Productivity information in vertical sharing agreements†

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That firms share information with their rivals is both well-known and studied. However, firms also often share information with their suppliers. The firm’s incentive to share productivity information with both its supplier and its rival is examined. It is found that by sharing productivity information a firm raises its own expected input price, which raises the rival’s expected input price. In contrast, sharing cost or demand information does not have this effect. Through this price effect information sharing can increase expected producer surplus in price competition, while in previous work the sharing of cost information always reduced producer surplus.

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It is well-known that firms systematically share information with their rivals,\(^1\) which has been thoroughly explored both theoretically and empirically.\(^2\) It is also well-known that firms will simultaneously share financial and productivity data with their input suppliers. For example, firms regularly share information with their unions: Fuess and Millea (2002) report that two-thirds of manufacturing establishments in Japan, and fifty-six percent of all establishments, share information, including profitability and productivity, with their unions. Likewise, Lee and Whang (2000) extensively catalog firms that share information with input suppliers, including suppliers with market power such as Procter & Gamble and Warner-Lambert. Moreover, there are many different types of information shared as, for instance, Chrysler shares the quality data of all its parts with all of its suppliers (Lee and Whang 2000).\(^3\) Finally, firms of course also disclose critical information to their banks. Not surprisingly, the effects of such a choice have been empirically studied in the labor, supply-chain and banking literatures.\(^4\)

Firms also share information to suppliers by using third parties to publicly disclose the information to all. A particularly interesting example is in the auto industry, where “every domestic and major foreign-owned” auto manufacturer in the U.S. discloses productivity information regarding “each vehicle, stamping and power-train plant” in the Harbour Report (www.harbourinc.com). More generally, antitrust policy sometimes requires nonexclusionary disclosure from the trade association and so to join a trade association requires that the firm disclose

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\(^1\) See Vives (1990) for an overview of information sharing in the US and OECD as well as antitrust policies with respect to them.


\(^3\) In addition to direct revelation, data can be shared by outsourcing its distribution to a third party, for example this practice has been adopted by Apple Computer’s plants in the western U.S. and some of their Asian suppliers, with Fritz Companies serving as the third party (Lee and Whang 2000).

\(^4\) The impact of this type of sharing has been studied empirically either by itself (e.g., Kleiner and Bouillon 1988, and Morishima 1991a, 1991b who find a positive effect on labor productivity) or as part of a set of explanatory variables (e.g., Ichniowski, et al. 1997), and has been discussed as a potential confounding factor in studies on firm behavior (e.g., Kelly 2004). In the supply-chain literature see Chen’s (2003) Handbook chapter on information sharing in the supply-chain. For examples in banking, see Herrera and Minetti (2006) and citations therein.
information to all (see the discussion in Vives 1990).

Firms may also indirectly share this information to the supplier when it discloses to a rival or some other firm. For example, the information may be sold for profit\(^5\) or the rival may attempt to use the information strategically to negotiate a better deal.\(^6\) Such a leakage of information may also be inadvertent – the inevitable consequence of the everyday interactions between the rival and the supplier or more specifically, implicitly communicated by the rival in its negotiations with the supplier over the input prices.\(^7\) Similarly, the intentional leakage of information by suppliers to the rivals is well-known and studied in various literatures including R&D (see the discussion in Bönte and Wiethaus 2005), supply-chain (see, e.g., Anand and Goyal 2004) and banking (see, e.g., von Rheinbaben and Ruckes 2004, and citations within).

Despite suppliers often being part of a firm’s decision to share information, with the notable exceptions of Li (2002) there has been little analysis of suppliers in previous theoretical work. In this pioneering research Li (2002) examined the incentives for firms to share cost or demand information with a monopoly supplier and (implicitly) its rivals. This model is extended to price competition for the common value case in Zhang (2002). More recently, Creane (2005) has examined the effect of discriminatory pricing and bargaining power on the incentive to share cost information. While cost and demand information are the standard variables in the information sharing literature (i.e., between horizontal rivals only), this paper considers instead the firm’s incentive to share information regarding a variable that is commonly studied in vertical relationships (including many of the examples above): productivity information.

At first glance, the distinction between cost and productivity information may seem trivial.

