refinement, we can pin down and characterize the unique equilibrium in the above example.

Proposition 1. No licensing is a TNE of the simple auction game if and only if $\pi_1(1) + \pi_j(1) \geq \pi_1(j) + \pi_j(j)$ for all $j = 2, \dots, K$. Suppose that no licensing is not a TNE. Then, an outcome (j^*, T^*) is a TNE with licensing ($j^* > 1$), if and only if $T_{j^*}^* = \max_{j \in K \setminus \{1\}} \{\pi_1(j) - \pi_1(j^*) + \pi_j(j) - \pi_j(j^*)\}$, $T_j^* = \pi_j(j) - \pi_j(j^*)$ for all $j \neq j^*$, and $\pi_1(j^*) + \pi_{j^*}(j^*) + \pi_j(j^*) \geq \pi_1(j) + \pi_j(j) + \pi_{j^*}(j)$ for all $j \neq 1$.

The last condition means that firm j^* is willing to challenge firm j by paying more if firm j gets the license. Suppose firm j receives the license. Then firm j^* 's payoff is $\pi_{j^*}(j)$, and firms 1 and j are jointly earning $\pi_1(j) + \pi_j(j)$. That is, the sum of these three firms' payoffs is $\pi_1(j) + \pi_j(j) + \pi_{j^*}(j)$. Now, if firm j^* receives the license, then the total profit of these three firms is $\pi_1(j^*) + \pi_{j^*}(j^*) + \pi_j(j^*)$. If this value exceeds $\pi_1(j) + \pi_j(j) + \pi_{j^*}(j)$, firm j^* can beat firm j . If firm j^* can beat all other potential licensees, firm j^* wins the licensing auction. As a corollary of the first part of Proposition 1 and Theorem 1 (joint profit increases), we can state the following.

Corollary 2. Under weakly concave demand, no licensing is not a TNE of the simple auction game if at least three firms remain in operation after licensing.

We call the licensee in a truthful Nash equilibrium outcome a **simple auction licensee**. Without negative externalities, a joint-profit-maximizing partner is a simple auction licensee. Taking externalities into account, a firm is a simple auction licensee if and only if such a transfer maximizes the joint profit of the licensor, the licensee, and any one potential licensee firm—that is, if the licensor's profit plus the licensee's gain from another potential licensee's not getting the license is greater than the licensor's profit and the gain of this other potential licensee. Comparing a simple auction licensee and a joint-profit-maximizing partner, it turns out that a simple auction licensee, if one exists, is at least as efficient as a joint-profit-maximizing partner.

Proposition 2. Under weakly concave demand, a simple auction licensee (if one exists) is at least as efficient as a joint-profit-maximizing partner.

From the characterization of TNE (Proposition 1), it is easy to see that a Nash equilibrium in pure strategy must satisfy many inequalities. If there are only two potential licensees ($K = 3$), then it is easy to show the existence of TNE and to characterize it. However, if there are more than two potential licensees, finding a Nash equilibrium (in pure strategies) is hard. Although we are unable to show the existence of a TNE in a simple auction game under weakly concave demand, we can show that it always exists if demand is linear and no firm shuts down after any transfers (see Creane, Ko, and Konishi 2012).

5.2 Menu Auction

When firm 1 (the most efficient firm) chooses to license its technology to a firm (the licensee) $j \in N = K \setminus \{1\} = \{2, \dots, K\}$, there is a negative externality from the technology transfer for the other firms not receiving the transfer (non-licensees $N \setminus \{1, j\}$). Non-licensees would like to influence the licensing decision and may be willing to offer money for firm 1 not to license to firm j . We try to capture such strategic interactions using the menu auction framework proposed by Bernheim and Whinston (1986).

A **menu auction game** Γ in our context is described by $(N + 2)$ tuples:

$$\Gamma \equiv \{N, (\pi_k)_{k \in N \cup \{1\}}\},$$

such that $\pi_k : A \rightarrow \mathbb{R}_+$ is firm k 's profit function, where A is the set of licensor's actions. That is, $\pi_k(a)$ denotes firm k 's profit when firm 1 chooses action $a \in A$. (In this section, A will be the set of potential licensees — who to license: $A = K$ and $a = 1$ means “no license.”) In the extensive form of the game, the potential licensees simultaneously offer contingent payments to the licensor, who subsequently chooses an action that maximizes her total payoff. A strategy for each potential licensee $k \in N$ is a transfer function $T_k : A \rightarrow [\underline{b}, \infty)$, which is a monetary reward (or punishment) of $T_k(a)$ to the licensor for selecting action $a \in A$, where \underline{b} is the lower bound for transfer (we set $\underline{b} = 0$ except for royalty case in Section 5). For each action a , potential licensee k receives a net payoff:

$$U_k(a, T) = \pi_k(a) - T_k(a),$$

where $T = (T_{k'})_{k' \in N}$ is a strategy profile. The licensor chooses an action that maximizes its total payoff: the licensor selects an action in the set $M(T)$ with:

$$M(T) \equiv \arg \max_{a \in A} \left[\pi_1(a) + \sum_{k \in N} T_k(a) \right].$$

The menu auction game is merely a game among potential licensees, although, strictly speaking, a tie-breaking rule among $M(T)$ needs to be specified for the licensor. An outcome of a menu auction game Γ is (a, T) . An outcome (a^*, T^*) is a **Nash equilibrium** if $a^* \in M(T^*)$ and there is no $k \in N$ such that $T_k : A \rightarrow [b, \infty)$ and $a \in M(T_k, T_{-k}^*)$ such that $U_k(a, T) > U_k(a^*, T)$. Unfortunately, with so many coordination problems among the many players (potential licensees), there are too many Nash equilibria in a menu auction game.

To get plausible predictions among the many allocations supported by Nash equilibrium, Bernheim and Whinston (1986) consider a reasonable refinement on the set of Nash equilibria and argue that “truthful strategies” are quite crucial in menu auctions. A strategy T_k is **truthful relative to \bar{a}** if and only if for all $a \in N$ either (i) $U_k(a, T) = U_k(\bar{a}, T)$ or (ii) $U_k(a, T) < U_k(\bar{a}, T)$ and $T_k(a) = 0$. An outcome (a^*, T^*) is a **truthful Nash equilibrium (TNE)** if and only if it is a Nash equilibrium, and T_k^* is truthful relative to a^* for all $k \in N$. Bernheim and Whinston (1986) show that an *efficient action* (it will be the industry-profit-maximizing licensee in our context) is chosen by the licensor in every TNE outcome in a menu auction: if (a^*, T^*) is a TNE, then we have $a^* \in \operatorname{argmax}_{a \in A} [\sum_{i \in N} \pi_i(a) + \pi_1(a)]$. Moreover, Bernheim and Whinston (1986) characterize the set of (potential licensees’) TNE equilibrium payoff vectors $u = (u_2, \dots, u_K) \in \mathbb{R}^N$. Let $S \subseteq N$ be a subset of potential licensees, and let

$$W_{N \setminus S}(\Gamma) \equiv \max_{a \in A} \sum_{k \in N \cup \{1\} \setminus S} \pi_k(a), \quad (2)$$

