Investment in risky innovations with multiple innovators

Anthony Creane*
Department of Economics
Michigan State University
East Lansing, Michigan 48824-1038
1-517-432-1068 (fax)
creane@pilot.msu.edu

Several firms compete to develop a new product. While most research assumes only a single firm develops the new product, here an uncertain number of firms develop the new product and compete. A firm has uncertainty about how many competitors it faces as well as whether it succeeds. Conditions for under investment always exist. However, restrictions on production costs can lead to deceptive results as linear marginal cost of production makes over investment likely. Investment subsidies (taxes) are most likely to raise expected welfare when the projects are likely to fail (succeed) in equilibrium and there are few (many) investing firms. *Keyword: investment, uncertainty*.

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1. Introduction

In 1997 firms around the world invested approximately $450 billion[1] in R&D attempting to develop new products and innovations. Some of these expenses may prove fruitless and the literature has modeled this uncertainty as a patent race in which one firm succeeds and the others fail. However, often when one firm succeeds, several other firms succeed with similar inventions or innovations. First, rivals may develop different products, or improvements on current products that compete with the firm's innovation. For example, in pharmaceuticals there was "the near simultaneous launch of two new over-the-counter ketoprofen products..." (Chemical Marketing Reporter, November 27, 1995) while other industries are known for this characteristic, e.g., "...the unbelievable food-industry propensity for simultaneous development of similar products" (Star Tribune (Minneapolis, MN) November 12, 1995). Second, the rivals may quickly appropriate the firm's new technology even if patented, as patent protection is often weak and easily circumvented.[2] For example, although Netscape briefly held a monopoly, the more relevant question may have been how many rivals would eventually compete with it. Third, the innovation may be one of process innovation or cost reduction. As there may be many ways to reduce cost, a successful innovator may have to compete with others with a new cost advantage. Finally, even if a firm is able to secure an enforceable patent for a new product, it still must compete with other inventions that are imperfect substitutes. Thus, a firm investing in R&D is not only uncertain about its own success, but it is uncertain about how many other firms will also succeed with similar inventions or innovations.

The paper here examines how having an uncertain number of firms succeeding affects investment decisions and as a result expected welfare. First, a very simple model is considered so as to emphasize the effects from a random number of firms succeeding compared to those from a single firm succeeding. The simplicity ensures that other inefficiencies do not exist: the market is ex post efficient. A more sophisticated model is then considered as successful firms compete in Cournot competition. Even though

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[1] National Science Foundation 1998.

[2] Levin, et. al. report that 57%/66% (31%/34%) of typical new unpatented (patented) process/product can be duplicated within a year. The median/modal number of firms capable was 6 to 10 (3 to 5).

excess investment-entry is the well-known result with certainty, in the uncertainty model here there can be insufficient investment. Finally, since innovations do not always drive out those who do not innovate, non-drastic cost reducing innovations are also considered.

The effects here differ from those in previous models in several ways. The most obvious is that unlike with a patent race (single-winner) model, a successful firm may face competition from one or more rivals. This has a key effect on the incentives to invest because here, unlike in single winner models, the firm conditional on its success expects more firms in the market than it expects unconditionally; in patent race models they are both one.[3] This result is simply illustrated in the example of section 2. Further, in patent race models all firms are attempting to acquire the same prize; one firm's increased investment may reduce other firms' chances. This duplication of resources can lead to excess investment in single winner models, while the effect of the model here can lead to insufficient investment because here one firm's increased investment *increases* the expected number of firms. The model here also differs from a single winner model in that the expected oligopoly environment can lead to excessive investment when a single winner model would lead to insufficient investment. If a sufficiently large number of firms succeed, then the private value for the marginal successful firm exceeds the social value. This is an extension of the well-known result regarding entry (von Weizsäcker 1980, Mankiw and Whinston 1986, Suzumura and Kiyono 1987), but instead in an environment of uncertainty and investment.

The effects here also differ from previous oligopoly models, as the number of firms is uncertain. A main result of this difference is that a firm's investment cannot affect the environment when it succeeds. That is, a firm that invests more increases its probability of success, but does not change the expected market structure conditional on its success. This effect is also captured by the simple example of section 2. (The firm does not care about the market conditional on its failure as it earns zero then.) On the one hand, this means that the typical inefficiency from strategic competition does not exist; the firm's choice

[3]Cursory musing might suggest that these are non-"rational" expectations/incorrect objectives. A simple example in section 2 clarifies that the expectations and objectives are correct.

variable does not affect the market price conditional on its success. In contrast, with, e.g., output competition a firm understands that contracting output raises the market price; this inefficiency does not exist here. However, the inability to affect the environment conditional on its success introduces other effects. As with the single-winner models or the integer effect in an entry model, the firm may not capture the entire social benefit of its success. The effect here differs from the integer effect on entry in that, instead of the marginal firm not entering and thereby not raising welfare, all the firms under invest. Finally, the uncertainty plays a direct role, as the profit or welfare gain from a single firm succeeding can be concave or convex in the number of successful firms. Thus, introducing a random number of firms that succeed can change the relative investment incentives.

Though there are several different effects present, whether excessive/insufficient invest occurs primarily rests on the equilibrium probabilities of success and the number of investors. To summarize, investment subsidies (taxes) are more likely to raise expected welfare when the equilibrium probabilities of success are low (high) and there are few (many) firms investing. Finally, steeper marginal costs of production encourage under investment. This implies that models that assume only linear marginal costs of production may incorrectly suggest that over-investment is very likely.

Previous Literature

Despite the fact that markets are often characterized by many innovators, there has been little analysis of this. Typically, models allow for only one successful firm that has monopoly power, and assumes that unsuccessful firms do not produce. One exception is Delbono and Denicolo (1991). Though only <u>one</u> firm succeeds, unsuccessful firms may produce. Sah and Stiglitz (1987) do allow a second firm to succeed, but assume Bertrand competition. This is analytically convenient as profits then drop to zero. However, the only relevant environment to the firm is when it succeeds and all other firms fail, as multiple innovators earning positive profits is not possible. In Davidson and Segerstrom (1998) also allows for a second firm to succeed, but that results from an R&D race that starts after the first innovation.

