A note on uncertainty and socially excessive entry*

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It is well known that under general conditions entry into imperfectly competitive markets usually is excessive. This note explores the effects of uncertainty on this result. That is, randomly some entrants who incur entry costs fail to enter a market. It is found the previous conditions may not hold when there is uncertainty. That is, with uncertainty entry may be socially <u>insufficient</u> by more than one firm. For example, in equilibrium four firms may attempt to enter, when it is socially optimal for seven firms to attempt to enter the market.

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1. Introduction

Introducing a product into a new market is typically a risky proposition. That is, many attempts to develop a new product fail. New technology sectors in particular are known for failed attempts at developing and introducing new products. Perhaps in response to this perceived risk, policy makers seem to believe that there is a need to subsidize the development (i.e., entry) of these new products. For example, President Bush has made budget proposals to give R&D tax credit for new technologies at roughly $6 billion a year (San Francisco Chronicle, 2001).

In contrast, in economic theory it is a well known that, outside of perfect competition, entry into markets usually is excessive as compared to the welfare optimum (e.g., Weizsäcker 1980 and Suzumura and Kiyono 1987). In particular, Mankiw and Whinston (1986) show that under very general conditions, entry into imperfectly competitive markets is usually excessive. This seemingly counter-intuitive outcome arises because with imperfectly competitive markets, much of a new entrant's sales come from the other firms contracting their output in response to the entry. In contrast, the social value of entry is derived from the increased in aggregate output it causes. Thus, the entrant's (private) incentive to enter is typically greater than the social incentive. That entry is generally excessive suggests that taxing entry, or using some other policy instrument that reduces the amount of entry, will generally raise welfare.

On the other hand, these models all assume a world of certainty – a firm is guaranteed to succeed in its entry attempt. Yet work examining investment in R&D under uncertainty also has found a similar result. For example, while many of the results in the influential papers by Loury (1979) and Lee and Wilde (1980) differ, both find that there is excessive investment in a winner-take-all competition. Combining these results in R&D under uncertainty with those regarding excessive entry would seem to imply that markets in which firms must make a risky fixed investment to enter are characterized by excessive entry. That is, policy makers have the optimal plan backwards.

This note examines whether introducing uncertainty can lead to too little entry. Contrary to previous results, a simple condition is derived under which there is insufficient rather than excessive entry. Further, the analysis reveals that a caveat to the previous excess entry results plays a new role with uncertainty. Specifically, with certainty a monopolist does not capture the entire social surplus its entry creates. Thus, it is possible that there can be insufficient entry rather than having excess entry. That is, there exist entry costs in which no firm would enter but social welfare would be greater if one firm entered. This is referred to as the "integer effect" – if it were possible for a fraction of a firm to enter, it would and it would be too large of a fraction relative to the social optimum. It turns out that under uncertainty, there can be several firms too few entering a market because the entrants cannot capture enough of the expected surplus if they succeed. The upshot is that the certainty result can be modified to a similar result with uncertainty: the <u>expected</u> number of successful entrants as compared to the expected number of successful entrants under the social optimum, is insufficient by at most one expected firm. Though the original result holds by modifying it in terms of the expected number of successful firms, the implication of discouraging entry does not, as the number <u>attempting</u> entry (as opposed to the number that succeed) may be well below the welfare optimum, e.g., more than two too few attempting to enter.

One aspect of this result is that insufficient entry is more likely the riskier an industry is. That is, as the probability of success decreases, the extent that entry is socially insufficient increases. If, as conventionally thought, new technologies and developing a new product are riskier than imitation, then these are the types of industries whose investment – research and development – should be encouraged while more established industries' investment should not. Thus, the results here partially reconcile the conflict between common beliefs regarding new industries and the well known excess entry result.

The effects of entry under uncertainty have been previously examined, most extensively in an important paper by Quirmbach (1993). In particular, Quirmbach compares the welfare of different types of competition among successful entrants. However, because Quirmbach's work ensued from a different literature that was unaware of the relevance of the excess entry result (which was not cited), it focuses on other issues and does not examine if the excess entry result holds. That is, the essential question of the excess entry literature, whether there could be two (or more) firms too few, is not addressed in Quirmbach.