\(^5\) For example, Wal-Mart ceased selling their information to a third party because without permission they resold the information and Newbury Comics had a similar problem (Anand and Goyal 2004).
\(^6\) Of course, when the rival strategically chooses not to reveal the information, the supplier still infers information, which could lead to a complete revelation equilibrium (Okuno-Fujiwara et al. 1990).
\(^7\) For example, the price it offers during a bargaining round. This point as it applies in the context of the model presented here is considered in more detail in section 5.
Indeed, if the effect of the supplier learning was eliminated, e.g., the input was supplied competitively, then it can be shown that the distinction is trivial: the results with productivity uncertainty are identical to the classic results with cost uncertainty. Furthermore, in Li (2002) the effects of a firm sharing cost information with its monopoly supplier as well as its rivals has little qualitative effects relative to the classic results which only have the rival learning. Hence, one might expect that examining the sharing of productivity information with a monopoly supplier and its rival would have no new effects.

Despite this intuition, there are new results with productivity information. First, agreeing to share productivity information raises the firm’s expected input price. In contrast, in Li (2002), Zhang (2002) and Creane (2005) where cost and demand uncertainty are examined, it had no such effect. The reason for this difference is because productivity shocks, unlike cost shocks, enter the supplier’s profit in a second way. Productivity shocks affect both the equilibrium level of output (as do cost shocks) and the amount of input needed per unit of output (unlike cost shocks). The second result is a consequence of the first: sharing productivity information has a “raising the rival’s costs” effect as it increases the rival’s expected input price. These effects on the expected input price are independent of whether the firms compete in prices or quantities.

The third new result is that information sharing affects the expected outputs. In contrast, in Li (2002), Zhang (2002) and Creane (2005) as well in the classic sharing models such as Vives (1990) in which there is no supplier, the expected equilibrium outputs are not affected by an information sharing agreement. This is because, as Vives (1990) observes, quadratic models with linear uncertainty – such as the classic model of information sharing and the model here – have certainty equivalence with respect to output and price. Moreover, this effect on the expected output is not driven simply by the introduction of an input supplier, since the certainty equivalence of output also holds in Li (2002), Zhang (2002) and Creane (2005).

Since information sharing raises both the firm’s and its rival’s expected price, the net effect
on output is not immediate. However, information sharing has a greater affect on the firm’s input price than on the rival’s, and so it decreases the firm’s expected output while weakly increasing the rival’s. Further, the total effect when both firms share is that expected outputs decrease. That is, expected output with complete information is less than that with incomplete information. These effects on output are independent of whether the firms compete in prices or quantities, while, in contrast, in previous work results often depended on the type of strategic competition.

The effect information sharing has on the expected input price can also affect the firm’s decision to share. Specifically, another new result is that in price competition producer surplus (supplier and downstream firms’ profits) increases with information sharing if the products are sufficiently undifferentiated. This is in contrast both to the firm’s incentive to share private cost information in the classic information sharing literature (Gal-Or 1986), i.e., without the supplier learning, and in Creane (2005) in which a price setting supplier also learns. That is, in both Gal-Or (1986) and Creane (2005) producer surplus always decreases with information sharing. Thus, if suppliers are allowed to pay up-front fees – and they do, for example, in retailing – then the supplier can compensate the downstream firms sufficiently to induce them to share productivity information.⁸

This increase in producer surplus is driven essentially by two effects: information sharing increases the expected downstream price and, seemingly contradictorily, the classic inefficiency in vertical relationships known as double marginalization (i.e., both the downstream firms and input supplier pricing above marginal cost). With complete information the double marginalization results in the downstream prices being greater than the prices that maximize producer surplus. Hence, since information sharing increases the expected prices, the incomplete information prices may generate greater producer surplus. However, there are two counter effects. First, the equilibrium prices with

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⁸ The practice of slotting fees – producers paying grocery stores up front fixed fees – has become an increasingly contentious issue (see FTC 2003). See also Mills (2004) for other examples in which retailers pay a price to the manufacturer with the manufacturer making a reciprocal fixed-type payment to the retailer.
incomplete information can be less than the prices that would maximize producer surplus. Second, as the products become less differentiated (downstream competition increases), the loss from double marginalization effect is reduced. As a result, if the products are sufficiently undifferentiated, then there is a gain from information sharing through this price effect that can overwhelm the loss normally generated from information sharing between the downstream firms (e.g., Gal-Or 1986). The role of downstream competition on these results implies that the type of competition (price or quantity) could affect the results. And in fact, if the firms compete in quantities, rather than prices, producer surplus (firms and supplier profits) always decreases with information sharing. In contrast, with cost uncertainty both when a price discriminating supplier also learns (for sufficiently homogenous goods Creane 2005), as well as in the classic literature (Gal-Or 1986), i.e., without the supplier learning, producer surplus increases with information sharing.