As far as is known, analysis of an uncertain number of multiple firms succeeding without resulting in zero profits only appears in Reynolds and Isaac (1992), Ericson and Pakes (1995) and Quirmbach

(1994). Reynolds and Isaac use Sah and Stiglitz's investment model to show that with Cournot competition Sah and Stiglitz's result may not hold. Though they do present examples examining the welfare effects of investment, they use a specific investment model and function, necessary for the comparison to Sah and Stiglitz. Ericson and Pakes present an elegant dynamic model of multiple firms investing in risky cost reduction, proving existence and the ergodicity of the equilibrium process. However, because of its complexity, all characterizations of the investment choice must come from simulations. Both, as well as Sah and Stiglitz, only use the first best welfare criterion.

In addition to delineating the investment effects instead of only providing examples, the paper presented here uses the second best welfare criterion: the government can affect investment, but not product competition. This approach has both pragmatic and theoretical advantages. Pragmatically, price controls on competition are not practical or possible in most countries, while industrial policy through investment subsidies is common. Theoretically, the first best is not very informative about the investment decision. First, efficient pricing ex post changes (increases) the social returns from investment as compared to those with Cournot competition ex post. As a result, the first best criterion may have more investment than the market outcome while the second best criterion may have less. Second, with the first best criterion there is no welfare gain from a second firm succeeding and so the optimal strategy only depends on the first firm succeeding. Finally, comparing the firms' investment levels to the expected welfare optimum with efficient pricing ex post does not provide insight into firm behavior, as firms are irrelevant in the first best criterion. The first best does not describe how the firm's marginal investment choice directly affects expected welfare.

Quirmbach does examine the second best with Cournot competition, but instead of firms choosing investment levels, they choose whether to enter with a fixed investment. Though general conditions are derived, they only hold when approximately one firm enters in equilibrium. The paper presented here, in addition to using a general investment function, differs from Reynolds and Isaac, Ericson and Pakes, and Quirmbach in that it analyzes the role of quadratic costs and both drastic and non-drastic cost reductions, and finds that both affect the results. The previous work also does not delineate the effects that affect investment, in particular overlooking the role that ex post strategic competition has on the welfare gain from success.

2. A very simple example of investing under uncertainty

Unguided intuition might suggest that the possibility of many firms succeeding does not change the incentives or the environment as compared to the standard patent race model. The following simple example shows the flaw in such intuition. Consider a two-period model. At the beginning of period one, an idea for a new product arises. There are two firms that have access to the same R&D technology. In period one, each firm independently and simultaneously invests an amount i to develop the product and succeeds with probability ϕ(i). Consider first the single winner model, only one firm wins the prize π, that yields a symmetric equilibrium in which wins with probability ½ If instead the firm splits π with probability 1, the expected gains are the same. However, this is not the same as when any, all or none may win the prize π. Consider the above equilibrium in which each firm has the same ½ probability of winning. *Note that the expected number of firms succeeding is still 1* [(½)(½)·2+2·(½)(½)·1+(½)(½)·0=1]. However, the uncertainty about the number of firms has changed the environment and incentives as expected gains for firm j are (½)·[(½)π/2 +(½)π]= 3π/8< π/2. This holds for any n. (Also, expected gains are less than π/2 for any symmetric probability less than one, though the expected number of firms then changes). So, firm j's expected gains from success have changed because, though its unconditional expectation is for only one firm succeeding, its expectation, <u>conditional on its success</u>, is for 1.5 firms succeeding. The firm does not care about its expectation conditional on its failure because then it earns zero. Finally, note that if the firm changes its investment level, this does not change its expectation that <u>conditional on its success</u> there are 1.5 firms

The results here not only differ in this manner, but also because the successful firms compete, which is examined next. Further, section 4 shows that, as a result of the successful firms competing, *excessive investment* occurs when the number of innovators and their equilibrium probability of success is high.

3. A simple model of investing under uncertainty

Consider now the effects of an uncertain number of firms in a more natural setting: demand is downward sloping. That is, consumer welfare is explicitly modeled. The model is as before, except that failure explicitly means that demand for the product is zero. The technology is characterized as $\phi(i) \in [0,1)$, $\phi(0)=0$,

$\phi' > 0$, $\phi'' < 0$, $\phi'(0) = \infty$. At the beginning of period two all firms and consumers observe which firms succeed. There is only one period of sales. Each consumer has unit demand and value a for the successful good. Consumers' values (a) are distributed uniformly on $[0, A]$, with total mass M. It is convenient to let $A/M \equiv b$. Given a single price P, there are $M - b^{-1}P = Q^d$ units demanded.

A firm can produce only one unit and has zero cost of producing the one unit. Thus, if there are m firms whose investment yields a successful product, then the market clearing price is $P = A - bm$. These assumptions are used to insure ex post efficiency if the firms succeed with probability one. In contrast, if non-Bertrand strategic competition exists in period two, then there would be inefficiency with certainty.

In period two, the successful firm earns $A - bm = p$. Firm j expects there to be, given the rivals' investments, $\sum_{k \neq j}^{n} \phi(i_k)$ rivals. Let the discount rate equal one. Expected profits, then, for a firm j investing i dollars, given its rivals' investment levels, are

$$\phi(i_j) \cdot [A - b - \textstyle\sum_{k \neq j}^{n} \phi(i_k)\, b] - i. \tag{1}$$

The firm chooses the investment level i that maximizes expression (1). For simplicity there are no positive or negative spillovers from investment. The first order condition is $\phi(i_j)' \cdot [A - b - \sum_{k \neq j}^{n} \phi(i_k)\, b] - 1 = 0$. Since the firms are identical ex ante and the number of firms that succeed is distributed binomially, in the symmetric equilibrium each firm invests the i that satisfies

$$\phi(i)' \cdot [A - b - (n-1) b \phi(i)] - 1 = 0. \tag{2}$$

For ease it is assumed that $A - bn \geq 0$.

To determine how the firm's choice compares to the welfare optimum, consider a government that maximizes the sum of expected aggregate profits and expected consumer surplus. The government is not viewed as having any special talent in evaluating investment projects. It, like consumers and firms, does not know ex ante which or how many firms will succeed. Since the price is efficient conditional on the number of successful firms, the government could only raise expected welfare by affecting the investment level.

If m firms succeed then welfare, W, equals $m(a - bm) + 0.5bm^2$. Thus, in a symmetric equilibrium with all n firms investing the same i, expected welfare equals

$$E[W] = \sum_{m=0}^{n} \binom{n}{m} \phi(i)^{m}(1-\phi(i))^{n-m}\{m(A - bm) + 0.5bm^2\} - ni. \tag{3}$$

By the properties of the binomial distribution equation (3) equals

$$E[W] = A\phi(i)n - 0.5b[n\phi(i) + n\phi(i)^2(n-1)] - ni. \tag{4}$$

The government chooses the i that maximizes (3), i.e., satisfies the first order condition

$$n\phi(i)'[A - b\phi(i)(n-1) - 0.5b] - n = 0. \tag{5}$$

<u>Proposition 1</u>: With unit capacity, firms invest less than the information-constrained welfare optimum.