where $\pi_k(a)$ is firm k ’s profit ($k = 1, \dots, K$) when the set of potential licensees is a . That is, $W_{N \setminus S}$ is the maximum amount of group $N \setminus S$ ’s total profit that can be achieved without help from S . Specifically, $W_{\emptyset}(\Gamma) = \max_{a \in A} \pi_1(a)$ denotes the profit that the licensor can achieve without having a licensee, and $W_N(\Gamma)$ denotes the total industry profits. Bernheim and Whinston (1986) show that the set of TNE payoffs of licensors u is the *Pareto-frontier* (for licensees, N) of the following set: for all $S \subseteq N$ such that $S \neq \emptyset$,

$$\mathcal{U}_{\Gamma} \equiv \left\{ u \in \mathbb{R}^N : \begin{array}{l} \sum_{k \in N} u_k \leq W_N(\Gamma) - W_{\emptyset}(\Gamma) \\ \sum_{k \in S} u_k \leq W_N(\Gamma) - W_{N \setminus S}(\Gamma) \quad \forall S \subseteq N, S \neq \emptyset \end{array} \right\}. \quad (3)$$

They also show that in menu auction games, the set of truthful Nash equilibria (TNE) and the set of coalition-proof Nash equilibria (CPNE) are equivalent in this utility space.

Now, we let the set of actions be the set of potential licensees, $A = N = K \setminus \{1\}$, to apply menu auction game to a licensee selection problem. The agent is the licensor firm 1. We call the licensee in a TNE of a menu auction a **menu auction licensee**. We can show the following result.

Proposition 3. A menu auction licensee is at least as efficient as a simple auction licensee (if one exists).

The underlying intuition of this proposition is that as a menu auction licensee is an industry-profit-maximizing partner, and a simple auction licensee is a three-firm-profit-maximizing partner. Thus, the negative externality of the technology transfer would make an industry-profit-maximizing firm more efficient than a three-firm-profit-maximizing partner to counteract the effect of the greater negative externality. Propositions 2 and 3 can be summarized as the licensing partners' efficiency ranking among different regimes in the following Theorem.

Theorem 3. Suppose that firm 1 is licensing technology to another firm. Under weakly concave demand, the licensing partner that maximizes the gains in their joint profit is weakly less efficient than the partner determined in a simple auction (if one exists), and the latter is weakly less efficient than the partner determined by a menu auction: i.e.,

$$\begin{array}{ccc} \text{menu auction} & \leq & \text{simple auction} \\ \text{licensee} & & \text{licensee} \end{array} \leq \begin{array}{c} \text{joint-profit-maximizing} \\ \text{partner} \end{array},$$

where firms are ordered by efficiency in a descending manner.

The following example illustrates that a joint-profit-maximizing (competitive equilibrium) partner, a simple auction licensee, and a menu auction licensee can be different.

Example 1 (revisited). Consider a market with five firms with marginal costs $c_1 = 0$, $c_2 = 0.05$, $c_3 = 0.1$, $c_4 = 0.14$, and $c_5 = 0.2$. The demand function is linear $P(Q) = 1 - Q$. Recall the table showing the resulting profits for all possible transfers:

$i = 1$	P	π_1	π_2	π_3	π_4	π_5
$j = 1$	0.24833	0.06167	0.03934	0.022	0.01174	0.00234
$j = 2$	0.24	0.0576	0.0576	0.0196	0.01	0.0016
$j = 3$	0.23167	0.05367	0.033	0.05367	0.0084	0.001
$j = 4$	0.225	0.05063	0.03063	0.01563	0.05063	0.00063
$j = 5$	0.215	0.04623	0.02723	0.01323	0.00563	0.04623

It is easy to see that firm 3 is the menu auction licensee, since it is the industry-profit-maximizing partner, as we have seen before. With the characterization in Proposition 1, we can confirm that firm 4 is the simple auction licensee. It is also easy to see that firm 5 is the joint-profit-maximizing partner (maximizes $\pi_1(j) + \pi_j(j) - \pi_1(1) - \pi_j(j)$ over $j = 2, \dots, 5$). Finally, for comparison, recall that firm 4 is the social-welfare-maximizing partner. \square

6 Conclusion and Extensions

We have analyzed which rival a licensor would choose as a partner when rivals are heterogeneous. We assume that licensing entails a complete technology transfer and show that licensing between any pair of firms would improve joint profits (thus licensing is profitable) as long as there are more than two firms in the industry. However, jointly profitable licensing can be welfare reducing. These results are in contrast with the ones in the duopoly case examined by Katz and Shapiro (1985), as well as the welfare results others have found with homogeneous rivals (Katz and Shapiro 1986; Sen and Tauman 2007). However, we show that licensing the most efficient firm's technology always improves social welfare, although a welfare-maximizing licensee might not be a joint-profit-maximizing licensee. Hence, when the licensee is determined by either a simple or a menu auction, the licensor might not select a welfare-maximizing licensee under these schemes, and the efficiency of the licensee can be ordered by the licensing method: a joint-profit-maximizing licensee is less efficient than a simple auction licensee, which in turn is less efficient than a menu auction licensee.

In the previous sections, we assume that at most one firm can get a license from the licensor, and that license contracts are restricted to fixed fees only. Although these simplifying assumptions are helpful in comparing licensees in different licensing schemes and criteria, it is obviously interesting to allow multiple licenses and more flexible licensing contracts (with royalties). We will analyze these extensions by simplifying the model, since these extensions clearly complicate the analysis. Thus, we will assume a linear demand $P = 1 - Q$, and we discuss what can happen if we depart from the benchmark. There are a licensor, firm 1, and $K - 1$ potential licensees. We order firms according to their marginal costs of production $c_1 < c_2 \leq \dots \leq c_K$, and the licensor firm 1's marginal cost is normalized to zero: $c_1 = 0$. Since firms whose marginal cost is less than market price $P = 1 - Q$ survive, the equilibrium demand is written as $-Q + \sum_{k=1}^K \max\{0, (1 - Q) - c_k\} = 0$ by adding up the first-order conditions. Suppose that firms $1, \dots, j$ stay in the market. Then, equilibrium allocation is described by

$$-Q + j(1 - Q) - \sum_{k=1}^j c_k = 0, \text{ or } P = 1 - Q = \frac{1 + \sum_{k=1}^j c_k}{j + 1}.$$

Note that firm $j + 1$'s exiting the market implies that if firm $j + 1$ remains in the market then c_{j+1} is not less than the market price, resulting in nonpositive profits:

$$\frac{1 + \sum_{k=1}^{j+1} c_k}{j + 2} \leq c_{j+1}, \text{ or } \frac{1 + \sum_{k=1}^j c_k}{j + 1} \leq c_{j+1}.$$

That is, firm $j + 1$ is out if and only if c_{j+1} is higher than the market price when j firms remain in the market. In this environment, we let firm 1 choose a subset of firms to license its technology to maximize its total profits (own profits and license fees). We will use menu auction as the rule of the game here, since there are widespread externalities among potential licensees and the menu auction by Bernheim and Whinston (1986) provides a tool for systematic analysis in such an environment by letting the licensor's action set $A \equiv 2^N$ be the set of all subsets of potential licensees $N = \{2, \dots, K\}$. We first illustrate the complications generated by allowing multiple licenses by the following example.

