Uncertain product quality, optimal pricing and product development

Short Title: Risky quality and product development

Anthony Creane ${ }^{\dagger 】}$<br>Department of Economics<br>Michigan State University<br>East Lansing, MI 48824-1038<br>1-517-432-1068 (fax)<br>creane@msu.edu

A firm is developing a new product. However, the firm is uncertain as to how consumers will perceive the product's desirability or quality. Using a general model of product quality, conditions for an increase in uncertainty to increase the optimal price are derived. General conditions are derived under which the firm prefers the less risky project, the one with lower quality variability. However, if at the optimal price the firm only has positive demand for high quality realizations, then the firm prefers a more risky project. As the uncertainty exists in the consumers' preferences, welfare effects can be determined, unlike in previous work examining uncertainty.

Keywords: vertical differentiation, risk, uncertainty.
AMS (MOS) classification: 90 Economics, operations research, programming, games

[^0]
## 1. Introduction

In developing a new product, most firms face a trade-off between the expected desirability of a new product and the riskiness of its success. Consider consumers' willingness-to-pay for clothing, as examined by Lazear [8]. For the most part, conformity in new styling (low riskiness) seems to be the rule of thumb, to which many minute variations on blue jeans or T-shirts as well as companies such as Lands' End and LL Bean attest. Though other chains such as GAP, Benetton and The Limited promote a cutting-edge and audacious image, inspection of the products reveals the un-daring. This is surprising as one may expect differentiation in such industries to be important for profitability. Instead, bandwagon effects are observed in which a known, successful formula is imitated relentlessly. These firms seem to be exhibiting risk-averse behavior in their development choices of new products.

What is the source of this behavior? The GAP, Benetton and the Limited would want to sell the next stylish or hip clothes if they knew which would be the next thing. Similarly, with a comedy movie, the studios would want it to be funnier to audiences. Yet success in hipness or funniness is not a sure thing. These firms do not know which will be the next big seller. The question that a firm faces, then, is whether it would develop a product with greater expected quality in these dimensions, even if it has a greater risk. If the firm never would make this tradeoff, then it, or its manager, would be observed picking less risky projects with lower expected quality even if it was risk-neutral. Naturally, this raises the additional issue of whether society would prefer the firms to choose these less risky projects.

That uncertainty in quality exists is not only a problem for firms developing new products. For example, most superstars (Rosen [15]) face the uncertainty of their own quality. Which newcomers become superstars is unpredictable. Although every superstar was once predicted to become one by someone, so were many non-superstars. Further, how long current superstars last is also uncertain -- a superstar is only as good as their last movie, publication, etc. There are, as well, "has beens" who make comebacks although they were considered "dead." Thus, though in expectation potential stars or
products can be ranked, that does not imply a lack of uncertainty. The question then is whether the amount of uncertainty in their potential affects their asking price in contracts even if they are risk neutral.

Not all firms, however, exhibit what appears to be risk aversion in quality. With clothing the obvious exception is the micro-market of haute couture. To a disinterested observer, Parisian designers, in presenting seemingly bizarre clothing for large sums of money, appear to take great risks as they may fail in their endeavor. However, the market for haute couture differs from the standard clothing market as it typically involves a one-of-a-kind item (as in Lazear [8]) with very few buyers while The Limited, etc. makes many sales to many buyers. A natural suspicion, then, is that differences in market structures can cause differences in observed firm behavior towards risk.

Independent filmmakers also seem to face great variation in their payoffs. The hits are legendary (e.g., Spike Lee's She's gotta have it) as well as the flops (Waterworld). More generally, attitudes towards risk taking towards product characteristics seem to vary across industries. For example, styling and physical characteristics in the auto-industry seem alike. Similarly, network TV programming appears to value copying similar formats. Yet, smaller cable networks seem to carry more innovative programming taking a greater risk so as to find a small, profitable niche.

In examining a firm's choice among projects with different risks, it could always be assumed that the firm, or its manager, is risk averse and thus sacrifice expected quality for lower risk, or risk loving and so choose "crazy" projects. However, assuming risk aversion or risk loving assumes the conclusion that the firm prefers either the less or more risky project. Instead, here risk is introduced into the actual characteristic of the product, while the firms are risk neutral in profits.

Consumers' preferences toward quality, or desirability, have been extensively modeled by, among others, Mussa and Rosen [13], Jaskold-Gabszewicz and Thisse [3,4], Shaked and Sutton [20,21], and Milgrom and Roberts [11]) as well as by Lazear[8], Rosen [15], etc. These monopoly models have uniformly adopted the same basic structure (known as vertical differentiation) towards
characterizing quality. However, in these models quality is always a sure thing. In section 2 , uncertainty is introduced into these classic models of quality. I examine how uncertainty regarding a product's quality affects the firm's pricing and its development of new products, i.e., I study the firm's willingness to trade off higher expected quality for a riskier project. The firm's willingness to trade off higher expected quality for lower risk, is then compared to society's willingness to trade off one for the other.

Introducing uncertainty into these vertical differentiation models implies that under general conditions, a firm, risk neutral in profits, prefers products with less risk. That is, given the choice, the firm may select a product with lower expected quality in exchange for less risky quality. The effect an increase in uncertainty has on price, however, is not clear-cut. For example, I show that if demand is not too concave (e.g., linear) and marginal costs are positive, then a less risky product results in a higher price. On the other hand, if the price-cost mark-up is sufficiently high and demand is strictly concave, then a less risky product lowers the price. Independently of whether the price goes up or down, the firm still prefers a product with less risk.

Though a firm prefers a less risky product, expected consumer and social welfare, with efficient (marginal cost) pricing, both increase with a more risky product. However, because of the varying effects uncertainty has on the firm's price, how an increase in risk affects expected consumers' welfare as well as social welfare, generally, is not clear. In particular, with a fixed price expected consumer welfare increases with an increase in risk, but if an increase in risk results in the firm setting a higher price, than an increase in risk has an ambiguous effect on expected consumer welfare. Even if an increase in risk reduces the firm's price and so expected consumer welfare increases, then the effects on social welfare are ambiguous since the firm is made worse off.

The above discussion is based on a model of a firm that has many sales to many buyers, e.g., such as LL Bean or network TV. For such firms, even the worse products have some sales although the products are very unprofitable. However, for firm with few buyers and one-of-a-kind products, such as
independent filmmakers or haute couture designers, there may be a strong possibility that their product generates no sales. To capture the idea of markets with few buyers or one-of-a-kind products, in section 3 the model is modified so that there is positive demand only for sufficiently high (above average) realizations of quality. Under this condition, the firm now prefers a project that would generate a riskier distribution. That is, the firm would be willing to sacrifice some expected quality for a riskier distribution. Further, under these conditions, the firm's optimal price increases with a more risky product. Because the firm's price always increases with risk, the expected consumer and social welfare results are even weaker since for a fixed price expected consumer welfare increases with an increase in risk.

Since the ambiguity of the welfare results stems from conflicting attitudes between the firm and consumers towards an increase in risk, the final section explores the effects of two-part tariffs. If the firm has positive demand for all realizations, then the firm prefers a more risky product that does not change the lowest quality possible. If the firm does not have positive demand for all realizations, then the firm again prefers an increase in risk. However, unlike with the price setting firm, the effect of an increase in risk on the optimal fixed fee is ambiguous. Finally, several observations are made reinterpreting the results in terms of the effects of learning.

## 2. The model

A firm develops a new product, but is uncertain as to how desirable consumers will find it. The firm must set its price without knowing exactly how much consumers desire the product. However, each consumer knows what he likes and chooses whether to buy the product at the given price. Consumers are heterogeneous and each consumer has unit demand for the good. Their utility from this consumption is $U=q \theta+x$. Here $q$ is the commonly valued quality parameter, $\theta$ is the consumer specific valuation and $x$ is a composite commodity of other goods. The amount each consumer spends on this good is small relative to his income, i.e., income effects are ignored. Consumers are distributed
according to their specific valuation $\theta$ by the distribution function $\mathrm{F}(\theta)$ on $\left[\theta_{\text {min }}, \theta_{\text {max }}\right]$, which is twice differentiable with $f(\theta)=F^{\prime}(\theta)$.