Though the following proof is overly simple, its outline will be used for later results.

Proof: Divide equation (5) by n to obtain

$$\phi(i)'[A - b\phi(i)(n-1) - 0.5b] - 1 = 0. \tag{5'}$$

Comparing the LHS of (5') to the LHS of (2), for all i, the difference is the bracketed expression. However, for all i, the bracketed expression of (5') is greater than the bracketed expression (2). Thus, at the i defined by (5') the LHS of (2) is strictly negative. The firm would choose a lower level of i.//

The inefficiency arises because the firm by increasing its investment does not affect its environment, including the expected price, conditional on it succeeding. Given that the environment is fixed, when the marginal firm succeeds, it cannot capture the entire benefit. Intuitively, this is similar to the integer problem: the marginal entrant does not enter the market because its profits are negative, even though social welfare is positive. This uncaptured consumer surplus can be seen in the difference of the two first order conditions. One difference between the integer problem and the effect here is that here all firms under invest instead of the marginal firm not entering. Another difference is that if instead all firms invest to be the sole winner (a patent race), then over investment can occur because of the duplication of resources, as an increased probability of success by one firm partly comes from a decreased probability by other firms (e.g., Lee and Wilde 1980). A final difference arises from an uncertain number of firms succeeding. If instead, as in a patent race model, the firms expect a fixed number to succeed (i.e. firms are naïve and do not see their investment affecting the number that succeeds), then the firm's investment level is the *same* as the social optimum.

Superficially, the investment model here and its inefficiency resemble typical models of strategic competition. However, because with uncertainty the firm's investment does not affect the price conditional on the firm succeeding, the inefficiencies differ. In contrast with, e.g., Cournot competition, a firm's output affects the market price. As another way to see the difference, assume that there is no uncertainty about the number of firms that will succeed and that if a firm increases its investment it increases the number of firms that succeed. In this case if a firm increases its investment, then it would lower the expected price. As a result, the integer-like effect in proposition 1 disappears. However, with this alternative model, the firm under-invests relative to the welfare optimum because it sees its investment affecting the relevant price. This is the cause of inefficiency in most strategic models.

4. R&D race with Cournot competition among the winners

The previous section's simple modeling of product competition provides insights into how having an uncertain number of firms succeeding affects the market. However, its non-standard ex post market structure makes it difficult to draw comparisons to known results. So the model is now modified so that the successful firms compete in quantities – Cournot competition, a standard approach in economic modeling. Cournot competition, however, creates difficulties as inefficiencies arise and tractable expected profit and welfare expressions do not exist for all n.

The ex ante competition is as before. In period one, n firms independently and simultaneously invest to develop a new product. All firms face the same R&D process. A firm that invests i dollars in period one faces a probability $\phi(i) \in [0,1)$, $\phi(0)=0$, $\phi'>0$, $\phi''<0$, $\phi'(0)=\infty$, that it can develop a product consumers want. Otherwise, the firm fails and cannot produce. Consumers' preferences are as before. All successful firms are identical.

In period two all firms and consumers observe which firms succeed. Each firm has costs $c \cdot q^2$, where $c \geq 0$. Firms independently and simultaneously set outputs. Aggregate output is Q $= \sum_{j=1}^{n} q_j$, so that the resulting price is $P = A - bQ$. Profits for the successful firm j, given m firms succeeding are

$$\pi_j = [A - b \cdot q_j \; - b\sum_{k \neq j}^{m} q_k \; - c \cdot q_j] \cdot q_j \tag{6}$$

Firm j chooses output q_j to maximize the above profits. Using the q_j defined by the first order condition and then imposing symmetry, the representative firm's output in a symmetric equilibrium is $q_j(m) = A/[b \cdot (m+1) + 2c]$ and profits are $\pi_j(m) = A^2 \cdot (b+c)/[b \cdot (m+1)+2c]^2$.

In period 1 expected profit for firm j given n firms and investment i_j is

$$E[\pi_j(n)] = \phi(i_j)\Pi_{k \neq j}(1-\phi(i_k))\cdot\pi_j(1) \; + \phi(i_j)\cdot\pi_j(2)\cdot\{\Sigma_{k\neq j}[\phi(i_k)\Pi_{l \neq j,k}(1-\phi(i_l))]\} + \ldots \phi(i_j)\Pi_{k \neq j}\phi(i_k)\cdot\pi_j(n) - i_j$$

Firms simultaneously and independently choose their investment levels. Firm j chooses i_j defined by the first order condition:

$$\phi'(i_j)\{\Pi_{k \neq j}(1-\phi(i_k))\cdot\pi_j(1) \; + \pi_j(2)\cdot\{\Sigma_{k\neq j}[\phi(i_k)\Pi_{l \neq j,k}(1-\phi(i_l))]\} + \ldots \Pi_{k \neq j}\phi(i_k)\cdot\pi_j(n)\} - 1 = 0$$

Since ex ante firms are identical, the symmetric equilibrium in investment is considered. Imposing symmetry, the optimal level of investment i_j for firm j is defined by

$$\phi'(i)\Big\{[1-\phi(i)]^{n-1}\pi(1) + (n-1)\phi(i)[1-\phi(i)]^{n-2}\pi(2) + \ldots + (n-1)\phi(i)^{n-2}[1-\phi(i)]\pi(n-1) + \phi(i)^{n-1}\pi(n)\Big\} - 1 = 0 \tag{7}$$

That is, the firm invests until the marginal expected profit from a dollar invested equals the marginal cost. Unfortunately as the value for π varies with m (the number of successful firms) in the denominator, there is not a simplified expression for the left-hand-side of (7), unlike in (2).

Consider a government that maximizes expected social welfare. The government cannot determine how the firms behave in the ex post market, but it can determine the investment level. That is, a second best investment level is derived, the optimal investment when ex post competition cannot be affected.