In the next section the basic environment is introduced. To ease comparison to previous work, the timing and structure assumed mimics the standard model in information sharing and in particular Li (2002). In section three, the market is characterized for when the firms compete in quantities and in section four price competition is considered. Section five considers two extensions. The first extension examines the supplier’s price setting power finding that as the firms have more bargaining power, information sharing is more likely in quantity competition. The second extension explores what occurs if a firm discloses to the supplier but not its rival. Section 6 concludes.

2. Environment

There is a monopoly input supplier that sells a necessary input for a downstream duopoly. It should be clear that the monopoly supplier could be a union and the input labor. The input supplier sets the input price, which for ease is referred to as the wage. All three firms maximize expected profits. There is uncertainty about each downstream firm’s productivity parameter and the issue here is each firm’s incentive to agree to share any information it may obtain in the future regarding this
parameter with the other firms.

The interaction is modeled in four stages with assumptions made to parallel the structure of previous work (Li 2002, Zhang 2002). In the initial stage each firm, before learning its parameter’s value, simultaneously and independently chooses whether to enter an enforceable contract to share with the supplier and its rival any information that it may learn. In the second stage, nature randomly chooses the parameters’ values and reveals to each firm only its own value. If a firm entered the contract, it discloses this information to the supplier and its rival. In the third stage, the supplier sets the wage for each firm, which is observed by both firms. In the final stage, the firms simultaneously and independently compete in a strategic variable.

Each firm has a specific productivity parameter \( \lambda_j \) \((j = a, b)\) that defines how many units of the input are needed to produce one unit of the good. Hence, productivity is decreasing in \( \lambda \). Initially the value of \( \lambda_a \) and \( \lambda_b \) are unknown but are independently drawn from a known distribution \( F_j(\lambda) \) on support \( [\lambda_{j1}, \lambda_{j2}] \) with the marginal density \( f_j(\lambda) \) and \( \lambda_{j2} \gg 0 \). Denote \( E[\lambda_j] = \lambda_{je} \) and \( \sigma_j^2 = E[\lambda_j^2] - \lambda_{je}^2 \).

For notational ease the mean and variance are assumed equal: \( \lambda_{je} = \lambda_{je} = \lambda_e \) and \( \sigma_j^2 = \sigma_e^2 = \sigma^2 \).

Let \( w_i \) be the wage the supplier sets for the input. Firm \( i \)'s cost function, then, is \( \lambda_i w_i q_i \). Thus, profits are \( \pi_i = (p_i - w_i \lambda_i)q_i \). Since the interest here is in an active oligopoly, it is assumed that the values on the parameters are such that outputs are positive in equilibrium. The supplier’s cost of supplying the input is normalized to zero. The suppliers’ profit then is \( \omega = w_a \lambda_a q_a + w_b \lambda_b q_b \).

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9 Many variations of the informativeness of the signal the firm receives or chooses to transmit have been examined in the information sharing literature without altering the basic results (see, e.g., Gal-Or 1986).

10 While the Robinson-Patman Act circumscribes differential pricing, since the 70’s, the number of cases brought by the FTC have declined to zero and the courts have made it increasing difficult for plaintiffs to succeed so that it is now an example of progressive contraction (Kovacic 2003). Specifically, since the Supreme Court ruling in Brooke (1993), which was brought under Robinson-Patman, “no predatory pricing plaintiff has prevailed on the merits in the federal courts” (Bolton, et al 2000). Some have argued (Stoll and Goldfein 2005) that Robinson-Patman will be further restricted in Volvo Truck which is currently before the Supreme Court. In addition, unions are not covered by Robinson-Patman so that different wage agreements can be reached.
3. Quantity competition/strategic substitutes

Consider first the market when the firms compete in quantities. Inverse demand is assumed linear: \( p_i = \alpha - q_i - \delta q_j \). Thus, profits are \( \pi_i = (\alpha - q_i - \delta q_j - w_i \lambda_i)q_i \). The standard approach is used to derive the equilibrium: first the equilibrium in the last stage is characterized for all possible information structures and then each previous stage is derived.