The construct of quality and preferences here is used in, among others, Mussa and Rosen [13], Jaskold-Gabszewicz and Thisse [3,4], Shaked and Sutton [20,21], and Milgrom and Roberts [11]. However, here there is uncertainty regarding quality. Moreover, this construct of consumers is a generalization of other models of uncertainty that assume identical consumers with unit demand. For example, Lazear [8] considers the pricing problem of a firm facing consumers with unit demand, but does not know the consumer's valuation. The interpretation of the uncertainty here is the same as in Lazear, but the assumption of heterogeneous consumer and examination of changes in risk yield new results.

Given a price $p$, consumer $\theta$ buys the good if he derives positive utility: $q \theta-p \geq 0$. Define the consumer with the lowest $\theta$ willing to buy the product with quality $q$ at price $p$ as $\underline{\theta}$, i.e., $\underline{\theta}=p / q$. With no loss of generality, the mass of consumers is normalized to one so that the demand, given a price $p$ and realized quality $q$, is $X^{d}=(1-\mathrm{F}(p / q))$. For example, if $\theta$ is distributed uniformly, then the demand is linear: $X^{d}=1-p / q$.

The firm maximizes expected profits. Marginal costs are assumed constant at $\overline{c .}$ Profit, given $p$ and the realized value $q$, are
$\Pi=[1-\mathrm{F}(p / q)](p-c)$.

The firm does not know the value of $q$, but instead knows the distribution of $q$, that is, a distribution $\mathrm{G}(q)$, with $\mathrm{g}(q)=\mathrm{G}^{\prime}(q)$ on the support $[\underline{q}, \bar{q}]$. The firm sets its price based on $\mathrm{G}(q)$, by maximizing

[^1]$$
\mathrm{E}[\Pi(p, \mathrm{G}(q))]=\int_{\underline{q}}^{\bar{q}}[1-\mathrm{F}(p / q)](p-c) g(q) d q
$$

That is, the firm chooses the $p$ that satisfies the first order condition
$\int_{\underline{q}}^{\bar{q}}[1-\mathrm{F}(p / q)-f(p / q)(p-c) / q] g(q) d q=0$.
As the optimal price depends on the distribution, $\mathrm{G}(q)$, and not the realized value of $q$, it is denoted by $p(\mathrm{G})$. It is assumed that the second order condition holds:

$$
\int_{q}^{\bar{q}}\left[-2 f(p / q) / q-f^{\prime}(p / q)(p-c) / q^{2}\right] g(q) d q<0
$$

It is assumed that optimization yields an interior solution. That is, the distribution of $q$ and $\theta$ are such that at $p(\mathrm{G})$ at least one consumer buys some of the product for all possible realizations of $q$ : $\theta_{\max }>p(\mathrm{G}) / q$. This does not rule out that some consumers are not served for some (or all) realizations of $q$. Though assuming an interior solution is a common assumption, in the following section the effects of relaxing it are explored.

The first order condition (1) suggests that the firm has preferences on the riskiness of the product. In other words, the firm may have preferences over the riskiness of a given product to be developed. That is, imagine that a firm is considering two different products to develop. Marketing research is done to learn about consumers' preferences towards the proposed products. Let $\mathrm{G}(q)$ and $\mathrm{H}(q)$ be the distributions that characterizes this market research for each product. A priori it might be believed that all else equal a profit-maximizing firm would develop the product with the higher expected quality, $q$, since profits are increasing in $q$. However, (1) suggests that the riskiness of the product's quality is also important to the risk-neutral firm. That is, the firm might develop the product with a lower expected mean if, in exchange, there was a significantly lower risk associated with the product's quality.

To investigate the firm's preference towards risk, the notion of the riskiness of a product's
quality needs to be clearly defined. For such a comparison to be interesting, it must be between products that have the same expected quality. The definition used here is Rothschild and Stiglitz's [16] economic interpretation of Blackwell [1]. Consider two products G and H with corresponding distributions $\mathrm{G}(q)$ and $\mathrm{H}(q)$.

## Definition: Product G is more risky than product H if and only if

i. $\quad \int_{\underline{q}}^{\bar{q}} q g(q) d q=\int_{\underline{q}}^{\bar{q}} q h(q) d q$
ii. $\quad \int_{\underline{q}}^{x} G(q) d q \geq \int_{\underline{q}}^{x} H(q) d q$ for all $x \in[\underline{q}, \bar{q}]$, with a strict inequality for some $x$.

The first condition requires that the expected quality between the two products is the same. The second condition, together with the first, implies that there is more weight in the tails of the first distribution of quality. This implies that with a concave (convex) function, the expected value of the function decreases (increases). To apply this result, the curvature of the profit function is needed.

Lemma 1: Iff $(\mathrm{p} / q)+2 \cdot f>0$, then the firm's profits are concave in $q$.

Proof: $d^{2} \Pi /(d q)^{2}=-(\mathrm{p}-\mathrm{c})\left(\mathrm{p} / q^{3}\right) \cdot\left[f^{\prime} \cdot(\mathrm{p} / q)+2 \cdot f\right]<0 . / /$

The condition that $f^{\prime}(\mathrm{p} / \mathrm{q})+2 \cdot f>0$, is the requirement that revenue is concave in price, which is a slight strengthening of the second order condition for profit maximization.

From lemma 1, it follows that the firm prefers to develop the less risky product.

Proposition 1: If $\mathrm{f}^{\prime} \cdot(\mathrm{p} / \mathrm{q})+2 \cdot \mathrm{f}>0$, then the monopolist's expected profit decreases with a more risky product.

Proof: Since profits are concave in $q$ by lemma 1, then by the definition of an increase in risk,
$\mathrm{E}[\Pi(p(\mathrm{G}), \mathrm{G}(q))]=\int_{\underline{q}}^{\bar{q}}[1-\mathrm{F}(p(G) / q)](p-c) g(q) d q<\int_{\underline{q}}^{\bar{q}}[1-\mathrm{F}(p(G) / q)](p-c) h(q) d q=\mathrm{E}[\Pi(p(\mathrm{G}), \mathrm{H}(q))]$.
By the definition of $p(\mathrm{G}), \mathrm{E}[\Pi(p(\mathrm{G}), \mathrm{H}(q))] \leq \mathrm{E}[\Pi(p(\mathrm{H}), \mathrm{H}(q))] . / /$

Risk-averse managers have been used as a motivation for firms choosing to develop less risky products (e.g., recently Zhang [24]). The result in this paper has a different flavor than these usual results. In particular, here less risky products are chosen because they have higher expected profits. Moreover, the riskiness comes from uncertainty about the product's characteristics.

This model is a generalization of other models. For example, the case of linear demand $\left(\mathrm{f}^{\prime}=0\right)$ corresponds to Rosen's [15] model of superstars. The demand for the star depends on the star's quality, where quality is modeled as it is here. Though in Rosen the quality is certain, since the actual success of a performer is rarely known ex ante, a natural issue is the effect of uncertainty has on contract decisions. For example, demand for long term contracts traditionally have been viewed as evidence of risk adverse preferences. However, a star's desire for a long-term contract may arise instead from how quality enters their pecuniary payoffs. Likewise, agents and, historically, movie studios, who lay a claim on the superstar's income stream, would prefer a potential superstar with lower expected stardom, but significantly less risk. That is, they would attempt to discover stars who were much of the same mode of other stars. Though this behavior would be profit maximizing, the lack of risk taking by agents might be mistakenly interpreted as the result of risk aversion.