Using the equilibrium quantities given m successful firms, welfare is the sum of aggregate profits $mA^2(b+c)/[b(m+1) \; +2c]^2$ and consumer surplus $b(mA)^2/2[b(m+1) \; +2c]^2$, W(m) = $A^2m[bm \; +2b \; +2c]/$

$2[b(m+1)+2c]^2$. The government chooses i to maximize the expected welfare from the n firms investing:

$$W(i;n) = (1-\phi(i))^n W(0) + n\phi(i)(1-\phi(i))^{n-1}W(1) + \binom{n}{2}\phi(i)^2(1-\phi(i))^{n-2}W(2)+\ldots+\phi(i)^n W(n) - n\cdot i.$$

Note that W(0)=0. To maximize expected welfare the government chooses the i that satisfies the first order condition $\partial W(i, n)/\partial i \equiv 0$ (which has been divided through by n):

$$\phi'(i)\Big\{(1/n)\cdot nW(1)[(1-\phi(i))^{n-1} -(n-1)\phi(i)(1-\phi(i))^{n-2}] + (1/n)\binom{n}{2}W(2)[2\phi(i)(1-\phi(i))^{n-2} -(n-2)\phi(i)^2(1-\phi(i))^{n-3}] +\ldots+ ((n-1)/n)\phi(i)^{n-1}W(n)\Big\} - 1 = 0. \tag{8}$$

Simple examination of (8) and (7) reveals that at ϕ=0, the integer effect for a monopoly causes under investment. Thus, for low enough equilibrium probabilities, under investment always occurs.

Lemma 1: At ϕ=0, the social benefit of the marginal investment is greater than the private benefit.

To determine how the socially optimal i compares to the market equilibrium i, note that for any i, the LHS of (8) differs from the LHS of (7) by the bracketed terms. Thus, following the proof of proposition 1, if for every i the bracketed term of (8) is greater than the bracketed term of (7), then the equilibrium level of i is less than the socially optimal level of i. Denote this difference, the bracketed term of (8) less the bracketed term of (7), by Δ.

From the equilibrium values given m, A^2 can be factored out of the difference Δ, so it depends on c, b, i and ϕ, $\Delta(c,b,i,\phi)$. For example, with five firms investing, the bracketed term of (7) is (with A^2 factored):[4]

$$\left[\frac{(1-\phi)^4}{(2b+2c)^2}+\frac{4\phi(1-\phi)^3}{(3b+2c)^2}+\frac{6\phi^2(1-\phi)^2}{(4b+2c)^2}+\frac{4\phi^3(1-\phi)}{(5b+2c)^2}+\frac{\phi^4}{(6b+2c)^2}\right](b+c) \tag{9}$$

and the bracketed term of (8) is (with A^2 factored):

$$(1-\phi)^3(1-5\phi)\frac{1.5b+c}{(2b+2c)^2}+(1-\phi)^2(2\phi-5\phi^2)\frac{8b+4c}{(3b+2c)^2}+(3\phi^2-5\phi^3)(1-\phi)\frac{15b+6c}{(4b+2c)^2}$$

[4]For two firms the expressions are much simpler: $(1-\phi)\frac{b+c}{(2b+2c)^2}+\phi\frac{(b+c)}{(3b+2c)^2}$ and $(1-2\phi)\frac{1.5b+c}{(2b+2c)^2}+2\phi\frac{(2b+c)}{(3b+2c)^2}$.

$$+\phi^3(5-6\phi)\frac{12b+4c}{(5b+2c)^2}+\phi^4\frac{17.5b+5c}{(6b+2c)^2}$$

If the difference $\Delta(c,b,i,\phi)$ is positive for all i, expected social welfare can be increased by increasing investment. Since ϕ is a continuous function and i does not appear directly in $\Delta(c,b,i,\phi)$, the analysis of the difference $\Delta(c,b,i,\phi)$ examines the effect of ϕ, instead of i, as ϕ is more convenient. First, the results can be characterized in terms of the probability of success at the welfare optimum, which is easier to interpret than is a level of i. The use of ϕ is also more general as a specific functional form does not have to be assumed on ϕ. Equivalently, the problem could be written as choosing probabilities with the cost being $\mathbf{C}(\phi)$, $\mathbf{C}'>0$, $\mathbf{C}''>0$ $\lim \mathbf{C}_{\phi\to 1}=\infty$.

If the difference Δ is decreasing in ϕ, then from lemma 1 (the difference is positive at $\phi=0$) there is an upper limit of the probability of success ϕ such that for any lower ϕ the difference Δ is positive. This can be used to characterize when under investment occurs. For example, if the upper limit is .80, then when the optimal level of investment yields a probability between 0 and .80, under investment occurs. This follows by the logic of the proof of proposition 1: if the difference Δ is positive at the expected welfare optimum, then the bracketed term of (8) is greater than the bracketed term of (7). Since the other terms are identical, the LHS of (7) must be negative. The firm would choose a lower level of investment, as the marginal benefit to the firm is less than that of the welfare planner (the difference Δ is positive) for any lower level of i. If the upper limit is 1, then under investment always occurs.

Unfortunately, an analytical expression for the difference Δ does not exist for general n since the expectation of both W and π depend inversely on the realization of m. However, conditions can be derived for a given n although calculations quickly become cumbersome, as the difference Δ is a polynomial in ϕ of the n^{th}-1 degree. The results for 2-5 firms had the same general characteristics, suggesting that solving for a greater n would not provide additional insights.

Proposition 2: With 5 (4,3,2) firms, if $c\geq 3.66b$ ($2.66b$, $1.66b$, $0.65b$), then under investment always occurs.

Proof: The values ($3.66b$, $2.66b$, $1.66b$, $0.65b$) are the (rounded up) unique value for the difference Δ to be zero at $\phi(i)$=1for n={5,4,3,2}. If the difference Δ is decreasing in ϕ over [0,1] when c is greater than these values, then the difference is always positive. The derivative at ϕ=1 is negative when c is greater than these values. (Calculations for the case of five firms is included in the appendix as it is overly long) Thus, if the derivative is increasing over [0,1], then the derivative must always be negative, i.e., the difference is decreasing in ϕ. The derivative can be shown to be increasing, i.e., the second derivative is positive, over [0,1] for n={4,3,2} with simple calculation by letting $x \equiv c/b$, and substituting out for c. For n=5, the second derivative can be shown to be positive over [0,1] when x=1 (c=b). As the coefficients on x are all positive between [0,1] then the second derivative is positive for any greater x.//

Proposition 2 reveals that as the slope of marginal cost (demand) becomes steeper (flatter) under investment is more likely to occur. Also, as n increases under investment becomes less likely. Since for ϕ=0 under investment always occurs, proposition 2 also implies that under investment is less likely the higher the equilibrium probability of success.