Example 2. Suppose that $K = 7$ with marginal costs being $c_2 = c_3 = c_4 = 0.2$, and $c_5 = c_6 = c_7 = 0.23$. In this case, originally, market equilibrium price is $\frac{1+3 \times 0.2+3 \times 0.23}{8}$. In this case, if firms 2, 3, and 4 get licenses, then firms 5, 6, and 7 will be driven out of the market (inactive), since the market price is driven down to 0.2. However, this is not the equilibrium choice of licensees in a menu auction, as is seen by the following table. The equilibrium choice does not drive any firm out of the market.

licensee(s)	price	industry profits	# of survivors
\emptyset	0.28625	0.11375	7
$\{2\}$	0.26125	0.14694	7
$\{5\}$	0.2575	0.14404	7
$\{2, 3\}$	0.23625	0.16887	7
$\{2, 5\}$	0.2325	0.16429	7
$\{5, 6\}$	0.22875	0.15946	6
$\{2, 3, 4\}$	0.20000	0.16000	4

Note that in every truthful Nash equilibrium in a menu auction, the industry profits are maximized. Thus, the equilibrium set of licensees is $\{2, 3\}$ (or $\{2, 4\}$ or $\{3, 4\}$, since firms 2, 3, and 4 are symmetric). Note that the set of equilibrium licensees does not achieve the lowest number of surviving firms in this example. That is, we can see that the predation itself is not the reason for the optimal set of the licensees. The TNE payoffs can be calculated easily in this example due to this symmetry. Firm 2 is replaceable, so the license fees are $(0.23625 - 0)^2 - (0.23625 - 0.2)^2 = 0.0545$ (they can also be derived by solving the TNE payoffs from the system (2) and (3)). With a slight modification, we obtain a very different result: if $c_5 = c_6 = c_7 = 0.24$ instead of 0.23, then we can again show that $\{2, 3\}$ is one of the equilibrium sets of licensees, while firms 5, 6, and 7 are predated and the number of survivors is four including nonlicensed firm 4. Note that licensing firms 2, 3, and 4 predated firms 5, 6, and 7 as well, but the licensor does not give

three licenses. Thus, in this case, the licensor chooses the smallest set of licensors that achieves the smallest number of active firms in the industry. \square

Although Example 2 is intriguing, it also shows that it is difficult to obtain general results in a multiple licensing case. Note that this example is one of the simplest ones with heterogeneous potential licensees and possibilities of predation. Adding more elements will complicate the analysis further. For example, adding fixed cost of operation (the fixed cost that needs to be paid in order to stay in the market) affects the results of licensing, since predation effect on the inefficient firms would be stronger — licensing superior technology to (possibly multiple) firms reduces the profits of nonreceptient firms of the technology, forcing them out of the market. This idea has been analyzed by Creane and Konishi (2009a) in the case where technology transfer is made without monetary transfer.¹¹ With a licensing fee, this motivation is strengthened further since potential licensees would compete over technology more vigorously when their survival in the industry is at stake. Moreover, the licensor (and surviving firms) might prefer licensing technology to a very inefficient firm to push the market price sufficiently low to predate many other firms. Once firms exit, the market price jumps up again, increasing the pie to be shared. Although this sort of motivation for licensing complicates the analysis even further, the general tendency of the results can be simplified — the licensor tends to choose a subset of licensees to drive out as many firms as possible.

We conclude this paper by providing an example that allows royalties in the licensing contract. Sen and Tauman (2007) and others analyze license contracts involving royalties and fixed fees. The usefulness of including royalties for the licensor is that it offers a tool to control a licensee's output level and the market price. That is, the licensor's action is written as $a = (S; (r_k)_{k \in S})$, where $r_k \in \mathbb{R}$ is firm k 's royalty. The set of actions is denoted by A , and potential licensee k 's strategy is $T_k : A \rightarrow [\underline{b}, \infty)$: i.e., $T_k(S; (r_{k'})_{k' \in S})$ is the licensee fee that firm $k \in N$ pays when the set of licensees is S and their royalties are $(r_{k'})_{k' \in S}$. However, because including royalties greatly complicates the problem, we will only consider a symmetric licensee case, identical marginal costs $c > 0$. Let $n \in \{0, 1, \dots, K - 1\}$ be the number of non-licensees. Using n , the number of licensees is described by $K - 1 - n$. For each case, the highest profit that the licensor and licensees can achieve together (joint profit $\Pi(n)$) is to jointly produce $Q(n)$ taking n firms best responses into account. It is easy to calculate the *Stackelberg allocation* for each $n < K - 1$ ($n = K - 1$ means no licensee). Let $P(n)$ be the market price, $Q(n)$ and $\Pi(n)$ be joint

¹¹There has been a recent increase in interest in how endogenous market structures can affect strategies. See, e.g., Etro (2006).

output and joint profit of the licensor and licensees, and $q(n)$ and $\pi(n)$ be the output and profit of each non-licensee.

1. If $c \leq \frac{1}{n+2}$, then $Q(n) = \frac{1+nc}{2}$, $q(n) = \frac{1-(n+2)c}{2(n+1)}$, $P(n) = \frac{1+nc}{2(n+1)}$, $\Pi(n) = \frac{(1+nc)^2}{4(n+1)}$, and $\pi(n) = \left(\frac{1-(n+2)c}{2n+2}\right)^2$.
2. If $c \geq \frac{1}{n+2}$, then $Q(n) = 1 - c$, $q(n) = 0$, $P(n) = c$, $\Pi(n) = c(1 - c)$, $\pi(n) = 0$.

Using the above formulas, we can determine the TNE menu auction license allocations of the following simple example.

Example 3. Suppose that $K = 5$ and potential licensees are all homogeneous: $c_2 = c_3 = c_4 = c_5 = 0.2$. This is equivalent to the Stackelberg allocation. The licensor chooses royalties to achieve such $Q(n)$ (implementing the Stackelberg allocation in Cournot-Nash equilibrium).

n	P	Q	q	Π	π	r	
0	0.5	0.5	0	0.25	0	0.5	Monopoly
1	0.3	0.6	0.1	0.18	0.01	0.2	
2	0.2333	0.7	0.0333	0.1633	0.0011	0	
3	0.2	0.8	0	0.16	0	-0.4	
4	0.3	0.3	0.1	0.09	0.01	N\A	Cournot

Note that $\Pi(n)$ corresponds to $W_{N \setminus S}(\Gamma)$ with $|S| = n$ in (2). If $n = 1$ (one non-licensee), then royalty and the marginal cost are the same ($r = c = 0.2$) and the market outcome looks identical to the Cournot outcome ($n = 4$). However, in the $n = 1$ case, three licensees' marginal costs are actually zero, so the licensor can exploit all royalty fees although production levels are the same. If $n = 2$, then the licensor does not charge royalties. If $n = 3$ (one licensee), then in this case, the royalty is negative: that is, the licensor gives the licensee a production subsidy. Note that the market price is just marginal cost, and all three non-licensees cannot produce anything. The licensor uses the licensee as a dummy to shut out other firms from the market. This is how subsidy is given to the licensee. Finally, if $n = 0$ ($W_{N \setminus S}(\Gamma) = W_N(\Gamma)$), then it is clearly a monopoly allocation. The licensor charges $P = r = 0.5$, forcing licensees to shut down. This means that the license-fixed fee needs to be negative. This can be done if the lower bound for transfer takes a negative value. We consider two cases: a flexible license contract case and a contracts with nonnegative fixed fees and royalties case. The TNE equilibrium payoff vectors (u_1, u_2, \dots, u_7) and license fee vectors can be analyzed by using (2) and (3) (u_1 can be calculated from $u = (u_2, \dots, u_7)$).