That a firm prefers a less risky product suggests that the riskiness of a product should also affect the firm's optimal price. One might suspect that if a firm prefers less risky products, then a more risky product causes the firm to price more conservatively. Intuitively, by lowering its price a riskaverse firm would be reducing its loss from low quality realizations. However, the effects of uncertainty on the pricing are not so straightforward and depend on the distribution $\mathrm{F}(\theta)$ as well as $\mathrm{G}(\mathrm{q})$ and costs.

Proposition 2: If 2.f.c $+\mathrm{f}^{\prime} \cdot \mathrm{c} \cdot \mathrm{p}^{2} / \mathrm{q}-\left(3 \cdot \mathrm{f}^{\prime}+\mathrm{p} \cdot \mathrm{f}^{\prime \prime} / \mathrm{q}\right) \cdot(\mathrm{p}-\mathrm{c}) \cdot \mathrm{p} / \mathrm{q}>(<) 0$, then the firm sets a lower (higher) price with a more risky product.

Though the proof follows directly from Rothschild and Stiglitz [16], it is useful for latter analysis.

Proof: The second derivative of $\mathrm{d} \Pi / \mathrm{dp}$ with respect to $q$ is
$-\left[1 / q^{3}\right] \cdot\left[2 \cdot f \cdot c+f^{\prime} \cdot c \cdot p^{2} / q-\left(3 \cdot f^{\prime}+p \cdot f^{\prime \prime} / q\right) \cdot(p-c) \cdot p / q\right]$.

Thus, if $2 \cdot f \cdot c+f^{\prime} \cdot c \cdot p^{2} / q-\left(3 \cdot f^{\prime}+p \cdot f^{\prime \prime} / q\right) \cdot(p-c) \cdot p / q>(<) 0$, then by the definition of a more risky product, the left-hand-side of (1) evaluated at price $p(\mathrm{H})$ under the distribution $\mathrm{G}(q)$ is negative (positive). $\mathrm{dE}[\Pi(p(\mathrm{H}), \mathrm{G}(q))] / \mathrm{dp}=\int_{\underline{q}}^{\bar{q}}[1-\mathrm{F}(p(H) / q)-f(p(H) / q)(p(H)-c) / q] g(q) d q<(>) 0 . / /$

Though the condition for proposition 2 may seem unintuitive, it captures many standard assumptions used in less general models. For example, if marginal costs are positive and the demand is linear $\left(f^{\prime}=0=f^{\prime}\right)$, then the firm's price decreases with a more risky product, as the second derivative (2) becomes $2 \cdot f \cdot \mathrm{c}>0$. Similarly, the firm's price also decreases with a more risky product, if demand is convex ( $f^{\prime} \leq 0$ ), and its third derivative non-positive ( $f^{\prime \prime} \leq 0$ ). In this case, the firm's first-order condition is concave in $q$ and an increase in risk decreases the price. Likewise, many other simple restrictions on the demand's concavity and third derivative ( $f^{\prime}$ ) make (2) positive, i.e., the first-order condition concave in $q$. This result seems to make intuitive sense; a riskier distribution has lowered the firm's profitability and so the firm is more tentative in its pricing.

Though standard assumptions, such as linear demand, yield the result that the firm's price decreases with a more risky product, other assumptions on demand yield the result that the firm's price increases with a more risky product. For example, if the price-cost mark-up, $(p-c)$, is sufficiently
large, $f^{\prime \prime} \geq 0$, and the demand is strictly concave $\left(-f^{\prime}<0\right)$, then the firm's price increases with a more risky price despite the decrease in expected profits. The assumption of concave demand is of particular interest since it also implies the second order condition. To conclude, the condition in proposition 2 shows that the effects of risk on the price are sensitive to the assumptions on demand.

That the price decreases with risk, e.g., with linear demand, has been found in previous work, which instead assumed risk aversion (e.g., Sandmo [18]) or strictly increasing costs with risk neutrality (Leland [7], Kimball [5]), rather than uncertainty of a product characteristic. Moreover, in previous work, the price increases with risk only under the special case of decreasing marginal costs. There is, however, an important conceptual difference between the model here and previous work. Here the source of uncertainty is modeled. In particular, the uncertainty arises in the product's characteristics, rather than in the output or price as in previous work. As a result, this is not a partial-partial equilibrium model and comparisons between the producer and consumer welfare and social welfare can be made. Alternatively, since consumers' taste drive demand, the firm's uncertainty can be understood as uncertainty regarding consumers' perception of the good. That is, $q$ can be taken as consumers' subjective evaluation of the product since ultimately all that matters for demand is what consumers believe.

## Welfare

Since the firm prefers to develop less risky products, a natural question is whether this is also true from the point of view of a welfare maximizer. That is, is the firm's decision efficient, or should the firm be encourage to develop riskier products or even less risky products? In determining the welfare optimum, several standards could be used. The traditional standard is the first best in which the welfare optimal, or efficient, price is set before the quality is revealed. (It is optimal, of course, to set the price equal to marginal cost regardless of the riskiness of the product.)

A second standard for welfare maximization is to find the optimal amount of risk when the
price is set to maximize profits. That is, whether the profit-maximizing firm should be encouraged to develop riskier products. This is a more useful standard as it allows the monopolist to set the price when determining the welfare effects of risk. At a practical level, a policy maker is unlikely to be able to set the price at which a firm sells its product, but through research incentives may be able to influence which product it develops. Though this standard may be more practical, it introduces two difficulties. First, since the firm's price responds to a change in risk, determining the effects is more difficult than with the efficient price, $p=c$, which is constant for all risk. Second, with efficient pricing there are zero profits so welfare is equivalent to consumer welfare. With a profit-maximizing firm, welfare comes from both consumer welfare and profits, which may be in conflict.

The social welfare measure is the total value generated by the consumption level less the cost of production. Recalling that the marginal consumer is defined by $\theta \cdot q-p=0$, consumer welfare and social welfare given price $p$ and $q$ are

$$
\begin{aligned}
& S(c, q)=\int_{p / q}^{\infty} q \theta f(\theta) \cdot d \theta-p[1-F(p / q)] \\
& W(c, q)=\int_{p / q}^{\infty} q \theta f(\theta) \cdot d \theta-c[1-F(p / q)]
\end{aligned}
$$

which are consumers total value less, respectively, expenditures and total costs. Following the outline for proposition 1 , it is easy to evaluate how social welfare, $\mathrm{E}[\mathrm{W}(p, \mathrm{G}(q))]$, changes with a more risky product under the first best optimum.

Proposition 3: Social welfare under the first best, i.e., with efficient pricing ( $p=c$ ), increases with a more risky product.

Proof: The second derivative of social welfare with respect to quality is

$$
\begin{equation*}
\mathrm{d}^{2} \mathrm{~W} /(\mathrm{d} q)^{2}=(p / q) \cdot \mathrm{f} \cdot\left(p / q^{2}\right)-(p-c) \cdot\left(p / q^{3}\right) \cdot\left[\mathrm{f}^{\prime} \cdot(p / q)+2 \cdot \mathrm{f}\right] . \tag{3}
\end{equation*}
$$

If $p=c$, then $\mathrm{d}^{2} \mathrm{~W} /(\mathrm{d} q)^{2}>0$. Hence, social welfare increases with a more risky product.//

Thus, from proposition 1 a firm prefers a less risky product with the same expected quality, and by proposition 3 , society prefers a riskier product.

Proposition 3 can be explained with the standard convexity argument. First, with efficient pricing $(p=c)$ welfare is equal to consumer welfare as profits are zero. Second, if output and prices were fixed, then consumer welfare would be linear in quality. However, since consumers can adjust their purchases (output) after observing the quality, then for a given quality realization they are better off than with a fixed output. Hence, consumer welfare is convex in $q$, and so in expectation consumers are better off with a more risky product.