Though the proof does not show the difference Δ to be decreasing in ϕ for all values, simulations suggest that the difference is always decreasing in ϕ and the effects of c, b and n on welfare are always the same. If the difference is always decreasing in ϕ, then by lemma 1 there is always an upper bound on ϕ such that under investment occurs. Thus, even when the conditions for proposition 2 are not met, under investment can occur in equilibrium. For example, with a pentapoly only if $c \geq 3.662b$ does under investment always occurs. However, if b=1 and c=3 (2), and if the probability of success at the expected welfare optimum is less then .88 (.68), then under investment occurs in equilibrium.

To provide insight for the relationships between the variable, the following table presents simulations for when b=1 and n=5. Costs (c) change along the horizontal axis, probabilities (in tens of percent) change along the vertical axis, b=1 and n=5. The difference Δ is normalized by the bracketed term of (7), which always is positive since it is the profit from being successful. Larger (smaller) slope b would decrease (increase) the range such that the difference Δ is positive.

[Insert table 1 here]

Effects on firm and welfare optimal investment choice

In the simple model all firms under invest because investment cannot influence the environment conditional on it succeeding and in that environment the welfare gain is greater than the firm's gain. Having instead strategic competition ex post does not change that investment cannot influence that environment. A firm knows that increasing its investment changes neither the expected number of firms nor the price conditional on it succeeding. However, greater investment by the firm does increase the number of firms the welfare maximizer expects, as it did in the simple model. The difference is that with strategic competition and a sufficiently large number of successful firms, the private gain to a successful firm exceeds the welfare gain. This is because the additional successful firm's output comes primarily from the other firms reducing their output (*business stealing* effect) and aggregate output increases by a relatively small amount (see von Weizsäcker, Mankiw and Whinston, Suzumura and Kiyono). One difference from these cited models is that here, instead of the business stealing effect causing excess entry, it causes <u>all</u> of the firms to over invest. The effects that the slopes of demand and costs have on the investment decision also reflect this effect. Steeper (flatter) marginal costs (demand) reduce the business stealing effect and so the integer effect dominates.

Two different effects, the business stealing and the integer effect, clearly affect the welfare optimum. However, the uncertainty regarding how many rivals succeed also may have a direct effect. This can be suggested indirectly by considering a firm that faces uncertainty in investing, but the number of other firms is certain. Simple calculation shows that profits with Cournot competition are convex in m, the number of firms that succeed. Thus, a small risk in m, a small uncertainty as to how many other firms succeed, raises the expected profit from entering. Simulations show that the change in consumer surplus from the firm entering is convex unless c is sufficiently large and m is sufficiently small. (In contrast, consumer surplus is concave in m if b is sufficiently larger than c and if m is restricted above 2. This is surprising since consumer surplus is convex in output.) The change in aggregate profit from the firm entering can be concave if m is sufficiently large or c sufficiently small. The welfare gain from a firm entering is convex in m (while welfare is concave).

Interpreting these curvatures is difficult, however. While introducing uncertainty clearly affects expected welfare, changes in ϕ from increased investment have unclear effects. Specifically, an identical change in ϕ for all firms cannot be characterized as a mean-preserving spread. However, increasing ϕ from zero to one-half does increase variance and increasing ϕ from one-half to one decreases variance.

5. Ex post oligopoly with non-drastic cost reduction

The firms have been characterized as developing a new product for consumers. However, with a slight modification they could have also been characterized as developing a cost reduction sufficiently large to drive out any firm that does not succeed. Such a cost reduction is known as a drastic cost reduction. Thus, it is natural to consider the other possibility: non-drastic cost reductions. With a non-drastic cost reduction, a successful firm lowers its costs, but an unsuccessful firm still produces.

Allowing unsuccessful firms to still produce is consistent with casual observation as non-innovating, old technology firms often compete with successful innovators. Considering non-drastic cost reduction also tests the generality of the previous results since in certainty models of R&D the outcomes often depend on whether cost reductions are drastic.[5] This is intuitive since allowing unsuccessful firms to compete changes the returns from innovating and not innovating.

Despite the introduction of a more complex ex post environment, the analysis actually becomes simpler if costs are assumed to be linear. The simplification results because with the non-drastic cost reduction, the number of firms producing remains constant. However, quadratic costs are also considered since it has theoretical implications, has affected previous results and is more general.

In the first period n firms can invest to try to reduce their costs. The probability function ϕ is as before. In second period m of these firms succeed and have a lower cost C_L. Those that fail have the same cost as before C_H, $C_H > C_L$. The cost reduction is non-drastic; all firms produce in equilibrium.

Except for the noted changes, the model remains the same: in period one, each firm independently

[5]For example, see the discussion in Tirole, pp. 391-392.

and simultaneously invests an amount to develop a cost reduction. All successes and failures are observed. In period 2, all firms that succeed are identical (have cost C_L) and all that fail are identical (have cost C_H). In period 2 firms independently and simultaneously set outputs. The outputs of those that succeed and have low cost are denoted with an L (H) superscript: q^L and q^H. Aggregate output is Q = $\sum_{j=0}^{n} q_j$, so that the resulting price is $P = A - bQ$.

Linear costs

Profits for a successful firm j, given m firms succeeding and n-m unsuccessful firms are

$$\pi_j^L = [A - bq_j^L - b(m-1)q^L - b(n-m)q^H - C_L]\cdot q_j^L \tag{10}$$

A successful firm j chooses output q_j^L to maximize the above profits. The successful firm realizes that the m – 1 other successful firms are maximizing the same expression and that n - m unsuccessful firms are each maximizing

$$\pi_k^H = [A - bq_k^H - b(m)q^L - b(n-m-1)q^H - C_H]\cdot q_k^H \tag{11}$$

Using this information and the first order conditions, and then imposing symmetry yields the following equilibrium outputs

$$q^L(m)= [A + (n-m)C_H - (n-m+1)C_L]/b(n+1) \tag{12}$$
$$q^H(m)= [A - (m+1)C_H+mC_L]/b(n+1).$$

where $q^L(m)$ holds for $m\in\{1,\ldots,n\}$ and $q^H(m)$ holds for $m\in\{0,\ldots,n\text{-}1\}$. From this, profits for a low (high) cost firm given m successful (low cost) firms are

$$\pi^L(m) = [A + (n-m)C_H - (n-m+1)C_L]^2/b(n+1)^2. \tag{13}$$
$$\pi^H(m) = [A - (m+1)C_H+mC_L]^2/b(n+1)^2$$

for the same values of m as in (12).