1. *Fully flexible license contracts*: Here, let us allow for negative fixed fees and negative royalties. With such full flexibility of license contracts, the menu auction TNE is characterized by: (i) the licensor gives all potential licensees licenses; (ii) the payoff vectors are $(u_1, u_2, u_3, u_4, u_5) = (0.13, 0.3, 0.3, 0.3, 0.3)$; and (iii) the license contract is $(T, r) = (-0.3, 0.5)$ such that all licensees shut down by receiving transfers 0.3 (negative fixed fees: assume $\underline{b} \leq -0.3$). This is derived by the following inequalities:

$$\sum_{k \in N = \{2, \dots, 5\}} u_k \leq W_N - W_\emptyset = 0.25 - 0.09 = 0.16 \quad (4)$$

$$\sum_{k \in S} u_k \leq W_N(\Gamma) - W_{N \setminus S}(\Gamma) = 0.25 - 0.16 = 0.09 \quad \forall S \subset N \text{ with } |S| = 3 \quad (5)$$

$$\sum_{k \in S} u_k \leq W_N(\Gamma) - W_{N \setminus S}(\Gamma) = 0.25 - 0.1633 = 0.0867 \quad \forall S \subset N \text{ with } |S| = 2 \quad (6)$$

$$\sum_{k \in S} u_k \leq W_N(\Gamma) - W_{N \setminus S}(\Gamma) = 0.25 - 0.18 = 0.07 \text{ for all } S \subset N \text{ with } |S| = 1 \quad (7)$$

The binding constraint is (5), and together with Pareto efficiency among licensees N ($\sum_{k \in N \cup \{1\}} u_k = W_N$), implies $u_2 = u_3 = u_4 = u_5 = 0.03$; thus firm 1 gets $0.25 - 0.12 = 0.13$.

2. *Restricted license contracts (nonnegative fixed fees and royalties)*: In this case, $W_N(\Gamma) = 0.25$ no longer holds, since without negative fixed fees, no licensee has an incentive to sign a contract that asks it to shut down its production. In this case, the most efficient license choice is to give licenses to three firms ($n = 1$): $W_N(\Gamma) = \Pi(1) + \pi(1) = 0.19$. Moreover, in the case of $n = 3$, $r \geq 0$ is imposed, so the best allocation for the licensor is simply to set $r = 0$. This implies a Cournot equilibrium with two zero marginal cost firms and three non-licensees, which generates $\Pi(3) = 0.142$. In this case, the modification of (7) will be binding:

$$u_k \leq W_N(\Gamma) - W_{N \setminus \{k\}}(\Gamma) = 0.19 - 0.18 = 0.01 \quad \forall k \in N$$

Thus, $u_2 = u_3 = u_4 = u_5 = 0.01$, and $u_1 = 0.15$. The menu auction TNE license contract is a combination of “no fixed fee” and a royalty of 0.2 to three licensees, leaving one firm unlicensed. This result may seem to contradict a result in Sen and Tauman (2007), since they show that there is no equilibrium with royalties only. This difference arises from a difference in the rules of the game: they consider a game in which the licensor announces a licensing policy, unlike our menu auction game.

By allowing fixed fees and production-prohibitive royalties (case 1), the licensor can effectively achieve a monopoly and licensing becomes an anti-competitive act. It is interesting to note that the licensor earns higher profit in the restricted case 2. This is because both $W_N(\Gamma)$ and $W_{\{k\}}(\Gamma)$ are modified by the restrictions, and the binding constraint changes from (5) to (7). This observation confirms that allowing different classes of licensing contracts complicates the results further. Moreover, the results are sensitive to the setup and restrictions of the game. \square

Appendix A

Proof of Lemma 1. First note that the equilibrium output of firm i , q_i , is expressed by equation

$$q_i = \frac{(P(Q) - c_i)}{-P'(Q)},$$

if $P(Q) > c_i$, and $q_i = 0$ if $P(Q) \leq c_i$. Recall that $c_1 \leq c_2 \leq \dots \leq c_K$. Summing up the first-order conditions for profit maximization over firms in subset $L \subset K$, and assuming that these firms produce positive outputs, we obtain

$$P'(Q)Q + LP(Q) = \sum_{\ell \in L} c_\ell,$$

where L also denotes the number of firms in L . If the solution of the above equation Q satisfies $P(Q) \geq c_\ell$ for all $\ell \in L$, and $P(Q) < c_k$ for all $k \in K \setminus L$, then Q is the equilibrium aggregate output. Rewriting the above equation, we obtain

$$P'(Q)Q + \sum_{\ell \in L} (P(Q) - c_\ell) = 0,$$

or

$$P'(Q)Q + \sum_{k \in K} \max\{0, P(Q) - c_k\} = 0.$$

The LHS of the above equation is continuous in Q , although it is not continuously differentiable since firms stop producing in order as Q increases.¹² However, for each $L \subset K$, the LHS is differentiable for Q satisfying $P(Q) \geq c_\ell$ for all $\ell \in L$, and $P(Q) < c_k$ for all $k \in K \setminus L$, and the derivative is

$$\frac{d(LHS)}{dQ} = P''(Q)Q + (L + 1)P'(Q)$$

¹²Firm i produces a positive output q_i earning a positive profit if and only if $P(Q) > c_i$, as is easily seen from the formula above.

Summing the strategic substitutability conditions up over firms $\ell \in L$, we obtain

$$P''(Q)Q + LP'(Q) \leq 0.$$

This implies that the LHS of the aggregated first-order condition is decreasing in Q since $P'(Q) < 0$. This implies that equilibrium aggregate output Q is uniquely determined for every marginal cost profile (c_1, \dots, c_K) .

Now, we conduct a comparative static analysis with respect to c_j . By the above analysis, it is easy to see that Q decreases as c_j increases if $P(Q) > c_j$, and Q is intact otherwise. \square

Proof of Lemma 2. In scenario (i) c_j goes down to c_i , while in scenario (ii) $c_{j'}$ goes down to c_i . Suppose that in scenario (i), firms $\ell \in L$ remain in operation: $q_\ell > 0$ (and firms $k \in K \setminus L$ chooses $q_k = 0$). Clearly, firms i and j will be in operation after technology transfer: $i, j \in L$. First consider the case where $j' \in L$ in scenario (i). Then, the aggregate output \hat{Q} in scenario (i) is described by (recall that firm j 's cost is c_i)

$$\begin{aligned} 0 &= P'(\hat{Q})\hat{Q} + \sum_{\ell \in L} (P(\hat{Q}) - c_\ell) \\ &= P'(\hat{Q})\hat{Q} + \sum_{\ell \in L \setminus \{j, j'\}} (P(\hat{Q}) - c_\ell) + (P(\hat{Q}) - c_i) + (P(\hat{Q}) - c_{j'}) \\ &< P'(\hat{Q})\hat{Q} + \sum_{\ell \in L \setminus \{j, j'\}} (P(\hat{Q}) - c_\ell) + (P(\hat{Q}) - c_i) + (P(\hat{Q}) - c_j). \end{aligned}$$

Since $P'(Q)Q + \sum_{k \in K} \max\{0, P(Q) - c_k\}$ is a decreasing function in Q , the equilibrium aggregate output \tilde{Q} in scenario (ii) satisfies $\hat{Q} < \tilde{Q}$.