To examine welfare under the second best, it is useful to divide the effects between consumer welfare and profits. Consumer welfare, however, depends both on the riskiness of the product and how the firm's price responds to the riskiness of the product. For example, if the firm raises its price in response to an increase in risk, consumer welfare is reduced by this price increase. In studying how risk affects consumer welfare, it is useful to introduce an intermediate case in which prices are constant but not efficient. Since the consumers' welfare expression is the social welfare expression, but with $p$ substituted for $c$ it immediately follows from (3) that,

## Corollary 3.i: For any fixed price, expected consumers' welfare increases with a more risky product.

Consumer welfare, however, is affected in another way, not captured by proposition 3, from an increase in risk. If the firm lowers (raises) its price with an increase in risk, then consumers have an additional benefit (loss) from the lower (higher) price. Thus, if the firm lowers its price in response to an increase in risk, e.g., linear demand (proposition 2), expected consumer welfare increases from an increase in risk. However, if the firm raises its price in response to an increase in risk, the effect on consumer welfare is ambiguous.

The effect that risk has on profits appears as the second term on the right-hand side of (3), which is the second derivative of the firm's profit with respect to $q$ (lemma 1). However, (3) does not capture the total effect of a change in risk on welfare because it does not take into account that the firm adjusts its price in response to a change in risk. (By proposition 1 expected profits are lower even with the price adjusting to the changed distribution.) Thus, social welfare effects under monopoly pricing are not clear since consumer welfare increases with risk for a given price while the firm's profits decreases. Hence, though developing riskier products should be encourage on efficiency grounds if there is efficient (marginal cost) pricing, this cannot be argued with a monopoly in general.

Though, generally, the effect of an increase in risk on welfare is not straightforward, with additional assumptions some results can be obtained. First, from (3), with a fixed price, there are conditions that imply that welfare decreases with risk.

Corollary 3.ii: For any fixed price greater than marginal cost, if costs are sufficiently small $(c \rightarrow 0)$ and demand is concave $\left(f^{\prime}>0\right)$, then social welfare under a monopoly decreases with a more risky product.

Proof: If $c=0$ and $f^{\prime}>0$, then the right-hand side of (3) becomes $-(p) \cdot\left(p / q^{3}\right) \cdot\left[f^{\prime} \cdot(p / q)+f\right]<0 . / /$

Interestingly, these conditions, along with $\mathrm{f}^{\prime \prime}=0$, are conditions for the firm's price to increase with risk. Thus, in this case, when the firm adjusts its price to the risk, consumer welfare then decreases. However, since the firm's profits increase from the price adjustment, no conclusions can be made about the welfare effect when the firm adjusts its price.

When the firm sets its optimal price, conditions can be derived such that an increase in risk increases welfare. The conditions exploit the assumptions that imply that the firm lowers its price in response to an increase in risk. When the firm lowers its price both it and consumers are better off. The firm is better off by profit maximization. Consumers are better off from the fact that consumer welfare increases with a price decrease. Thus, the condition that, for a fixed price, an increase in risk
increases welfare, together with the condition that an increase in risk reduces the price, generates the desired result.

Corollary 3.iii: If $2 \cdot \mathrm{f} \cdot \mathrm{c}+\mathrm{f}^{\prime} \cdot \mathrm{c} \cdot \mathrm{p}^{2} / \mathrm{q}-\left(3 \cdot \mathrm{f}^{\prime}+\mathrm{p} \cdot \mathrm{f}^{\prime \prime} / \mathrm{q}\right) \cdot(\mathrm{p}-\mathrm{c}) \cdot \mathrm{p} / \mathrm{q}>0$ and $(p / q) \cdot \mathrm{f} \cdot\left(p / q^{2}\right)-(p-c) \cdot\left(p / q^{3}\right) \cdot\left[\mathrm{f}^{\prime} \cdot(p / q)\right.$ $+2 \cdot \mathrm{f}]>0$, then social welfare under a monopoly increases with a more risky product.

Proof: From proposition 3, if $(p / q) \cdot f \cdot\left(p / q^{2}\right)-(p-c) \cdot\left(p / q^{3}\right) \cdot\left[\mathrm{f}^{\prime} \cdot(p / q)+2 \cdot \mathrm{f}\right]>0$, then for a given price welfare increases. From proposition 2, if $2 \cdot f \cdot c+f^{\prime} \cdot c \cdot p^{2} / q-\left(3 \cdot f^{\prime}+p \cdot f^{\prime} / q\right) \cdot(p-c) \cdot p / q>0$, then the firm's price decreases. By revelation, expected profits must increase from this price decrease. Expected consumer surplus also increases with a price decrease.//

Under the conditions of corollary 3.iii a monopolist would increase welfare if it developed more risky products.

There is an extensive literature examining the effects of uncertainty, e.g., Turvovsky, et al. (1980). However, typically the uncertainty is modeled as price or quantity uncertainty, without modeling the cause of the price or quantity variability. This incomplete modeling results in conundrums, such as Waugh [23] finding that consumers like price uncertainty and Oi [14] finding that firms also like price uncertainty. As noted by Massell [10], the source of the contradiction is that only one side of the market (supply or demand) is modeled. By not modeling the source of the uncertainty, welfare conclusions cannot be drawn. More recent work has used the concepts of riskier distributions to examine exogenous demand uncertainty. Rothschild and Stiglitz [17] show that with strictly convex costs uncertainty in output demand can result in lower capital usage. Kimball [5] demonstrates that, with strictly convex cost, demand uncertainty can result in higher prices set by the firm.

The model here diverges from the previous literature in several ways. First, here the implication is that a firm wants to develop a less risky product. That is, the model here implies that the firm would choose a less risky distribution. Further, this choice of a less risky distribution has welfare
effects. In contrast, previous work does not consider the welfare implications of choosing a less risky price or quantity distribution as it would be a nonsensical question in their context. Second, the source of the uncertainty is modeled, and therefore welfare effects of an increase in risk can be examined. That is, the price or quantity risk arises from a known source unlike previous work. Third, in both Rothschild and Stiglitz [17] and Kimball [5] only strictly convex cost case is considered. In fact, with constant marginal costs as assumed in this paper, their effects disappear. Closer in spirit to the model here, recently, Eeckhoudt et al [2] found that tax codes could induce risk neutral firms to not choose riskier investment projects, which lowers social welfare.

## 3. Complete failures

The results so far are quite intuitive, as they only require a standard assumption for the firm to prefer a less risky distribution of quality: revenue is concave in price -- a strengthening of the second order condition on profit maximization. This would seem to indicate that firms prefer to develop products whose expected quality is less risky, safer. Yet, sometimes very risky behavior is observed . The current model can only explain this behavior with an unusual assumption, that revenue is convex in price. This suggests that markets in which firms do exhibit risky behavior should be examined to determine if their characteristics are consistent with the assumptions of the model.

In the previous section an interior solution was assumed. That is, at the optimal price the firm has positive demand for all realizations of $q$, i.e., at least one consumer buys a positive amount for all $q$. For many products, assuming that a positive amount is always demanded may be quite reasonable. Firms generally believe that while there is some chance that their product might fail, there are always some units sold; even the Edsel sold some units. However, for some products this may be a poor assumption. In particular one-of-a kind goods such as art, haute couture or independent films that need a distributor may never have even one unit sold. In this case, at the optimal price there is zero demand for low $q$. This particular type of market, one-of-a kind products, has been studied in stylized models
such as in Rosen [15] or Lazear [8]. In this section, the assumption that at the optimal price there is positive demand for all realizations of $q$ is relaxed, i.e., for some $q$ the firm may have no sales.