In period 1, firms simultaneously and independently choose their investment levels. Expected profits depend on how many firms succeed, but now if the firm fails it still earns profits. Expected profits for firm *j* given *n* firms investing are

$$E[\pi_i(n)] = \phi(i_j)\Big\{\Pi_{k\neq j}(1-\phi(i_k))\pi_j^{L}(1) \;+\; \pi_j^{L}(2)\{\Sigma_{k\neq j}[\phi(i_k)\Pi_{l\neq j,k}(1-\phi(i_l))]\} + \ldots\; \Pi_{k\neq j}\phi(i_k)\pi_j^{L}(n)\Big\} \;+$$

$$(1-\phi(i_j))\Big\{\Pi_{k\neq j}(1-\phi(i_k))\pi_j^{H}(0) + \pi_j^{H}(1)\{\Sigma_{k\neq j}[\phi(i_k)\Pi_{l\neq j,k}(1-\phi(i_l))]\} + \ldots\; \Pi_{k\neq j}\phi(i_k)\pi_j^{H}(n-1)\Big\} \;-\; i_j \qquad (14)$$

Firm *j* chooses the investment level i_j to maximize profits (14). The first order condition from (14) defines this i_j. Symmetry is then imposed ($i_j = i_k$) and then substituting (13) into (14) and using the properties of the binomial distribution results in

$$\phi'\Big\{n\times(C_H-C_L)\Big\{2A \;+\; C_H(n-2)(1-\phi) - C_L(n - 2\phi(n-1))\Big\}/\, b(n+1)^2\Big\} - 1 = 0 \qquad (15)$$

Equation (15) is relatively simple because in the expectation *m* only enters in the numerator.

As before, the government maximizes expected social welfare, and cannot determine how the firms behave in the ex post market, but can influence the investment decision. Given that the firms compete in Cournot competition, welfare given *m* firms succeeding is

$$W(m) = m[A + (n\text{-}m)C_H - (n\text{-}m\text{+}1)C_L]^2/b(n\text{+}1)^2 + (n\text{-}m)[A - (m\text{+}1)C_H + mC_L]^2/b(n\text{+}1)^2 \;+$$
$$(nA\text{-}\,(n\text{-}m)C_H - mC_L)^2/2b(n\text{+}1)^2,$$

that is aggregate profits of those that succeed and fail, plus consumer surplus. With symmetric levels of investment, expected consumer surplus is,

$$\{1/2b(n+1)^2\}\Big\{\binom{n}{0}\phi^0(1-\phi)^n n^2(A-C_H)^2 + \binom{n}{1}\phi^1(1-\phi)^{n-1}(nA - (n-1)C_H + 1C_L)^2 + \ldots +$$
$$\binom{n}{n}\phi^n(1-\phi)^0[nA - 0\cdot C_H + nC_L]^2\Big\} \qquad (16)$$

By the properties of the binomial distribution (16) can be written as an overly long, yet analytical expression that can be found in the appendix as equation (16').

The government chooses *i* to maximize the expected welfare from the *n* firms investing:

$$W(i,\, n) = (1-\phi(i))^n W(0) + n\phi(i)(1-\phi(i))^{n-1}W(1) + \binom{n}{2}\phi(i)^2(1-\phi(i))^{n-2}W(2) + \ldots + \phi(i)^n W(n) - n\times i. \qquad (17)$$

Substituting in the profit expressions and consumer surplus into (17) and by the properties of the binomial distribution, (17) can be written as an expression found in the appendix as (16') + (17'), where (17') is aggregate profits. The first order condition for the government is (dividing through by n)

$$\phi'\left\{(C_H\text{-}C_L)\left[A(2n+4)+C_H(2n^2+6\phi\text{-}5\text{-}4n^2\phi\text{-}2n\phi)-C_L(2n^2+6\phi\text{-}1\text{-}4n^2\phi\text{-}2n\phi)\right]/2b(n+1)^2\right\}-1=0 \qquad (18)$$

As in the previous sections, note that for given i, the LHS of (18) and (15) only differ by the bracketed term. Comparing the two and using the proof from proposition 1 gives a sufficient condition for under investment.

Proposition 3: Under investment occurs in equilibrium if, for all probabilities,

$$(C_H\text{-}C_L)\left[A(4\text{-}2n)+C_H(6\phi\text{-}5+4n\text{-}6n\phi)-C_L(2n+6\phi\text{-}1\text{-}6n\phi)\right]/2b(n+1)^2>0. \qquad (19)$$

Proposition 3 can be used to derive two results: a necessary condition for (19) to be positive for all ϕ and the maximum number of firms there can be in the market and still have under investment possible. These two results indicate that for cost reduction under investment is highly unlikely with linear cost.

Corollary 1: In a market with a fixed number (greater than one) of firms that compete in quantities, firms always under-invest relative to the information-constrained welfare optimum level only if $n<2$.

Proof: Expression (19) is decreasing in ϕ if $(1\text{-}n)\cdot(C_H\text{-}C_L)<0$. Thus, restrict attention to when ϕ=1 since the LHS of (19) must be positive when ϕ=1 for n>1, if the LHS of (19) is to be positive for all ϕ and under investment is to occur. Note that there is a unique n that equates the LHS of (19) to zero, n_z:

$$n_z=\frac{-(C_L(1-6\phi)+C_H(6\phi-5)+4A)}{C_L(6\phi-2)+C_H(4-6\phi)-2A}$$

If the LHS of (19) is decreasing in n at n_z, then the expression is positive only for n less than or equal to n_z. At ϕ=1, the LHS of (19) is decreasing in n at n_z. To find the maximum n note that n_z is decreasing in C_H at ϕ=1. Taking the limit of n_z as C_H approaches C_L yields 2.//

Note that this is a necessary, but not sufficient condition for under investment always to occur. For probabilities less than one, the LHS of (19) can be positive for n>2. However, only for probabilities close to zero and very high C_H, (the relationship reverses for low ϕ) can the LHS be positive for n> 3.

<u>Corollary 2:</u> For under investment to be possible, the most firms there can be is 3.225.