Second, consider the case where $j' \notin L$ in scenario (i). Since $c_j < c_{j'}$ and $P(\hat{Q}) < c_{j'}$, the aggregate output \hat{Q} in scenario (i) is

$$\begin{aligned} 0 &= P'(\hat{Q})\hat{Q} + \sum_{\ell \in L} (P(\hat{Q}) - c_\ell) \\ &= P'(\hat{Q})\hat{Q} + \sum_{\ell \in L \setminus \{j\}} (P(\hat{Q}) - c_\ell) + (P(\hat{Q}) - c_i) \\ &\leq P'(\hat{Q})\hat{Q} + \sum_{\ell \in L \setminus \{j, j'\}} (P(\hat{Q}) - c_\ell) + \max\{0, P(\hat{Q}) - c_j\} + (P(\hat{Q}) - c_i). \end{aligned}$$

Thus, as before, the equilibrium aggregate output \hat{Q} in scenario (ii) satisfies $\hat{Q} < \tilde{Q}$ if $P(\hat{Q}) - c_j > 0$, and $\hat{Q} = \tilde{Q}$, otherwise. \square

Proof of Theorem 1. The proof utilizes an artificial market. This device is useful because transferring technology partially can reduce the joint profit. We first replace firm

j with an artificial (public: not profit-maximizing) firm i' with marginal cost c_i , but control its output level so that the joint profit between firms i and i' increases monotonically. After that, we return to the original economy.

Consider an *artificial* market parametrized by $\alpha \in [0, 1]$, in which firm j ($c_i < c_j$) is replaced by an artificial firm i' that satisfies (i) $c_{i'} = c_i$, (ii) $q_{i'}(\alpha) = \alpha q_i(\alpha)$, and (iii) $(q_k(\alpha))_{k \neq i'}$ is a solution of the system of equations: $q_k(\alpha) = \max \left\{ 0, \frac{P(Q(\alpha)) - c_k}{-P'(Q(\alpha))} \right\}$ for all $k \neq i'$ and $Q(\alpha) = \sum_{k \neq i'} q_k(\alpha) + \alpha q_i(\alpha)$. That is, although the output decision by firm i' is linked with that of firm i , firms $k \neq i'$ do not use this information by choosing the best response to $Q_{-k}(\alpha) = \sum_{\ell \neq k} q_\ell(\alpha)$ (the standard Cournot behavior, not the Stackelberg one). Note that when $\alpha = 1$, $Q(1)$ is the aggregate Cournot equilibrium output after the complete technology transfer from firm i to firm j , since the best response by firm i' is identical to the one by firm i when $\alpha = 1$.

In the following, we will show that in this artificial market, the joint profit of firms i and i' , $\Pi^J(\alpha) = (1 + \alpha)\pi_i(\alpha) = \frac{(1+\alpha)(P(Q(\alpha)) - c_i)^2}{-P'(Q(\alpha))}$, increases monotonically as α goes up (step 1). Then, we connect this artificial economy with the original economy before technology transfer (step 2).

(Step 1) The best response by firm $k \neq i'$ is described by

$$q_k(\alpha) = \min \left\{ 0, \frac{P(Q(\alpha)) - c_k}{-P'(Q(\alpha))} \right\}.$$

Since firm i will be in operation after technology transfer, we have

$$q_i(\alpha) = \frac{P(Q(\alpha)) - c_i}{-P'(Q(\alpha))};$$

thus we can write

$$q_{i'}(\alpha) = \alpha \times \frac{P(Q(\alpha)) - c_i}{-P'(Q(\alpha))}.$$

Let $L(\alpha) \equiv \{k \in K : q_k(\alpha) > 0\}$. As before, we denote the cardinality of $L(\alpha)$ by $L(\alpha)$ as well. Summing up these equations, we have

$$\sum_{\ell \in L(\alpha)} q_\ell(\alpha) = (1 + \alpha) \frac{P(Q(\alpha)) - c_i}{-P'(Q(\alpha))} + \sum_{\ell \in L(\alpha) \setminus \{i, i'\}} \frac{P(Q(\alpha)) - c_\ell}{-P'(Q(\alpha))},$$

or

$$P'(Q(\alpha))Q(\alpha) + (L(\alpha) - 1 + \alpha) P(Q(\alpha)) - \left(\sum_{\ell \in L(\alpha)} c_\ell - (1 - \alpha)c_i \right) = 0.$$

Totally differentiating the above, we have

$$(P''(Q(\alpha))Q(\alpha) + P'(Q(\alpha)) + (L(\alpha) - 1 + \alpha)P'(Q(\alpha)))dQ + (P(Q(\alpha)) - c_i)d\alpha = 0$$

so that

$$\frac{dQ}{d\alpha} = \frac{P(Q(\alpha)) - c_i}{-P''(Q(\alpha))Q(\alpha) - (L(\alpha) + \alpha) P'(Q(\alpha))}.$$

Since $Q(\alpha) = \sum_{\ell \in L(\alpha)} q_\ell(\alpha)$ and $P''(Q)q_\ell + P'(Q) \leq 0$ holds for all $\ell \in L(\alpha) \setminus \{i'\}$, we have

$$\begin{aligned} & -P''(Q(\alpha))Q(\alpha) - (L(\alpha) + \alpha) P'(Q(\alpha)) \\ &= - \sum_{\ell \in L(\alpha) \setminus \{i, i'\}} (P''(Q(\alpha))q_\ell(\alpha) + P'(Q(\alpha))) - (1 + \alpha) (P''(Q(\alpha))q_i(\alpha) + P'(Q(\alpha))) \\ &> 0. \end{aligned}$$

The inequality is strict as long as there is at least one firm with a different marginal cost from others (i.e., if $P''(Q)q_k + P'(Q) = 0$ holds then $P''(Q)q_\ell + P'(Q) < 0$ must hold due to the strategic substitutability assumption). That is, for each $L \subset K$ with $L = L(\alpha)$ for some range of $\alpha \in [0, 1]$, $\frac{dQ}{d\alpha} > 0$ holds for the range of α . This implies that $Q(\alpha)$ monotonically increases as α increases, resulting in monotonic reduction of $P(Q(\alpha))$. Since firms shut down their production in order from higher marginal cost ones (if any firm does), the set of active firms $L(\alpha)$ shrinks in a nested manner: $L(\alpha') \subseteq L(\alpha)$ for all $\alpha' > \alpha$.