2. The Model

The model has two stages. In stage one, a large number of firms choose whether to invest i to enter a new market. There is a probability ρ that each will succeed. If a firm succeeds, then in stage two it can sell its product. If it fails, then it cannot sell. In the second stage the m firms that succeed compete in Cournot fashion. Let q_k denote the output of the k^{th} successful firm. Each firm has an identical cost function of the form $c \cdot q^2$. The inverse demand for the product is represented by $P = A - bQ(m)$, where $Q(m) = \sum_1^m q_k$. It follows that consumer surplus is $C = bQ^2/2$. Straightforward calculation obtains that the symmetric equilibrium output is $q_k(m) = A(b+c)/[b(m+1)+2c]$, the price is $P(m) = A(b+2c)/[b(m+1)+2c]$, profits are $\pi_k(m) = A^2(b+c)/[b(m+1)+2c]^2$, and consumer surplus is $C(m) = 0.5bm^2A^2/ [b(m+1)+2c]^2$. Let $\boldsymbol{n^e}$ denote the number of firms that enter in equilibrium and $\boldsymbol{n^*}$ denote the socially optimal number of firms.

Mankiw and Whinston consider a generalized version of this model, except that entry is certain. They showed that the equilibrium amount of entry is usually socially excessive if in the second stage firms compete strategically. Modified for the model here, it can be stated as, for n entrants that succeed with probability one:

$$\textbf{If } Q(n')>Q(n), \lim Q(n)=\mathrm{N}<\infty;\ q(n')<q(n);\ P(n)>\ 2cq(n),\ \forall n'>n, \textbf{ then } n^e \geq n^*-1. \qquad (1)$$

The condition on aggregate output is to insure that entry raises output and the entry equilibrium is well defined. The condition on firm output is that a business stealing effect occurs; entry reduces each firm's output. Finally there is a condition that price is above marginal cost.

The issue here is if introducing uncertainty can change this result, i.e., in contrast to Mankiw and Whinston's result is it possible that $\boldsymbol{n}^*$ (the socially optimal number of firms attempting entry) be greater than $\boldsymbol{n}^e$+1 (the equilibrium number plus one)? To fix this idea consider an example: could the equilibrium under uncertainty be characterized with, say, five firms entering ($\boldsymbol{n}^e$ =5), but the welfare optimum would have more seven or more firms enter? Such a case would "violate" Mankiw and Whinston's proposition. This would require establishing a condition under which a firm's expected profit with n entrants is less than the expected welfare gained from the n^{th}+1 entrant. Since the expected profit from entry is strictly positive if investment costs are zero (i=0), it then immediately follows if this condition holds, then there exist investment costs i such that at most $n-1$ firms enter in equilibrium, but at least n+1 would be the welfare optimal number of entrants. Returning to the example, if the expected profits for the sixth entrant is less than the increase in welfare from the seventh firm entering, then there exists an i such that the sixth firm does not enter (i.e., at most only five firms enter), but welfare still increases with the seventh firm entering. To determine this, we need the expressions for expected profits given n firms (a level) and welfare gain from an *additional* firm (a change in level).

Firm k's expected profit if it is the nth (for the example, n=5) entrant is

$$E[\pi_k(n)] = (1-\rho)\cdot 0 + \rho \cdot \sum_{m=0}^{n-1} \frac{(n-1)!}{m!(n-1-m)!} \rho^m (1-\rho)^{n-1-m} \left[\frac{A^2(b+c)}{(b+b(m+1)+2c)^2} \right] - i.$$

(2)

The first term is the firm's probability of failure (1–ρ) times its profit (zero). The second term is the firm's probability of success (ρ) times its expected profits given that it succeeds, which depends on the number of other entrants that succeed (x). Specifically, the term $\rho^m(1-\rho)^{n-1-m}$ $(n-1)/m!(n-1-m)$ is the probability of m other firms succeeding. Finally, note that the random variable m appears in the denominator, and so there is no simplified form for the expectation. As a result, there is not an explicit function for the n such that the RHS of (2) equals zero. That is, there is not an explicit function for the equilibrium n (which is also a difficulty in Quirmbach 1993, etc.). Further, the derivative of the RHS of (2) is not well defined and so comparative statics cannot be done (e.g., via the implicit function theorem).