<u>Proof:</u> From the proof of corollary 2, for any n (19) reaches its maximum when ϕ=0. Setting ϕ=0, if expression (19) is decreasing with n at n_z, then n_z is the maximal n. Taking the derivative of (19) and evaluating it at n_z obtains $-3.5C_L + 4.5C_H - 3A$, which is obviously increasing in C_H. Thus, if this derivative is negative at the maximum value of C_H, then it is always negative and n_z is the maximum value of n with under investment. Substituting in the condition for high cost output to be positive ($A - nC_H + (n-1)C_L > 0$) at n_z, the derivative of (19) is negative at the maximum value of C_H. Finally, to find the maximum value of n_z note that it is <u>increasing</u> in C_H if ϕ=0. Thus, substituting this high cost output constraint into n_z yields n_z=3.225.//

Under investment is very unlikely with non-drastic innovation if costs are linear. Since under investment is less likely with <u>drastic</u> innovation if costs are linear, the result is not entirely surprising. Yet this stronger result with non-drastic innovation is consistent with the results in a certainty model by Creane (1999) which suggests that the excess entry effect is stronger with non-drastic innovation than with drastic innovation. Further, Tandon (1984) and Okuno-Fujiwara and Suzumura (1993) find excess entry and over investment with certain cost reduction. Interestingly, uncertainty in the number of successful firms does not introduce an effect here as the gain in consumer surplus and the gain in welfare are both linear.

Quadratic Cost

With drastic cost innovation, making costs slightly quadratic increased greatly the range under which under investment would occur. It is natural then to examine how the cost assumption affects the results for non-drastic cost reduction. To do this, costs are now assumed to be quadratic. Although this introduces more general and intuitive costs, the results are no longer as easily analyzed.

Profits for the successful firm j, given m firms succeeding and n-m unsuccessful firms are

$$\pi_j^L = [A - bq_j^L - b(m-1)q^L - b(n-m)q^H - C_L \gamma q_j^L] \cdot q_j^L \qquad (20)$$

Successful firm j chooses output q_j^L to maximize the above profits. Each n- m unsuccessful firm is maximizing

$$\pi_k^H = [A - bq_k^H - b(m)q^L - b(n\text{-}m\text{-}1)q^H - C_H q_k^H]\cdot q_k^H \tag{21}$$

Using this information and the first order conditions, and then imposing symmetry yields the following equilibrium outputs.

$$q^L(m) = [A\,(b+2C_H)]/[\mathrm{b}^2(n+1)+2C_Lb(n\text{-}m+1)+2C_Hb(m+1)+4C_LC_H] \tag{22}$$

$$q^H(m) = [A\,(b+2C_L)]/[\mathrm{b}^2(n+1)+2C_Lb(n\text{-}m+1)+2C_Hb(m+1)+4C_LC_H]$$

where $q^{\mathrm{L}}(m)$ holds for $m\in\{1,\ldots,n\}$ and $q^{\mathrm{H}}(m)$ holds for $m\in\{0,\ldots,n\text{-}1\}$. From this, profits for a low (high) cost firm given m successful (low cost) firms are, for the same values of m as in (22),

$$\pi^{\mathrm{L}}(m) = [A^2(b+2C_H)^2(b+C_L)]/[\mathrm{b}^2(n+1)+2C_Lb(n\text{-}m+1)+2C_Hb(m+1)+4C_LC_H]^2. \tag{23}$$

$$\pi^{\mathrm{H}}(m) = [A^2(b+2C_L)^2(b+C_H)]/[\mathrm{b}^2(n+1)+2C_Lb(n\text{-}m+1)+2C_Hb(m+1)+4C_LC_H]^2.$$

In period 1, expected profit for firm j given n firms is still represented by expression (14) but with the definitions of profits from (23). Firm j chooses the investment level i_j to maximize profits (14). The first order condition from (14) defines this i_{j}. Symmetry is then imposed ($i_j=i_k$) which yields an expression that does not have a simpler form:

$$\phi'(i)\left\{\sum_{m=0}^{n-1}\binom{n-1}{m}f^m(1-f)^{n-1-m}\frac{A^2(b+2C_H)^2(b+C_L)}{[b^2(n+1)+2C_Lb(\mathrm{n}\text{-}(m+1)+1)+2C_Hb((m+1)+1)+4C_LC_H]^2}\right. -$$

$$\left.\sum_{m=0}^{n-1}\binom{n-1}{m}f^m(1-f)^{n-1-m}\frac{A^2(b+C_H)(b+2C_L)^2}{[b^2(n+1)+2C_Lb(n\text{-}m+1)+2C_Hb(m+1)+4C_LC_H]^2}\right\} - 1=0 \tag{24}$$

Note that the first term runs from m=0 (all other firms fail) and thus the denominator has to be adjusted to (m+1) as it is conditional on the firm succeeding.

The government chooses the i that maximizes expected social welfare, given that the firms compete in quantities. Welfare, given m successful firms, is aggregate profits plus consumer surplus, or

$$\mathrm{W}(m) = m[A^2(b+2C_H)^2(b+C_L)]/[\mathrm{b}^2(n+1)+2C_Lb(n\text{-}m+1)+2C_Hb(m+1)+4C_LC_H]^2$$

$$+ (n\text{-}m)[A^2(b+2C_L)^2(b+C_H)]/[\mathrm{b}^2(n+1)+2C_Lb(n\text{-}m+1)+2C_Hb(m+1)+4C_LC_H]^2$$

$$+(b/2)\Big\{m[A\,(b+2C_H)]/[b^2(n+1)+2C_Lb(n\text{-}m+1)+2C_Hb(m+1)+4C_LC_H]$$
$$+\,(n\text{-}m)[A\,(b+2C_L)]/[b^2(n+1)+2C_Lb(n\text{-}m+1)+2C_Hb(m+1)+4C_LC_H]\Big\}^2$$

With symmetric levels of investment, expected consumer surplus is,

$$\sum_{m=0}^{n}\binom{n}{m}f^m(1-f)^{n-m}\left\{m\frac{A(b+2C_H)}{b^2(n+1)+2C_Lb(n\text{-}m+1)+2C_Hb(m+1)+4C_LC_H}\right.$$
$$\left.+(n-m)\frac{A(b+2C_L)}{[b^2(n+1)+2C_Lb(n\text{-}m+1)+2C_Hb(m+1)+4C_LC_H]^2}\right\}^2 \qquad (25)$$

Expected aggregate profits is n times expected profits for an individual firm. The government chooses i to maximize the expected welfare from the n firms investing:

$$\mathrm{W}(i,n)=(1\text{-}\phi(i))^n\mathrm{W}(0)+n\phi(i)(1\text{-}\phi(i))^{n\text{-}1}\mathrm{W}(1)+\binom{n}{2}\phi(i)^2(1\text{-}\phi(i))^{n\text{-}2}\mathrm{W}(2)+\ldots+\phi(i)^n\mathrm{W}(n)-n\cdot i. \qquad (17)$$

with the definitions (23,25) substituted into (17) and into the first order condition

$$\phi'(i)\Big\{\text{-}\mathrm{W}(0)(1\text{-}\phi(i))^{n\text{-}1}+\mathrm{W}(1)[(1\text{-}\phi(i))^{n\text{-}1}\text{-}(n\text{-}1)\phi(i)(1\text{-}\phi(i))^{n\text{-}2}]+\ldots+((n\text{-}1)/n)\phi(i)^{n\text{-}1}\mathrm{W}(n)\Big\}-1=0. \qquad (26)$$

As before if the expected welfare gain from additional investment (the bracketed term of (26)) is greater than the profit gain (the bracketed term of (24)) for all ϕ, then under investment always occurs. Let denote Δ the bracketed term of (26) less the bracketed term of (24).