Now, we show $\Pi^J(\alpha) = \frac{(1+\alpha)(P(Q(\alpha))-c_i)^2}{-P'(Q(\alpha))}$ increases as α increases. We consider (To save space, we omit $Q(\alpha)$ for $P(Q(\alpha))$.)

$$\begin{aligned} & \frac{d\Pi^J}{d\alpha} \\ &= \frac{(P - c_i)^2}{-P'} + (1 + \alpha) \times \frac{2(P - c_i)P'(-P') + P''(P - c_i)^2}{(-P')^2} \times \frac{P - c_i}{-P''Q - (L(\alpha) + \alpha) P'} \\ &= A \times \left[(-P')(-P''Q - (L(\alpha) + \alpha) P') + (1 + \alpha) \left\{ -2(-P')^2 + P''(P - c_i) \right\} \right] \\ &= A \times \left[\{(L(\alpha) + \alpha) - 2(1 + \alpha)\} (-P')^2 + (-P'') \{-P'Q - (1 + \alpha)(P - c_i)\} \right] \\ &= A \times \left[(L(\alpha) - 2 - \alpha) (-P')^2 + (-P'') \{(-P')(Q - (1 + \alpha)q_i) - (1 + \alpha)(P'q_i + P - c_i)\} \right] \end{aligned}$$

where $A = \frac{(P-c_i)^2}{(-P')^2(-P''Q-(L(\alpha)+\alpha)P')} > 0$. We can determine the sign of $\frac{d\Pi^J}{d\alpha}$. Note that $P' < 0$ and $P'' \leq 0$. Since $L(\alpha) \geq 3$, $L(\alpha) - 2 - \alpha \geq 0$ must follow, and the first term in the bracket of the last line is nonnegative for all $\alpha \in [0, 1]$. Since $L(\alpha) \geq 3$ with interior solution, we have $Q > (1 + \alpha)q_i$, and $P'q_i + P - c_i = 0$ holds by firm i 's first-order condition. This implies that the second term is positive. Thus, we can conclude that $\frac{d\Pi^J}{d\alpha} > 0$ holds for all $\alpha \in (0, 1)$.¹³

¹³Strictly speaking, Π^J is not continuously differentiable (the right and left derivatives are different) at α with $P(Q(\alpha)) = c_k$ for some $k \in K$, though it is a continuous function. However, it is clear that Π^J is monotonically increasing in α .

(Step 2) Now, we show that the equilibrium allocation with firm j is mimicked by an equilibrium allocation in our artificial market at a certain $\hat{\alpha} \in (0, 1)$. Let $(\hat{P}, (\hat{q}_k)_{k=1}^K)$ be the Cournot equilibrium allocation before firm j received a complete technology transfer. Let $\hat{\alpha} = \frac{\hat{q}_j}{\hat{q}_i}$. Since $c_j > c_i$, we have $\hat{q}_i > \hat{q}_j \geq 0$ and $0 < \hat{\alpha} < 1$. Thus, $(\hat{P}, (\hat{q}_k)_{k=1}^K) = (P(\hat{\alpha}), (q_k(\hat{\alpha}))_{k=1}^K)$ holds, and the initial equilibrium allocation is mimicked by the equilibrium in an artificial market with $\alpha = \hat{\alpha}$. Since $\hat{q}_j = \hat{\alpha}\hat{q}_i = \hat{\alpha}q_i(\hat{\alpha})$, we have

$$\begin{aligned}\hat{\pi}_i + \hat{\pi}_j &= (\hat{P} - c_i) \hat{q}_i + (\hat{P} - c_j) \hat{q}_j \\ &= (P(\hat{\alpha}) - c_i) q_i(\hat{\alpha}) + (P(\hat{\alpha}) - c_j) \hat{\alpha} q_i(\hat{\alpha}) \\ &< (P(\hat{\alpha}) - c_i) q_i(\hat{\alpha}) + (P(\hat{\alpha}) - c_i) \hat{\alpha} q_i(\hat{\alpha}) \\ &= \Pi^J(\hat{\alpha}).\end{aligned}$$

Since $\Pi^J(\alpha)$ is monotonically increasing in α , we have $\Pi^J(\hat{\alpha}) < \Pi^J(1)$. Since $\Pi^J(1)$ is the same as the joint profit by firms i and j after the complete technology transfer from firm i to firm j , we can conclude that the joint profit by firms i and j must increase after the complete technology transfer. \square

Proof of Theorem 2. By Lemma 1, we know that if a technology transfer is made from a technologically superior firm to a technologically inferior firm, the equilibrium aggregate output Q increases. Now consider firm k . If C decreases keeping c_k constant, Q increases while q_k shrinks. We can represent the relationship between Q and q_k (through changes in C behind) as follows:

$$q_k(Q) = \frac{P(Q) - c_k}{-P'(Q)}.$$

Denote the original (before transfer) equilibrium by “hat,” and the new equilibrium by “tilde.” Since firm j ’s marginal cost c_j only goes down from $\hat{c}_j = c_j$ to $\tilde{c}_j = c_i$ keeping all other marginal costs constant, we have $\hat{Q} < \tilde{Q}$ and $\hat{q}_k > \tilde{q}_k$ for all $k \neq j$.¹⁴ Then, we have $\hat{q}_j < \tilde{q}_j$ and $\tilde{q}_j - \hat{q}_j > \tilde{Q} - \hat{Q}$.

Since the social welfare is written as

$$SW = (\text{total benefit}) - (\text{total cost}) = \int_0^Q P(Q') dQ' - \sum_{k=1}^K c_k q_k,$$

we have

$$\widetilde{SW} = \int_0^{\tilde{Q}} P(Q') dQ' - \sum_{k=1}^K c_k \tilde{q}_k$$

¹⁴If firm k is not in operation before the transfer, then $\hat{q}_k = \tilde{q}_k = 0$.

$$\begin{aligned}
&= \int_0^{\hat{Q}} P(Q')dQ' + \int_{\hat{Q}}^{\tilde{Q}} P(Q')dQ' - \sum_{k \neq j} c_k \tilde{q}_k - c_1 \tilde{q}_j \\
&= \int_0^{\hat{Q}} P(Q')dQ' + \int_{\hat{Q}}^{\tilde{Q}} P(Q')dQ' - \sum_{k \neq j} c_k \tilde{q}_k - c_1(\tilde{Q} - \hat{Q}) - c_1(\tilde{q}_j - (\tilde{Q} - \hat{Q})) \\
&= \int_0^{\hat{Q}} P(Q')dQ' - \sum_{k \neq j} c_k \tilde{q}_k - c_1(\tilde{q}_j - (\tilde{Q} - \hat{Q})) \\
&\quad + \int_{\hat{Q}}^{\tilde{Q}} P(Q')dQ' - c_1(\tilde{Q} - \hat{Q}).
\end{aligned}$$