3.A The fourth stage

In the fourth (last) stage each firm simultaneously sets output. From the maximization of \((\alpha - q_i - \delta q_j - w_i \lambda_i)q_i\), firm i’s output (best response) given \( q_j \) is

\[
q_i = (\alpha - \delta q_j - \lambda_i w_i)/2. \tag{1}
\]

The equilibrium outputs will depend on the firm’s and its rival’s information. As a result, there are four structures to be characterized: when both firms shared information in the second stage, when neither did and when one shared but the other did not.

When both firms share information, the game is a standard Cournot game as each firm knows the other’s productivity and wage. From (1) the equilibrium outputs are:

\[
q^C_a(w_a, w_b) = [\alpha \gamma - 2\lambda_a w_a + \delta \lambda_b w_b]/\Delta, \quad q^C_b(w_a, w_b) = [\alpha \gamma - 2\lambda_b w_b + \delta \lambda_a w_a]/\Delta,
\]

where the C superscript represents complete information, \( \gamma \equiv (2 - \delta) \) and \( \Delta \equiv 4 - \delta^2 \). The equilibrium outputs are analogous to the case when there are uncertain costs, with marginal costs replacing \( \lambda_i w_i \). Profits and producer surplus (supplier and downstream profits) in the fourth stage are

\[
\pi^C_a = (\alpha - q^C_a - \delta q^C _b - w_a \lambda_a)q^C_a, \quad \omega^C = w_a \lambda_a q^C_a + \lambda_b w_b q^C_b, \quad \text{etc.}
\]

When neither firm shares information, the game is a standard Cournot game of incomplete information with respect to the other’s productivity (instead of what is typically examined, cost). Based on each firm’s information, the Bayesian-Nash equilibrium outputs are

\[
q^I_a(w_a, w_b) = [\alpha \gamma - 2\lambda_a w_a + \delta \lambda_a w_b]/\Delta - \delta^2(\lambda_a - \lambda_a) w_a/2\Delta,
\]
\[ q_b^f(w_a, w_b) = \frac{[\alpha \gamma - 2\lambda_b w_b + \delta \lambda_c w_a] \Delta - \delta^2 (\lambda_c - \lambda_b) w_b}{2\Delta}, \]

with the profits \( \pi_a^f \), etc. following analogously to when there is complete information.

Finally, consider what occurs when only one firm shares information. Assume for concreteness that firm \( a \) shares information (firm \( b \) learns \( a \)'s cost) while firm \( b \) does not. This is a Cournot game of asymmetric information. The Bayesian-Nash equilibrium outputs are

\[ q_a^S(w_a, w_b) = \frac{[\alpha \gamma - 2\lambda_a w_a + \delta \lambda_c w_b] \Delta}{\Delta}, \]

\[ q_b^N(w_a, w_b) = \frac{[\alpha \gamma - 2\lambda_b w_b + \delta \lambda_a w_a] \Delta - \delta^2 (\lambda_c - \lambda_b) w_b}{2\Delta}, \]

where \( S \) indicates share and \( N \) not share. Note that in the first stage that for given \( w_a \) and \( w_b \), expected outputs in the fourth stage are the same for all information structures: \( \bar{q}_i = \frac{[\alpha \gamma - 2\lambda_i w_i - \delta \lambda_c w_j]}{\Delta}. \)

3.B The third stage

With the equilibrium outputs for each information structure determined for the fourth stage, the third stage wage that the supplier sets can now be derived for each information structure. To begin with, if information was shared by both firms in the second stage, then the supplier knows the values of \( \lambda_a \) and \( \lambda_b \), and can calculate the stage four equilibrium outputs given wages, i.e., \( q_a^C(w_a, w_b) \) and \( q_b^C(w_a, w_b) \). Thus, it chooses \( w_a \) and \( w_b \) to maximize its profits

\[ \omega^C = q_a^C(w_a, w_b) \lambda_a w_a + q_b^C(w_a, w_b) \lambda_b w_b. \]

From the first order conditions, the optimal wages are

\[ w_a^C = \frac{\alpha}{2\lambda_a} \quad w_b^C = \frac{\alpha}{2\lambda_b}. \]

With the wages determined, outputs in the fourth stage can be computed:

\[ q_{a,U}^C = \frac{\alpha}{2(2 + \delta)} \quad q_{b,U}^C = \frac{\alpha}{2(2 + \delta)} \]

where the subscript \( U \) indicates that the (Upstream) supplier has set the wage. These outputs can be
substituted into the profit definitions \( \pi^C_{i,U} \) to obtain stage four equilibrium profits

\[
\pi^C_{i,U} = [q_{i,U}^C]^2
\]

\[
\omega^C_U = \alpha^2/2(2 + \delta)
\]

with which producer surplus can be calculated, \( PS^C_U = \alpha^2(3+\delta)/2(2+\delta)^2 \).