With the inclusion of unsuccessful firms' costs and profits, the difference Δ could not be shown to be decreasing, not even for given n or with restrictions such as C_L=1, which is needed to ensure uniqueness. However, simulations indicate that the difference Δ decreases in ϕ. If it does, then an upper bound on ϕ exists, and over investment can be characterized by this ϕ. Thus, it is assumed that Δ decreases in ϕ.

Even assuming that Δ is decreasing, the analysis is still complicated by the additional variable C_H, which interacts with b, c, n, and C_L. To simplify, n is fixed since a change in n has the same effect with a non-drastic cost reduction as with drastic cost reduction. A firm chooses i that satisfies (24) for n=5. This overly long expression can be found in the appendix (24'). Similarly, the expected welfare optimal level of i

satisfies the derivative of expected consumer surplus (in the appendix (25')) and expected aggregate profits (in the appendix (26')) equated to zero. Dividing (25')+(26')=0 by 5, yields (26) for when n=5.

In the linear case under investment is not possible with more than three firms and with certainty society may want no firms to invest in certain non-drastic cost reduction. Thus, it might be expected that over investment always occurs with five firms, or at least sufficient conditions do not exist for under investment to occur for all probabilities. However, if the difference Δ decreases in ϕ, which simulations indicate to be true, then this expectation is incorrect. Thus, results for the linear case can be misleading.

<u>Lemma 2</u>: For n=5, if the difference Δ is decreasing in ϕ, C_H=1.25C_L (1.5, 2, 3, 4), and $C_L \geq 1.678b$ (1.835, 2.092, 2.445, 2.674), then under investment occurs in equilibrium.

Thus, with quadratic costs of production over investment is <u>more</u> likely with non-drastic innovations than with drastic innovations, in striking contrast to the results for linear costs of production.

In the following table simulations are for a case (b=1 and C_L=1) when both under and over investment could occur in equilibrium. Though C_L=1, which is less than the C_L needed for lemma 2, under investment may occur in equilibrium. C_H changes across the horizontal axis and ϕ (in ten percents) changes down the vertical axis.

[Insert table 2]

From table 2 under investment can occur if the optimal amount of investment results in probabilities less than ½, which is surprisingly consistent with table 1 for C_L=1. More generally the results for non-drastic innovation with quadratic cost agree with those for drastic innovation with quadratic cost. For example, in both under investment is more likely as the probability decreases, and a decrease in the slope increases the range of probabilities under which under investment occurs. (If the slope becomes 10, then only for very low probabilities at the welfare optimum and high C_H does under investment occur.) Thus, the basic effects on investment from there being an unknown number of firms succeeding are consistent for both drastic and non-drastic innovations if marginal costs of production are quadratic. If marginal costs are linear, then under investment can occur with drastic innovation but is unlikely with non-drastic innovations.

The effects for a single firm uncertain about whether it will succeed are more difficult to ascertain. Profits, not surprisingly, are generally convex in the number of successful firms. The change in consumer surplus is convex in the number of successful firms, though consumer surplus is concave. However, a general characterization of the concavity of aggregate profits or welfare is not tractable and simulations show that it can be concave or convex.

6. Conclusion

This paper has examined the incentives and welfare effects from investing in risky R&D when the number of competitors that will succeed is also unknown. Some broad effects arise from this environment. First, what is relevant to the investing firm is the market when it succeeds. As a result, the firm does not see its investment affecting the expected number of firms or market price conditional on its success; the conditional expectation is not the same as the unconditional expectation. Second, there are always conditions for the firms to under invest. Third, production costs play an important role as the assumption of linear marginal costs make over investment very likely, while steeper marginal costs and drastic cost reductions make under investment likely. These results suggest that Ericson and Pakes' simulations showing that the market has excess investment/entry relative to the first best might be misleading. The results here indicate that with instead, quadratic production costs, or a drastic cost reduction, or a less efficient investment technology, there may be under investment/entry. Finally, under investment is likely when equilibrium probabilities of success are low and there are few firms investing. Expected welfare can be raised by investment subsidies (taxes) when the investments are most likely to fail (succeed) and there are few (many) investors.

There are several effects that cause investment inefficiencies. However, though the firms compete strategically in investment, the investment inefficiency here is not the typical inefficiency from strategic competition. Inefficiency in most models of strategic competition arises because the firm knows that by changing its choice it affects the market price and output. For example, a Cournot firm knows that by increasing output slightly or making a *certain* cost reduction, the price falls. The firms have some market power over price and as a result, the marginal value to consumers (the price) is greater than the marginal value to the firm (marginal revenue slopes downward as a result of the market power). In contrast, with the uncertainty models here, when a firm increases its investment it does not see a corresponding decrease in the price when it succeeds, though the welfare maximizer does see a decrease in the expected price.

The investing firm being unable to affect the environment conditional on it succeeding does introduce other effects. First, an integer like effect exists since the firm cannot influence its environment when it succeeds and so cannot capture the entire benefit of its success. This differs from the integer problem with entry in certainty models since with uncertainty all the firms under invest while with certainty the inefficiency is from one firm not entering the market. This also differs from uncertainty with a single firm succeeding (e.g., a patent race), as in the latter over investment can result from the duplication of resources as one firm succeeding may simply displace another. In the paper here, a firm succeeding simply increases the total number of successful firms. Second, though the firm's investment cannot influence its environment conditional on succeeding, it does increase the expected number of firms from the social perspective. For a sufficiently large number of firms the *nth* firm's gain from succeeding exceeds the social gain (von Weizsäcker, Mankiw and Whinston, and Suzumura and Kiyono). Thus, the marginal social gain from the increased expected number of firms can be less than the firm's marginal gain, which equals the marginal investment cost. Finally, the random number of firms succeeding affects the firm's and society's expected welfare as profits and welfare are almost always either strictly convex or strictly concave.

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