If for some realizations of $q$ the firm has zero demand, then the choice of $p$ determines the minimum $q$ for a positive number of units to be sold. That is, since $\theta_{\text {max }}$ is the consumer with the highest valuation, then given a price $p=\tilde{p}$, the minimum $q$ necessary for positive demand is defined by $q \cdot \theta_{\text {max }}-\tilde{p}=0$, or $q$ minimum is $\tilde{p} / \theta_{\text {max }}$. Thus, in choosing $p$ the firm chooses the revenue, for a given $q$, as well as the probability of a sale. That is, given a quality distribution $\mathrm{G}(q)$, the firm now chooses $p$ to maximize

$$
\int_{p / \theta_{\max }}^{\bar{q}}[1-\mathrm{F}(p / q)](p-c) g(q) d q .
$$

Assuming that the second order condition holds ${ }^{2}$ the first order condition defines the $p$, denoted by $p(\mathrm{G})$, which maximizes the above expression,

$$
\begin{equation*}
\int_{p(G) / \theta_{\max }}^{\bar{q}}[1-\mathrm{F}(p(G) / q)-f(p(G) / q)(p(G)-c) / q] g(q) d q=0 . \tag{4}
\end{equation*}
$$

(Note that the derivative of the lower limit of the profit expression, $-\left(1 / \theta_{\max }\right)\left[1-\mathrm{F}\left(\theta_{\max }\right)\right](p-c)$, is zero.).
Since profits are constant (at zero) for realizations of $q$ below $p(\mathrm{G}) / \theta_{\min }$ and concave above, profits are no longer globally concave. Thus, increases in risk no longer imply a decrease in expected profits. However, given that profits are zero for the lower portion of the support, there are natural conditions under which the firm now prefers a riskier distribution. Perhaps the most natural is

Proposition 4: Let $p(H)$ be the price that maximizes profits given a distribution $H$. If $p(H)$ is such that the minimum $q$ for positive demand is greater than the average $q$, i.e., $p(H) / \theta_{\max } \geq \int q \cdot h(q) d q$, then the

[^2]monopolist's expected profit increases with a more risky product.

Proof: Let $G$ be more risky than $H$. If $p(H) / \theta_{\max } \geq \int q \cdot h(q) d q$, then since $[1-\mathrm{F}(p / q)](p-c)$ is increasing in $q$ and by the definition of a more risky product $\int_{p(H) / \theta_{\max }}^{\infty}[1-\mathrm{F}(p(H) / q)](p(H)-c) g(q) d q \geq$ $\int_{p(H) / \theta_{\max }}^{\infty}[1-\mathrm{F}(p(H) / q)](p(H)-c) h(q) d q . \quad$ By definition of $p(\mathrm{G}), \int_{p(G) / \theta_{\text {max }}}^{\infty}[1-\mathrm{F}(p(G) / q)](p(G)-c) g(q) d q \geq$ $\int_{p(H) / \theta_{\max }}^{\infty}[1-\mathrm{F}(p(H) / q)](p(H)-c) g(q) d q . / /$

The intuition behind proposition 4 is straightforward. With the truncated distribution and the minimum being above the average, a more risky distribution pushes weight on to higher realizations of $q$ which increases expected profit. For example, from the introduction, designers of haute couture do not care what average people may think of their clothing. Instead, their market is the small market of the wealthy or editors of fashion magazines. What the designers are looking for is originality: something unique that captures the fancy of someone who can afford to have it and so sell one unit at a very high price. Thus, attempting to make fashions more bizarre may simply be an attempt to introduce risk. Likewise, for independent movie directors pleasing a few movie critics who value quirkiness and originality so that the movie sells may generate convex payoffs and makes riskier film projects more profitable.

Though the proposition requires that the minimum $q$ be greater than the average, this is not a necessary condition for a firm to prefer an increase in risk. There also are conditions under which the minimum $q$ is less than the average and the firm still prefers the increase in risk. However, these conditions are quite unintuitive and so, instead of presenting them, in the next subsection examples are presented in which expected profits increase with risk even though the minimum $q$ is below the average.

The conditions for a firm to prefer an increase in risk also suggest that an increase in risk is likely to increase the firm's optimal price. If marginal profit is increasing in $q$, then with an increase in risk there is more weight on higher $q$ and so more weight on realizations with larger marginal profit. Thus, a riskier product implies that if $\mathrm{G}(q)$ is riskier than $\mathrm{H}(q)$, then the first order condition at $p(\mathrm{H})$ with the distribution of $\mathrm{G}(q)$ is now positive and so $p(\mathrm{G}) \geq p(\mathrm{H})$. Thus, all that is needed to show that the price increases in risk is that marginal profit is increasing in $q$, which requires either an even greater strengthening of the second order condition, or an assumption such as $c=0$. Specifically, the derivative of marginal profit for given $q, 1-\mathrm{F}(p / q)-f(p / q) \cdot(p-c) / q$, with respect to $q$ is $\left(p / q^{2}\right) \cdot(2 f+$ $\left.f^{\prime}(p-c) / q-c \cdot f / p\right)$, which differs from the second order condition by $-c \cdot f / p$. To summarize,

Proposition 5: If $p(H) / \theta_{\max } \geq \int q \cdot h(q) d q$ and $2 f+f^{\prime}(p-c) / q-c f / p \geq 0$, then the firm sets a higher price with a more risky product.

Proof: Since by the definition of a more risky product the expected qualities are equal, if $p(H) / \theta_{\max } \geq \int$ $q \cdot h(q) d q$, and $2 f+f^{\prime}(p-c) / q-c \cdot f / p \geq 0$, then the derivative of expected profits at $p(H)$ increases:

$$
\int_{p(H) / \theta_{\max }}^{\infty}[1-\mathrm{F}(p(H) / q)-f(p(H) / q)(p(H)-c) / q] g(q) d q \geq \int_{p(H) / \theta_{\max }}^{\infty}[1-\mathrm{F}(p(H) / q)-f(p(H) / q)(p(H)-c) / q] h(q) d q
$$

$$
=0 . ?
$$

In terms of social welfare it is still difficult to make comparisons, even though for a fixed price both social welfare and profit increase with an increase in risk, because the firm's optimal price changes with changes in risk. Social welfare now is
$\mathrm{E}[\mathrm{W}(p, \mathrm{G}(q))]=\int_{p / \theta_{\max }}^{\bar{q}}\left\{\int_{\mathrm{p} / q}^{\infty} q \theta(\theta) \cdot \mathrm{d} \theta-c[1-\mathrm{F}(p / q)]\right\} g(q) d q$

[^3]Though social welfare is constant at zero for some realizations of $q$, as is profits, if social welfare is convex in $q$ everywhere else, then social welfare is convex in $q^{4}$ Thus, convexity (concavity) arguments, though they cannot be used when examining expected profits, can still be applied to the effects of a riskier distribution on social welfare for a given $p$. As a result, proposition 3, which examines the case of efficient pricing, can be strengthened slightly. That is, since social welfare was already shown to be convex in $q$ at the efficient price and consumers' welfare was shown to be convex in $q$ at a fixed price, they still are.

Proposition 3': Social welfare under efficient pricing and expected consumers' welfare under any fixed price increase with a more risky product, even if there is zero demand for some realizations of $q$.

When the optimal price for the firm is such that the minimum $q$ for positive demand is greater than the average $q$, then for a fixed price the monopolist now also prefers an increase in risk (proposition 4) as consumers always have (proposition $3^{\prime}$ ). Thus, for a fixed price society benefits from a more risky product. This is in sharp contract to the earlier result (corollary 3.ii) in which social welfare can decrease with a more risky product.

Corollary 3.iv: If the monopolist's price p is such that the minimum q for positive demand is greater than the average $q$, i.e., $p\left(H / \theta_{\max }\right) \geq \int q \cdot h(q) d q$, then for fixed $p$, social welfare increases with a more risky product.

Proof: Since social welfare is increasing in $q$, the logic follows the first step in proposition 4's proof.

Thus, the conditions for corollary $3 . i i$ no longer imply that welfare decrease with a more risky product when the minimum $q$ for positive demand is greater than the average $q$. Furthermore, if for any $q$ there

[^4]is zero demand, then the conditions for corollary 3.ii do not imply that welfare decrease with a more risky product. This is because the conditions imply that welfare is concave in $q$ for a fixed price when there is positive demand for every $q$. However, if there is zero demand for some $q$, then welfare is constant for those $q$ 's. Thus, welfare is not concave everywhere.