The last two terms are obviously positive since $P(\tilde{Q}) > c_1$. Thus, we have

$$\begin{aligned}
\widetilde{SW} - \widehat{SW} &> \int_0^{\hat{Q}} P(Q')dQ' - \sum_{k \neq j} c_k \tilde{q}_k - c_1(\tilde{q}_j - (\tilde{Q} - \hat{Q})) - \widehat{SW} \\
&= \int_0^{\hat{Q}} P(Q')dQ' - \sum_{k \neq j} c_k \tilde{q}_k - c_1(\tilde{q}_j - (\tilde{Q} - \hat{Q})) - \int_0^{\hat{Q}} P(Q')dQ' + \sum_{k=1}^K c_k \hat{q}_k \\
&= \sum_{k=1}^K c_k \hat{q}_k - \sum_{k \neq j} c_k \tilde{q}_k - c_1(\tilde{q}_j - (\tilde{Q} - \hat{Q})) \\
&= \sum_{k \neq j} c_k (\hat{q}_k - \tilde{q}_k) + c_j \hat{q}_j - c_1 \left(\tilde{q}_j - \sum_{k=1}^K (\tilde{q}_k - \hat{q}_k) \right) \\
&= \sum_{k \neq j} (c_k - c_1) (\hat{q}_k - \tilde{q}_k) + (c_j - c_1) \hat{q}_j > 0.
\end{aligned}$$

Hence, we conclude $\widetilde{SW} > \widehat{SW}$. \square

Appendix B: Licensing Equilibria

We first characterize the set of Nash equilibria. Although firm 1 is not a bidder, we let $T_1 \equiv 0$ for notational convenience.

Lemma 3. In a simple auction, an outcome (j^*, T^*) is a Nash equilibrium if and only if

- (a) $\pi_1(j^*) + T_{j^*}^* \geq \pi_1(j) + T_j^*$ for all j .
- (b) If $j^* > 1$, then $\pi_1(j^*) + \pi_j(j^*) + T_{j^*}^* \geq \pi_1(j) + \pi_j(j)$ for all $j \neq j^*$.
- (c) If $j^* > 1$ and $T_{j^*}^* > 0$, then $\pi_1(j^*) + T_{j^*}^* = \pi_1(j) + T_j^*$ for some $j \neq j^*$ and $\pi_1(j^*) + T_{j^*}^* = \pi_1(\tilde{j}) + T_{\tilde{j}}^*$ implies $\pi_{j^*}(j^*) - T_{j^*}^* \geq \pi_{j^*}(\tilde{j})$.

Proof. Consider a Nash equilibrium outcome (j^*, T^*) . Condition (a) is obvious from the structure of the game. For (b), suppose we have some $j \neq j^*$ such that $\pi_j(j^*) < \pi_j(j) - [(\pi_1(j^*) + T_{j^*}^*) - \pi_1(j)]$. Then firm j can offer $\tilde{T}_j = \pi_1(j^*) + T_{j^*}^* - \pi_1(j) + \varepsilon_j$ for some $\varepsilon_j > 0$ so that $U_j(j, \tilde{T}_j, T_{-j}^*) \geq U_j(j^*, T^*)$ and $U_1(j, \tilde{T}_j, T_{-j}^*) \geq U_1(k, \tilde{T}_j, T_{-j}^*)$ for all k . What remains is condition (c). If there is no j such that $\pi_1(j^*) + T_{j^*}^* = \pi_1(j) + T_j^*$, then from condition (a), we have $\pi_1(j^*) + T_{j^*}^* > \pi_1(j) + T_j^*$ for all j . Then firm j^* can offer $\tilde{T}_{j^*} = T_{j^*}^* - \varepsilon_{j^*}$ for some $\varepsilon_{j^*} > 0$ so that $U_{j^*}(j^*, \tilde{T}_{j^*}, T_{-j^*}^*) \geq U_{j^*}(j^*, T^*)$ and $U_1(j^*, \tilde{T}_{j^*}, T_{-j^*}^*) \geq U_1(k, \tilde{T}_{j^*}, T_{-j^*}^*)$ for all k . If there is some \tilde{j} with $\pi_1(j^*) + T_{j^*}^* = \pi_1(\tilde{j}) + T_{\tilde{j}}^*$ such that $\pi_{j^*}(j^*) - T_{j^*}^* < \pi_{j^*}(\tilde{j})$, then firm j^* can deviate to $\tilde{T}_{j^*} = 0$ so that $U_{j^*}(\tilde{j}, \tilde{T}_{j^*}, T_{-j^*}^*) \geq U_{j^*}(j^*, T^*)$ and $U_1(\tilde{j}, \tilde{T}_{j^*}, T_{-j^*}^*) \geq U_1(k, \tilde{T}_{j^*}, T_{-j^*}^*)$ for all k . Therefore, any Nash equilibrium satisfies all three conditions.

Suppose to the contrary that an outcome (j^*, T^*) satisfies the three conditions but is not a Nash equilibrium. Condition (a) implies that firm 1 selects firm j^* . First, consider the case where j^* has incentive to deviate from $T_{j^*}^*$ to \tilde{T}_{j^*} . It is clear that $\tilde{T}_{j^*} < T_{j^*}^*$ because $\tilde{T}_{j^*} \geq T_{j^*}^*$ would still make firm j^* be the licensee with no less payment. However, condition (c) implies that when j^* reduces payment, there exists $\tilde{j} \neq j^*$, $\pi_1(\tilde{j}) + T_{\tilde{j}}^* = \pi_1(j^*) + T_{j^*}^*$ with $\pi_{j^*}(j^*) - T_{j^*}^* \geq \pi_{j^*}(\tilde{j})$ to be chosen as the licensee, which violates the condition that j^* will deviate. Now consider $j \neq j^*$ deviates from T_j^* to \tilde{T}_j . Then we have $\tilde{T}_j \geq 0$ such that $\pi_1(j) + \tilde{T}_j \geq \pi_1(k) + T_k^*$ for all $k \neq j$ and $\pi_j(j) - \tilde{T}_j > \pi_j(j^*)$. From condition (b), we have $\pi_1(j^*) + \pi_j(j^*) + T_{j^*}^* \geq \pi_1(j) + \pi_j(j)$. Hence, we have $\pi_1(j^*) + T_{j^*}^* - \tilde{T}_j > \pi_1(j)$. From condition (c), we have for some $\tilde{j} \neq j^*$, $\pi_1(j^*) + T_{j^*}^* = \pi_1(\tilde{j}) + T_{\tilde{j}}^*$, so that $\pi_1(\tilde{j}) + T_{\tilde{j}}^* - \tilde{T}_j > \pi_1(j)$, which contradicts the condition that j deviates. \square

Proof of Proposition 1. First suppose that no licensing is a TNE. Then, $\pi_1(j) + T_j^* \leq \pi_1(1)$ and $\pi_j(j) = \pi_j(1) + T_j^*$ holds for all $j \neq 1$. Thus, $\pi_1(1) + \pi_j(1) \geq \pi_1(j) + \pi_j(j)$ holds. Conversely, if $\pi_1(1) + \pi_j(1) \geq \pi_1(j) + \pi_j(j)$ holds for all $j \neq 1$, then $\pi_1(j) + T_j^* \leq \pi_1(1)$ and $\pi_j(j) = \pi_j(1) + T_j^*$ holds.