If neither firm shared information in stage two, then the supplier’s profit in the fourth stage is

\[
\omega^I = \omega^I_a (w_a, w_b) \lambda_a w_a + \omega^I_b (w_a, w_b) \lambda_b w_b.
\]

In the third stage, though the supplier does not know the values of \( \lambda \), it understands how it enters into its fourth-stage profits and take the expectation of its fourth stage profits. Thus, the supplier chooses \( w_a \) and \( w_b \) to maximize the expectation of the fourth-stage profits

\[
E[\omega^I] = \bar{q}_a \lambda_a w_a + \bar{q}_b \lambda_b w_b - (w_a^2 + w_b^2) \sigma^2/2
\]

which obtains

\[
w^I_a = \alpha \lambda_a^2/2 \Lambda_e^2
\]

\[
w^I_b = \alpha \lambda_b^2/2 \Lambda_e^2
\]

where \( \Lambda_e^2 = \lambda_e^2 + (2 + \delta) \sigma^2/2 \). Note that if \( \sigma=0 \), then \( \Lambda_e^2 = \lambda_e^2 + (2 + \delta) \sigma^2/2 = \lambda_e^2 \). If there were no uncertainty (\( \sigma=0 \)), then the wage would be the complete information wage evaluated at the expected productivity level \( w^I_a = \alpha/2 \lambda_e \). With these wages the fourth stage outputs can be calculated for given \( \lambda_a \) and \( \lambda_b \) (expressed so as to ease comparison):

\[
q^I_{a,U} = q^C_{a,U} + \alpha (\lambda_e^2 - \lambda_e \lambda_a + \sigma^2)/\Lambda_e
\]

\[
q^I_{b,U} = q^C_{b,U} + \alpha (\lambda_e^2 - \lambda_e \lambda_b + \sigma^2)/\Lambda_e
\]

Using \( q^I_{a,U} \) and \( q^I_{b,U} \) the corresponding profits can be calculated

\[
\pi^I_{i,U} = [q^I_{i,U}]^2
\]

\[
\omega^I_U = \alpha^2 \lambda_e^2/(2+\delta) \Lambda_e^2.
\]
Finally, consider the supplier’s problem when firm $a$ shared its cost observation in stage two, but firm $b$ did not. The supplier’s profit in the fourth stage then is

$$\omega^{SN} = q^S_a(w_a, w_b) \lambda_aw_a + q^N_b(w_a, w_b) \lambda_bw_b.$$  

Since the supplier has the same information that firm $a$ does, it can calculate that firm $a$ will set output $q^S_a(w_a, w_b)$. However, the supplier does not know the value of $\lambda_b$ so taking the expectation given the $\lambda_a$, in stage four it expects its profits to be

$$E_{\lambda,a}[\omega^{SN}] = q^S_a(w_a, w_b) \lambda_aw_a + E_{\lambda,a}[q^N_b(w_a, w_b)]\lambda_bw_b - w_b\sigma^2/2.$$  

From the maximization the optimal wages are

$$w^SN_a = \alpha(2\lambda_e + \gamma\sigma^2)/4\lambda_a(\lambda_e^2 + \sigma^2) \quad w^SN_b = \alpha/2(\lambda_e^2 + \sigma^2).$$  

The fourth stage outputs then are

$$q^S_{a,U} = \alpha/2(2 + \delta) \quad q^N_{b,U} = q^C_{b,U} + \alpha(\lambda_e^2 + \sigma^2 - \lambda_b\lambda_a)/4(\lambda_e^2 + \sigma^2).$$  

with the corresponding profits

$$\pi^S_{a,U} = [q^S_{a,U}]^2 = [q^C_{a,U}]^2,$$  

$$\pi^N_{b,U} = [q^N_{b,U}]^2,$$  

$$\omega^SN_U = \alpha^2(4\lambda_e^2 + \gamma\sigma^2)/8(\lambda_e^2 + \sigma^2)(2+\delta).$$  

The case when firm $b$ shares, but firm $a$ does not, follows symmetrically.