Despite the congruity of firm and consumer preferences towards an increase in risk with a fixed price when the minimum $q$ for positive demand is greater than the average $q$, the welfare effect from an increase in risk is still ambiguous. The ambiguity arises because the monopolist raises its price with an increase in risk, increasing the probability that no consumers are served. Thus, general conditions such as in corollary 3 .iii for social welfare to increase with a more risky product do not exist when the minimum $q$ for positive demand is greater than the average $q$.

Even if, for the profit maximizing $p$, social welfare is increasing with a more risky product, it is still likely that the firm's value from an increase in risk would differ from society's. Thus, if an investment stage existed before the price setting stage, the firm's level of investment would differ from the social optimum. However, in this case at least the firm and society would have the same preference towards a change in risk.

## Examples

Some simple examples may provide additional intuition for these results. Assume that both types, $\theta$, and quality, $q$, are uniformly distributed. Assume that marginal $\operatorname{cost}, c=1$. If $q=20$, then the optimal price, $p(\mathrm{G})$, equals 10.5 , and expected profits are 4.5125 . If the support of $q$ ranges from 19 to 21 , then the optimal price is 10.49 and expected profits are 4.51 . As the support expands, $p(\mathrm{G})$ falls until the support is $[10,30]$ to 9.60 and expected profits fall to 4.07 . As the support expands further, the optimal price starts to increase and there now exists zero demand for low realizations of $q$. Generally, even though the minimum $q$ is well below the average $q$, an increase in the support increases the price. Expected profits continue to fall as the support expands to [2,38], where profits fall to 3.721 while the
optimal price rises to 11.64. If the support extends further, then profits rise slightly. For the largest possible positive support $([0,40])$ expected profits are 3.728 and the optimal price is 12.21 . Though profits are never greater under uncertainty than with certainty in this example, it is easy to create examples in which this happens. In particular, if marginal cost is 10 , then with certainty the optimal price is 15 and profits are 1.25 . As the support expands, price and profits initially decline. However, at $[14,26]$ the price starts to rise and goes above the certainty price at [13,27]. Profits start to rise at $[12,28]$ and rise above certainty profits at $[7,13]$ where the price is 17.09 . At [ 0,40 ] profits reach 1.54 and the optimal price is 19.16. As marginal costs approach the average quality, it takes a smaller support to generate higher profits than the certainty profits.

## 4. Two-part tariffs

If at least one consumer buys a positive amount for all $q$, then with fixed prices, an increase in the riskiness of quality benefits consumers (proposition 3) but harms the firm (proposition 1). These conflicting preferences towards risk make it difficult to determine if social welfare increases or decreases with an increase in risk, even for fixed prices. However, if the firm could capture part of the consumers' benefit from the increased risk, then both firms and consumers may prefer greater risk and so social welfare effects can be determined. Since in the current model each consumer consumes only one unit, capturing the consumers' benefit is difficult. Instead, in this section a single representative consumer model is considered. This allows the firm to set a two-part tariff in an attempt to capture the consumers' benefit from an increase in the riskiness of quality. To provide continuity with the previous sections, the assumptions made in this section generate as a special case the previous sections' demand. That is, the demand in this section can be interpreted as the demand that would arise if the previous sections' demand were generated by a single consumer.

The basic structure and timing are as in the previous section. The amount spent on the good is small relative to income and quasi-linear utility is assumed so that income effects are ignored. For
quality $q$, if the consumer buys $x$ units the consumer obtains value $\mathrm{V}(q, x)$. It is assumed that $\mathrm{V}(q, x)$ is thrice differentiable for positive $x$ and $q, \partial \mathrm{~V}(q, x) / \partial q>0$ and $\partial^{2} \mathrm{~V}(q, x) / \partial q \partial x>0$, i.e., for given $x$, both the total and the marginal value are increasing in quality. The preferences specified here are a generalized version with those specified in, e.g., Rosen [15] or e.g., quasi-linear utility of the form $V(q)$ $=q\left(1-(1-x)^{2}\right) / 2.5$

The utility maximizing consumer observes $q$ and a (per unit) price $p$ and chooses the amount $x^{*}$ $=x(q, p)$ to maximize his net value: $\mathrm{V}(q, x)-p \cdot x$, i.e., $\partial \mathrm{V}\left(q, x^{*}\right) / \partial x-p=0$, subject to $\mathrm{V}\left(q, x^{*}\right)-p \cdot x^{*} \geq 0$. Given the distribution $\mathrm{G}(q)$, but before the firm knows the realization of $q$, the firm chooses the price. The firm has per unit cost of $c$. If the firm sets only a per unit price, then by applying the additional assumptions of section 2 on the demand, the results there can be applied here. By proposition 1 the firm prefers a reduction in risk and by proposition 2 the firm's price increases with an increase in risk.

Now let the firm set a fixed fee in addition to the per unit price. Let $T(x)=A+p \cdot x$ be the two part tariff that the firm offers, where $A$ is the fixed fee. As before, it is assumed that the firm sets the tariff without knowing the consumer's preference toward the product, but the consumer observes the quality before choosing to buy the product? ${ }^{7}$ It is optimal for the firm to set the per-unit price equal to $c$. To see why, consider any per-unit price and fixed fee $\{p, A\}$, where $p>c$. Consider now the marginal quality $q^{\prime}: \mathrm{V}\left(q^{\prime}, x^{*}\right)-p \cdot x^{*}-A=0$. That is, for any lower realized $q$ the consumer would not

[^5]${ }^{7}$ Another possibility is that the firm sets a menu of output-fixed fee combinations, essentially practicing second degree price discrimination where $q$ becomes the unknown type of the consumer (Maskin and Riley [9]). Since this is not possible in the general model of section 2, this will be developed in future research. However, since such types of revelation mechanisms typically obtain lower expected profits than the certainty case, the results in section 2 suggest that if the structure was set so that sales occurred for all realizations of $q$ then the firm would prefer a reduction in risk. Further, the results in section 3 suggest that if the firm only serves above average realizations of $q$, then the firm would prefer an increase in risk.
buy the product. The firm, by lowering $p$ to $c$ can increase $A$ to capture the additional benefit to the consumer from the lower price. This yields higher profits for $q^{\prime}$. Since V is increasing in $q$, for any higher realization of $q$, the consumer would buy and the firm earns higher profits.

The firm's problem is to choose $A$ so as to maximize profits. By choosing $T=\{c, A\}$ the firm has implicitly defined the minimum $q, q^{\prime}$, necessary for the consumer to buy: $\mathrm{V}\left(q^{\prime}, x\left(q^{\prime}, c\right)\right)-\mathrm{c} \cdot x\left(q^{\prime}, c\right)-A$ $=0$ and so its probability of having a sale: $1-\mathrm{G}\left(q^{\prime}\right)$. That is, the problem can be thought of as the firm choosing $q^{\prime}$ and so determining $A\left(q^{\prime}\right)=\mathrm{V}\left(q^{\prime}, x\left(q^{\prime}, c\right)\right)-\mathrm{c} \cdot x\left(q^{\prime}, c\right)$. With this fixed fee $A\left(q^{\prime}\right)$, the consumer pays $A\left(q^{\prime}\right)$ plus $c \cdot x(q, c)$ for $q$ greater than $q^{\prime}$. As the firm incurs costs $c \cdot x(q, c)$ for $q$ greater than $q^{\prime}$, the firm chooses $q^{\prime}$ to maximize:
$\mathrm{E}[\Pi(\mathrm{G})]=\int_{q^{\prime}}^{\bar{q}}\left\{A\left(q^{\prime}\right)+c \cdot x(q, c)-c \cdot x(q, c)\right\} g(q) d q=\int_{q^{\prime}}^{\bar{q}}\left\{\mathrm{~V}\left(q^{\prime}, x\left(q^{\prime}, c\right)\right)-c \cdot x\left(q^{\prime}, c\right)\right\} g(q) d q$

Using the definition of $x(q, p)$, the derivative of expected profits with respect to $q^{\prime}$ is
$\partial \mathrm{E}[\Pi(\mathrm{G}(q))] / \partial q^{\prime}=-\left\{\mathrm{V}\left(q^{\prime}, x\left(q^{\prime}, c\right)\right)-c x\left(q^{\prime}, c\right)\right\} g\left(q^{\prime}\right)+\int_{q^{\prime}}^{\bar{q}}\left\{\partial \mathrm{~V}\left(q^{\prime}, x\left(q^{\prime}, c\right)\right) / \partial q^{\prime}\right\} g(q) d q$.
Since the first best price is set equal to marginal cost and the fixed fee is zero, the effect of an increase in risk has on social welfare under the first best is the same as in section 2.