Second, we consider the case with licensing. Let (j^*, T^*) be a TNE. In a TNE, we have $T_j^* = \pi_j(j) - \pi_j(j^*)$ for all $j \neq j^*$. From condition (a) of Lemma 3, we have $\pi_1(j^*) + T_{j^*}^* \geq \pi_1(j) + T_j^*$ for all $j \neq 1$ so that $T_{j^*}^* = \max_{j \in K \setminus \{1, j^*\}} \{\pi_1(j) + \pi_j(j) - \pi_j(j^*) - \pi_1(j^*)\}$. This implies condition (b) of Lemma 3. By condition (c) of Lemma 3, we have $\tilde{j} \neq j^*$ such that $\pi_1(j^*) + T_j^* = \pi_1(\tilde{j}) + T_{\tilde{j}}^*$ and $\pi_{j^*}(j^*) - T_{j^*}^* \geq \pi_{j^*}(\tilde{j})$. Hence, we have $\pi_1(j^*) + \pi_{j^*}(j^*) - \pi_{j^*}(\tilde{j}) \geq \pi_1(\tilde{j}) + \pi_{\tilde{j}}(\tilde{j}) - \pi_{\tilde{j}}(j^*)$. Since $\pi_1(\tilde{j}) + \pi_{\tilde{j}}(\tilde{j}) - \pi_{\tilde{j}}(j^*) \geq \pi_1(j) + \pi_j(j) - \pi_j(j^*)$ for all $j \neq j^*$, we have $\pi_1(j^*) + \pi_{j^*}(j^*) + \pi_j(j^*) \geq \pi_1(j) + \pi_{j^*}(j) + \pi_j(j)$ for all $j \neq 1$.

Consider $\pi_1(j^*) + \pi_{j^*}(j^*) + \pi_j(j^*) \geq \pi_1(j) + \pi_{j^*}(j) + \pi_j(j)$ for all $j \neq 1$. Define

$T_j^* = \pi_j(j) - \pi_j(j^*)$ for all $j \neq j^*$ and $T_{j^*}^* = \max_{j \in K \setminus \{1\}} \{\pi_1(j) + \pi_j(j) - \pi_j(j^*)\} - \pi_1(j^*)$. It is easy to check that all conditions for a Nash equilibrium are satisfied. \square

Proof of Proposition 2. Let $j^* \in \operatorname{argmax}_{j \in K} [(\pi_1(j) + \pi_j(j)) - (\pi_1(1) + \pi_j(1))]$ be a joint-profit-maximizing partner. Suppose to the contrary that there exists $k > j^*$ with $c_k > c_{j^*}$ such that $\pi_1(k) + \pi_k(k) + \pi_{j^*}(k) > \pi_1(j^*) + \pi_k(j^*) + \pi_{j^*}(j^*)$. Since we have $\pi_1(j^*) + \pi_{j^*}(j^*) - \pi_{j^*}(1) \geq \pi_1(k) + \pi_k(k) - \pi_k(1)$, it is easy to see $\pi_{j^*}(k) - \pi_k(j^*) > \pi_{j^*}(1) - \pi_k(1) > 0$. First consider $\pi_{j^*}(k) = 0$. Then $\pi_k(j^*) = 0$, which is a contradiction. Second, consider $\pi_k(j^*) = 0$. Then we have $P(Q_{j^*}) \leq c_k$ where Q_j is the equilibrium aggregate output when firm j obtains the license. However,

$$\begin{aligned} \pi_{j^*}(1) - \pi_k(1) &= \frac{(2P(Q_1) - c_{j^*} - c_k)(c_k - c_{j^*})}{-P'(Q_1)} \\ &> \frac{(2P(Q_1) - c_{j^*} - c_k)(P(Q_{j^*}) - c_{j^*})}{-P'(Q_k)} \\ &\geq \frac{(P(Q_k) - c_{j^*})^2}{-P'(Q_k)} = \pi_{j^*}(k) \end{aligned}$$

where the second inequality comes from $-P'(Q_1) < -P'(Q_k)$ and the third inequality comes from $P(Q_1) > P(Q_{j^*}) \geq P(Q_k)$. This is a contradiction. Finally, consider $\pi_{j^*}(k) \geq 0$ and $\pi_k(j^*) \geq 0$. We have

$$\begin{aligned} \pi_{j^*}(k) - \pi_k(j^*) &= \frac{(P(Q_k) - c_{j^*})^2}{-P'(Q_k)} - \frac{(P(Q_{j^*}) - c_k)^2}{-P'(Q_{j^*})} \\ &< \frac{(P(Q_k) - c_{j^*})^2}{-P'(Q_{j^*})} - \frac{(P(Q_{j^*}) - c_k)^2}{-P'(Q_{j^*})} \end{aligned}$$

since $-P'(Q_{j^*}) < -P'(Q_k)$ by weak concavity of P and Lemma 2. Then we have

$$\begin{aligned} &\frac{(P(Q_k) - c_{j^*})^2}{-P'(Q_{j^*})} - \frac{(P(Q_{j^*}) - c_k)^2}{-P'(Q_{j^*})} \\ &= \frac{(P(Q_k) + P(Q_{j^*}) - c_{j^*} - c_k)(P(Q_k) - P(Q_{j^*}) + c_k - c_{j^*})}{-P'(Q_{j^*})} \\ &< \frac{(2P(Q_1) - c_{j^*} - c_k)(c_k - c_{j^*})}{-P'(Q_1)} = \pi_{j^*}(1) - \pi_k(1) \end{aligned}$$

since $2P(Q_1) \geq P(Q_k) + P(Q_{j^*})$, $-P'(Q_1) < -P'(Q_{j^*})$ and from equilibrium conditions we have $c_k - c_{j^*} = [-P'(Q_k)]Q_k - [-P'(Q_{j^*})]Q_{j^*} + K[P(Q_{j^*}) - P(Q_k)]$ so that $0 \leq P(Q_k) - P(Q_{j^*}) + c_k - c_{j^*} \leq c_k - c_{j^*}$. Hence, we have $\pi_{j^*}(k) - \pi_k(j^*) < \pi_{j^*}(1) - \pi_k(1)$, which is a contradiction. \square

Proof of Proposition 3. Denote with j^S and j^M a simple auction licensee and a menu auction licensee. By the property of a TNE in a menu auction, we have $\sum_{h \in K} \pi_h(j^M) \geq \sum_{h \in K} \pi_h(j^S)$. First, suppose $j^S = 1$. By Proposition 1, we have $\pi_1(1) + \pi_{j^M}(1) \geq \pi_1(j^M) + \pi_{j^M}(j^M)$. Hence, we have $\sum_{h \in K \setminus \{1, j^M\}} \pi_h(j^M) \geq \sum_{h \in K \setminus \{1, j^M\}} \pi_h(1)$. This implies that $j^M = 1$.

Now suppose, $j^S > 1$. By Proposition 1, we have $\pi_1(j^S) + \pi_{j^M}(j^S) + \pi_{j^S}(j^S) \geq \pi_1(j^M) + \pi_{j^S}(j^M) + \pi_{j^M}(j^M)$. Hence, we have $\sum_{h \in K \setminus \{1, j^S, j^M\}} \pi_h(j^M) \geq \sum_{h \in K \setminus \{1, j^S, j^M\}} \pi_h(j^S)$. This implies that $j^M \leq j^S$. \square

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