The welfare from m firms producing is $m\pi_k(m) + C(m) = mA^2(b+bm+2c)/2(b+bm+2c)$. As the question is when less than n firms would enter in equilibrium, how many more would be socially optimal, let y be an additional number of entrants beyond the nth entrant and let n_y denote the $n + y^{th}$ firm. The welfare gain from the n_y^{th} firm ($n + y$ firms) entering is

$$E[\Delta W(n_y)] = \sum_{0}^{n_y} \frac{n_y!}{m!(n_y-m)!} \rho^m (1-\rho)^{n_y-m} \frac{mA^2}{2} \frac{2b+bm+2c}{(b+bm+2c)^2} -$$

$$\sum_{0}^{n_y-1} \frac{(n_y-1)!}{m!(n_y-1-m)!} \rho^x (1-\rho)^{n_y-1-m} \frac{mA^2}{2} \frac{2b+bm+2c}{(b+bm+2c)^2} - i, \qquad (3)$$

with the terms following analogously to those in the expected profit expression.

Returning to the example, Mankiw and Whinston's condition does not hold under uncertainty if the profits for the sixth firm $E[\pi(6)]$ are less than the welfare game of the seventh entrant. That is, if $E[\Delta W(6_1)] > E[\pi(6)]$, then there exist investment costs $i \in (E[\pi(6)], E[\Delta W(6_1)])$ such that the welfare optimal amount of entry is at least seven firms, while only five firms enter in equilibrium. More generally, whenever it is true that $E[\Delta W(n_y)] - E[\pi(n)] > 0$ for $y \geq 1$, then there exist i that contradicts Mankiw and Whinston's proposition, i.e., such that at least two firms too few attempt to enter the market. A second question to be ask is how large could y be, that is, by how much could entry be socially insufficient.

Unfortunately, as with expected profits, a random variable is in the denominator of expectation of welfare; it is a hypergeometric function. As a result, the RHS of (3) is not differentiable either, nor does there exist an explicit solution for n when differencing the RHS of (3) to determine the optimal n. For the analysis then, two different lines of attack are taken to characterize the outcome. First, a model of ex post competition that does not generate a hypergeometric function is considered: the unit-output model which is describe below. This model is approximately the limit case of the Cournot model as c becomes large and has the attractive property that an explicit solution exists for either the welfare maximizing n or equilibrium entry n. However, this model does not strictly meet Mankiw and Whinston's condition. Nevertheless, with the results of the limit case, the Cournot competition model can then be reconsidered for a range of n and y to see if the result in the unit output model extends to Cournot competition. Finally, examples are considered to show that the results hold approximately even for small c (i.e., $c = 1$ or less).

Consider then a model in which each successful firm can sell one unit on the market or none. The total number of units that firms supply is cleared at the efficient price. With this unit-output model and linear demand we have $P = A - bm$, $Q(m) = m$, $\pi_k(m) = A - bm$, $C(m) = 0.5bm^2$, and so W(m) = $Am - bm^2/2$. It is straightforward to show that under certainty, the free entry equilibrium equals the socially optimal level of entry. Hence, under certainty Mankiw and Whinston's result (weakly) holds. This is intuitive as this model's difference from Mankiw and Whinston's conditions is that the business stealing effect condition is not met (the business stealing effect only weakly holds here instead of strictly).

With uncertainty, the RHS of (2), the expected profits for the n^{th} entrant becomes

$$\mathrm{E}[\pi_k(n)] = (1-\rho)\cdot 0 + \rho\cdot\sum_{m=0}^{n-1}\frac{(n-1)!}{m!(n-1-m)!}\rho^m(1-\rho)^{n-1-m}\left[A-b-bm\right]-\mathrm{i}. \tag{4}$$

The RHS of (4) simplifies to $\mathrm{E}[\pi_k(n)] = \rho[(A - b - \rho b(n-1)] - i$. For the welfare optimum, the RHS of (3) becomes:

$$\mathrm{E}[\Delta W(n_y)] = \sum_{0}^{n+y}\frac{(n_y)!}{m!(n_y-m)!}\rho^m(1-\rho)^{n+y-m}[Am-bm^2]-$$

$$\sum_{0}^{n_y-1}\frac{(n_y-1)!}{m!(n_y-1-m)!}\rho^m(1-\rho)^{n+y-1-m}[Am-bm^2]-i. \tag{5}$$

The RHS of (5) simplifies to $\rho[A - b(1/2 - \rho(n + y - 1))] - i$. Algebraic manipulation of $\mathrm{E}[\pi_k(n)]$ and $\mathrm{E}[\Delta \mathrm{W}(n_y)]$ obtains immediately the probability ρ needed such that $\mathrm{E}[\Delta \mathrm{W}(n_y)] > \mathrm{E}[\pi_k(n)]$:

Proposition 1: In the unit output model with uncertainty, if $\rho < 1/2y$, then there exists A and i such that it is socially optimal for $n + y$ firms to enter but only $n - 1$ firms enter in equilibrium.