The effects of an increase in risk on firm and consumer welfare when the firm sets the profit maximizing tariff are more complex then with a price setting firm. Some effects, though, can be determined for two particular cases: when the profit-maximizing fixed fee implies that the consumer buys for all quality realizations (the corner solution) and when the profit-maximizing fixed fee implies that the consumer only buys when the realized quality is greater than the expected quality.

Consider first the case in which the consumer buys for all realizations of quality, i.e., the profitmaximizing minimum $q$ is the lower bound of the support, $q$. For this case, it is convenient to divide an
increase in risk into two types. The first type, type $A$, does not effect the support of $q$. That is, the lowest possible $q$ realization, $q$, does not change. The second type, type $B$, does effect the support of $q$, expanding it.

If the consumer buys for all realizations of quality, then, holding the fixed fee constant, the consumer still buys for all realizations of $q$ with a type $A$ increase in risk. It follows then that the profit maximizing fixed fee and expected profits are non-decreasing in a type $A$ increase in risk. To see this, notice that the firm could always leave the fixed fee unchanged and expected profits would remain constant. Thus, if the firm increases the fixed fee with a type $A$ increase in risk, then expected profits must increase.

The effect this increase in risk has on expected welfare is less clear. If the fixed fee does not change with a type $A$ increase in risk, then expected consumer welfare increases by corollary 3.i, and so does welfare since expected profits are non-decreasing. If the fixed fee does increase with a type $A$ increase in risk, the effect on expected consumer welfare is ambiguous and hence so is welfare.

With a type $B$ increase in risk, the support of quality realizations expands, reducing $q$, the lower bound. Thus, holding the fixed fee constant the consumer no longer buys for all realizations of $q$ with a type $B$ increase in risk. As a result, with a type $B$ increase in risk, if the profit maximizing fixed fee does not increase, then the firm's expected profit decreases. This is because either the fixed fee decreases or there is a positive probability that the firm has no sales.

While expected profits decrease with a type $B$ increase in risk if the fixed fee does not increase, expected consumer welfare increases. To see this, note that holding the fixed fee constant, consumer welfare is zero at $q$ and then convex in $q$ for $q>q$. Thus, for a given fixed fee expected consumer welfare increases with risk. Since the fixed fee does not increase, expected consumer welfare increases and the welfare effects are ambiguous. If instead, the profit maximizing fixed fee increases with a type $B$ increase in risk, then the effect on expected consumer welfare and on the firm's expected profit cannot be determined without additional assumptions.

Consider the second case: the consumer only buys when the realized quality is greater than the expected quality. This case is similar to that studied in section 3, except that here, in addition to the product's price, the consumer also pays a fixed fee. In particular, the effect on profits from an increase in risk is nearly identical to those derived in section 3. If the consumer only buys when the realized quality is greater than the expected quality, then holding the fixed fee constant, an increase in risk increases the probability that the consumer buys the product. Thus, holding the fixed fee constant, expected profits increase with an increase in risk. Since the fixed fee maximizes expected profits, an increase in risk increases expected profits.

In contrast, the effect an increase in risk has on the fixed fee, and hence the minimum quality realization such that the consumer buys is not analogous to the effect when the firm only set a price. In section 3, the profit-maximizing price, and hence the minimum quality realization such that the consumer buys, increases with an increase in risk. However, when the firm sets a two-part tariff the minimum quality may decrease. This is because the derivative of profit with the two-part tariff at the optimum (5) has an additional term compared to the derivative of profits at the optimum in (4). This difference in the expressions arises because the derivative of the lower limit in (5) is not zero as in (4). As a result, with a two-part tariff the derivative of expected profits, evaluated at the optimum for the less risky distribution, might decrease with a more risky distribution of $q$, whereas it always increases if the firm only only a price.

The effect an increase in risk has on social welfare when the consumer only buys for quality realizations greater than the expected quality is more difficult to determine when a two-part tariff is set than when only a price is set. This is because while welfare is globally convex given a price, it is not globally convex given a two-part tariff. For a given price, consumer demand is continuous in $q$. Thus, at the minimum quality realization such that the consumer buys, consumer welfare and profits are zero and continuous in $q$. However, with a positive fixed fee, consumer demand is no longer continuous in q. At the lowest $q$ such that the consumer pays the fixed fee, there is a discrete increase in the quantity
bought. At this marginal $q$ the firm's profit goes from zero to the amount of the fixed fee. Thus, welfare has a discrete increase at this point, though thereafter it is continuous and convex. As a result, welfare is no longer globally convex in $q$ even holding the fixed fee constant. In addition, an increase in risk affects the profit maximizing fixed fee creating additional effects that make welfare comparisons even more difficult to determine.

The following proposition summarizes these results.

Proposition 6: With two part tariffs, if the minimum quality for positive demand is greater than the unconditional average, the firm prefers an increase in risk. For any fixed two-part tariff, expected consumer welfare increases with an increase in risk. ${ }^{\square}$

Some informational results can be inferred when a firm uses two part tariffs and the representative consumer is risk neutral (see Schlee [19]) by considering how consumer welfare and profits depend on the timing of the observation of quality. That is, the realization of $q$ may be thought of as the updated belief as to the quality $q$. With the two-part tariff, if the consumer observes quality before paying the two-part tariff, then the consumer obtains positive surplus for some $q$ and never obtains negative surplus. In expectation the consumer has positive surplus. In contrast to the positive consumer surplus, consider what occurs if the consumer must pay the fix fee before observing the quality. If the consumer is risk neutral, then the firm sets the fixed fee to capture the expected consumer welfare. Similarly, if both the consumer and the firm observe the quality, then the firm sets

[^6]the two-part tariff to capture the consumer's entire welfare. In these latter two cases, expected consumer welfare is zero. Thus, the consumer values gaining the information that neither has, that is, observing the quality that the firm does not observe, but does not value gaining the information if the firm already has the information. Put differently, if it has the information it values preventing the firm from obtaining the information. The firm is indifferent between neither the firm nor the consumer learning or both learning. However, it is worse off if the consumer obtains the information. Society, however, prefers common knowledge: either both not know or both know. These simple results required some strong assumptions, i.e., risk neutrality or complete learning, which future research could relax.

## 5. Conclusion

There is an extensive literature examining research and development of new products. Typically in these models it is assumed that if the firm is successful, then its product's characteristics are deterministic. Instead, here I consider the stochastic properties of possible development projects and the effect of these stochastic properties on the firm's selection of projects. Under general conditions it was found that between two projects with the same expected quality, the firm would prefer the less risky project. Interestingly, this result is sensitive to an important cases: if the firm can sell only a product with a high realization of quality, then the firm prefers to develop the riskier one.

The model here is different than previous work since the uncertainty in the consumer's preferences is explicitly modeled in this paper. As a result, statements can be made regarding the welfare effects of risks. In general, the effect on consumer welfare from an increase in risk typically has the opposite sign of the effect on firm's profits. Thus, the general welfare effects from an increase in risk are ambiguous.

The results here also are used to extend some aspect of previous models of uncertainty. For example, Lazear [8] also examines optimal pricing when demand is uncertain. Lazear, however, is
interested in one-of-a-kind items, so that model is a special case of the model here. Also, stock-outs (the sole unit is sold) are a feature of Lazear's model, but not here. The results here also extend work that explores the value of "variability," e.g., Turnovsky, et al. [22] by introducing uncertainty into preferences rather than assuming random output.

Recently, Schlee [19] and Kehoe [6] have explored other effects from there being uncertainty regarding product quality. Schlee considers the value of information that is publicly observed by both the firm and buyers, before any decision is made. Therefore, unlike here, the monopolist's demand curve is non-stochastic when it sets its price since consumers' preferences are fixed as the common beliefs about quality have been updated. Kehoe [6] focuses instead on a firm's optimal pricing when consumers do not know the quality of the product. Kehoe derives general conditions for the price to decrease as consumers become less certain (but does not examine welfare). Although consumer, instead of producer, uncertainty about quality is examined, a firm's price is more likely to decline in consumers' uncertainty, the smaller the initial uncertainty. Thus, starting with certainty, the introduction of some consumer (Kehoe) or producer (here) uncertainty can lead a firm to reduce its price.
[1] D. Blackwell, Equivalent comparisons of experiments, Annals of Mathematical Statistics, 24 (1953) 265-272.
[2] L. Eeckhoudt, C. Gollier, and H. Schlesinger, The no-loss offset provision and the attitude towards risk of a risk-neutral firm, Journal of Public Economics, 65 (1997) 207-17.
[3] J. Jaskold Gabszewicz and J.-F. Thisse, Price competition, quality and income disparities, Journal of Economic Theory, 20 (1979) 340-359.
[4] J. Jaskold Gabszewicz and J.-F. Thisse, Entry (and Exit) in a differentiated Industry, Journal of Economic Theory, 22 (1980) 327-338.
[5] M.S. Kimball, The effect of demand uncertainty on a precommitted monopoly price, Economic Letters, 30 (1989) 1-9.
[6] M.R. Kehoe, Quality uncertainty and price in monopoly markets, Journal of Industrial Economics, 44 (1996) 25-32.
[7] H.E. Leland, Theory of the firm facing uncertain demand, American Economic Review, 62 (1972) 278-291.
[8] E.P. Lazear, Retail pricing and clearance sales, American Economic Review, 76 (1986) 14-32.
[9] E. Maskin and J. Riley, Monopoly with incomplete information, Rand Journal of Economics, 15 (1984) 171-196.
[10] B.F. Massell, Price stabilization and welfare, Quarterly Journal of Economics, 85 (1969) 284-298.
[11] P. Milgrom and J. Roberts, Price and advertising signals of product quality, Journal of Political Economy, 94 (1986) 796-821.
[12] L.J. Mirman, Uncertainty and Optimal Consumption Decisions, Econometrica, 39 (1971) 179-85.
[13] M. Mussa and S. Rosen, Monopoly and product quality, Journal of Economic Theory, 18 (1978) 301-317.
[14] W. Oi, The desirability of price instability under perfect competition, Econometrica, 29 (1961) 5864.
[15] S. Rosen, The Economics of superstars, American Economic Review, 71 (1981) 847-858.
[16] M. Rothschild and J.E. Stiglitz, Increasing risk: I, a definition, Journal of Economic Theory, 2 (1970) 225-243.
[17] M. Rothschild and J.E. Stiglitz, Increasing risk: II, its economic consequences, Journal of Economic Theory, 3 (1971) 66-84.
[18] A. Sandmo, On the theory of the competitive firm under price uncertainty, American Economic Review, 61 (1971) 65-73.
[19] E. Schlee, The value of information about product quality, Rand Journal of Economics, 27 (1996) 803-15.
[20] A. Shaked and J. Sutton, Relaxing price competition through product differentiation, Review of Economic Studies, 49 (1982) 3-13.
[21] A. Shaked and J. Sutton, Natural Oligopolies, Econometrica, 51 (1983) 1469-1484.
[22] S.J. Turnovsky, H. Shalit, and A. Schmitz, Consumer's surplus, price instability, and consumer welfare, Econometrica, 48 (1980) 135-152.
[23] F.V. Waugh, Does the consumer benefit from price instability? Quarterly Journal of Economics, 58 (1944) 602-614.
[24] G. Zhang, Ownership Concentration, Risk Aversion and the Effect of Financial Structure on Investment Decisions, European Economic Review, 42 (1998) 1751-78


[^0]:    ${ }^{\dagger}$ I am grateful to J. Meyer, W. Mullin, E. Schlee and J. Wooldridge for their helpful discussions and participants at the Michigan State Theory seminar, the Midwest Mathematical Economics meetings and the Southeast Theory meetings for their comments on a preliminary version of this paper. I thank L. Mirman and T. Jeitschko for their helpful comments and suggestions on this revision. All errors are mine.

[^1]:    ${ }^{1}$ Previous work has relied on strictly convex cost to create effects from demand uncertainty. That is, with the assumption of linear costs, as is assumed here, their results would disappear.

[^2]:    ${ }^{2}$ This is a stronger assumption than for proposition 1 as there is the additional term of the lower limit which is positive: $\left(1 / \theta_{\max }\right) \cdot \mathrm{f}\left(\theta_{\max }\right) \cdot\left(\theta_{\max } / \mathrm{p}\right) \cdot(\mathrm{p}-\mathrm{c}) \cdot \mathrm{g}\left(\mathrm{p} / \theta_{\max }\right)$.

[^3]:    ${ }^{3}$ Note that the derivative at the lower limit, is evaluated at the lower limit, and so, is zero $\left(1-\mathrm{F}\left(\theta_{\max }\right)=0\right)$.

[^4]:    ${ }^{4}$ Note that at the lowest $q, q=p / \theta_{\min }$ demand is zero and continuous as $q$ increases.

[^5]:    ${ }^{5}$ This also corresponds to the quadratic utility case in Schlee [19].
    ${ }^{6}$ Note that, defining $\mathrm{x}^{\prime}$ as $\partial \mathrm{V}\left(q, x^{\prime}\right) / \partial x=0$, the consumer's value is $\left.\int_{0}^{x}[\partial \mathrm{~V}(q, t) / \partial x] \mathrm{dt}=\underset{x^{\prime}-x(q, 0)}{\int_{x^{\prime}-x(q, p)}[\partial \mathrm{V}}(q, t) / \partial x\right] \mathrm{dt}=$ $\left.\underset{x(q, 0)}{x(q, p)} \int_{x}^{x}\left(q, x^{\prime}-u\right) / \partial x\right](-1) \mathrm{du}=\int_{x(q, p)}^{x(q, 0)}\left[\partial \mathrm{V}\left(q, x^{\prime}-u\right) / \partial x\right] \mathrm{du}$ which is consumers' welfare in the previous sections.

[^6]:    ${ }^{8}$ Another issue is whether the buyer pays the fixed fee before the buyer observes $q$. For example, a consumer may pay to join a food club before learning the true quality of products, which is observed before buying. This is important because the buyer now pays the fixed fee if it is less than his expected surplus from buying the product at the contract price. As a result, if the buyer is risk neutral, then the firm can set its fixed fee equal to the expected consumer welfare for a given price. It can be shown that it is still optimal to set the per-unit price to marginal cost. With the per-unit price equal to marginal cost $(p=c)$, the optimal fixed fee, $A$, equals expected consumer welfare given this price, $A^{*}=\mathrm{E}[S(c, q)]=\mathrm{E}\left[\mathrm{V}\left(q, x^{*}\right)-c \cdot x^{*}\right]$ since the consumer is risk neutral. By proposition 3, since the per-unit price equals marginal cost, the firm prefers an increase in risk since it is capturing expected welfare.