Thus, in this model with uncertainty the result that $n^e \geq n^* - 1$ does not hold. Specifically, if $\rho < ½$ then there exist parameter values such that $y \geq 1$: only $n - 1$ firms enter, but it is socially optimal that $n + 1$ firms enter. That is, $n^e < n^* - 1$. For example, as $\rho \to ½^-$, there exist parameters values where six firms enter a market in equilibrium, but it is socially optimal for eight to enter. Likewise as $\rho \to \frac{1}{4}^{-}$, there exist parameters such that only one firm enters the market, but it is socially optimal for four to enter the market. Finally, note that the restriction on A is only to rule out corner solutions.

That it takes sufficient uncertainty (small enough ρ) for Mankiw and Whinston's result not to hold in this setting of uncertainty accords with intuition; as $\rho \to 1$ the model approaches the certainty model. The implication is that very "risky" markets, i.e., ones in which very few investors succeed, is most likely to be characterized by insufficient entry even though the firms are profit maximizes. Note also that the condition is independent on the equilibrium number of entrants.

While for this simple model the condition is intuitive, whether it holds in the Cournot model is examined next. To do this requires a case by case analysis of $E[\Delta W(n_y)] - E[\pi_k(n)]$, that is, solving for ρ such that $E[\Delta W(n_y)] - E[\pi_k(n)] = 0$ for each n and y. Not surprisingly, if c is sufficiently large, the critical ρ approaches $1/2y$:

Proposition 2: With Cournot competition and uncertainty, for $n \in \{1,\ldots,20\}$ and $y \in \{1,\ldots,20\}$, if $\rho < 1/2y$, then for sufficiently large c there exists A and i such that it is socially optimal for $n + y$ firms to enter but only $n - 1$ firms enter in equilibrium.

Although the result has the requirement of sufficiently large c, for small c the condition $1/2y$ works as a fair approximation. To see this consider the following examples in which $A = 10$ and $b = 1/10$. The proposition states that ion 2, Mankiw and Whinston's result fails under uncertainty ($y = 1$) for ρ arbitrarily close to .5, but if $c = 1$, the condition can fail if $\rho = .46$. Similarly, for it to be socially optimal that four enter when only one enters in equilibrium, the proposition requires sufficiently large c for $\rho < .25$: with $c = 1$, ρ must be less than .24. However, as n increases larger c is required. For example, for there to exist an i such that ten firms enter in equilibrium but it is socially optimal for twelve to enter requires that $\rho \leq 1/3$. For it to be socially optimal for thirteen to enter requires $\rho \leq .2$. This is consistent with the hypothesis that the uncertainty amplifies the integer problem; all else equal, under certainty increasing c makes the integer problem more likely to occur. However, note that this does not imply that there must be in expectation a monopoly ex post. For instance, consider the example above in which ten firms enter in equilibrium and $\rho = 1/3$, in the entry equilibrium the expected ex post market structure has three firms. Finally, the condition also suggests that a simple modification can be made to the Mankiw and Whinston result for it to hold with uncertainty: that the result is stated in terms of <u>expected</u> number of successful entrants. That is, it is true that $E[n^e] \geq E[n^*] - 1$. However, the implications in terms of the optimal policy for entry are very different.

3. Conclusion

It is a well-known result that in imperfectly competitive markets socially excessive entry is the norm and at most there can be one firm too few entrants. This result, though, is in an environment of certainty. This notes shows that if entry is an uncertain process then the result is weakened. Under certain conditions entry is socially insufficient (as opposed to be excessive) by more than one firm. For example, a welfare planner could want four firms to enter a market when only one does so in equilibrium. The analysis here also suggests that the original result with certainty holds with uncertainty if it is modified to be in terms of <u>expected</u> number of successful entrants, instead of the number of entrants. However, since the number attempting entry can still be too small, the standard view of excessive entry may need to be ameliorated when there is uncertainty.

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