Consider first the effect that sharing has on the wages. Unlike previous work examining the sharing of cost information in vertical relations, here the expected wage with incomplete information does not equal that with complete information. In particular, (proof in the appendix)

**Proposition 1:** If the firms compete in quantities, then a firm by sharing productivity information increases both its expected wage and its rival’s expected wage. The expected wage with complete information is greater than the expected wage with incomplete information.
To understand why productivity information affects wages while cost uncertainty does not, note that cost information only affects input demand through the firms’ output choice. In contrast, productivity uncertainty affects input demand both through the firms’ output and, holding constant output, how much input the firms demand. For example, as the firm becomes more productive it needs less input to produce a level of output, but its strategic relationship with the rival downstream firm leads it to expand its output and hence its input demand. As a result, productivity enters into the supplier’s profits non-linearly and the expected wage is affected by the firm’s information sharing decision.

An interesting part of proposition 1 is the “raising rival’s cost” aspect of information sharing: a firm by agreeing to share information raises its rival’s expected wage. This arises from the supplier’s pricing taking into account the interaction between the firms. For instance, consider what occurs when one firm chooses to share information but its rival does not. By sharing the firm will face in expectation a higher wage, which benefits the rival because this shifts the firm’s best response function. More importantly, this leads to an increase in the rival’s expected output, which leads the supplier to raise its wage to the rival. That is, although the supplier does not know the rival’s productivity, in anticipating the effect the other firm’s sharing has, the supplier raises the rival’s wage. This raises the issue of whether this raising of rival’s wage is significant enough to off-set any harm the firm accrues from the higher wage it received from sharing information. This issued is examined in the next subsection.

Turning to the question of the effect sharing has on output, again previous work in vertical information sharing (Li 2002, Creane 2005), and horizontal information sharing (Vives 1990) finds that sharing has no effect on expected output. Indeed, if there were no supplier, i.e., the wage remained constant, then it can be shown (omitted) that the expected output would remain constant in this case as well. This is because in all of these cases, as well as in the model here, the uncertainty enters the firms’ profit expression linearly and profits are a quadratic function. However, here
because the uncertainty now affects another player in a non-linear manner – the input supplier – the expected wage increases and as a result, a comparison of the expectation of $q^1_{i,u}$ to $q^C_{i,u}$ yields

**Proposition 2:** If the firms compete in quantities, then sharing productivity information decreases the firm’s expected output and weakly increases its rival’s expected output. The expected output with complete information is less than the expected output with incomplete information.

Interestingly, though by sharing information a firm raises its rival’s expected wage, it does not reduce the rival’s expected output. In fact, if the rival is not sharing information then the rival’s output increases, as the increase in the sharing firm’s wage dominates the rival’s wage increase. This suggests that a firm by sharing information will increase the rival’s expected profit. In such a case, then even if sharing information lowers a firm’s profits, expected producer surplus, if not downstream profits, could be greater if both shared than if neither shared.

3.C The first stage

In the first stage the firms chose whether to enter a contract to share information. If transfers between agents are possible, then there are several reasons why a firm might do this. First, it might be a dominant strategy for a firm to unilaterally share information (i.e., profits always increase regardless of what the rival decides). Second, if sharing has a positive effect on the rival’s profits, then joint profits may be greater if they both share than if neither does, and so a quid pro quo contract may be signed when there is a prisoner’s dilemma (see the discussion in Vives 1990). Finally, even if the firms’ profits are lower with sharing, producer surplus may increase; the supplier’s gain from the firms sharing may be greater than the firms’ joint loss. In such a case, the supplier would be willing to pay up front fees to one or both firms to induce them to share the information.

Starting with the case of complete information (i.e., both firms agree to share information), in stage one the calculation of the expected profits and producer surplus in stage four is straightforward.
from the profit functions derive in section 3.B (\(\pi^C_{i,U}\) and \(\omega^C_U\)):

\[
E[\pi^C_{i,U}] = \left[\frac{\alpha}{2(2 + \delta)}\right]^2 = \left[E[q^C_{a,U}]\right]^2
\]

\[
E[\omega^C_U] = \frac{\alpha^2}{2(2 + \delta)}.
\] (3)

Turning to the case of incomplete information taking the expectation of \(\pi^I_{i,U}\) and \(\omega^I_U\) yields

\[
E[\pi^I_{i,U}] = \left[E[q^I_{a,U}]\right]^2 + \left[\alpha \lambda_e \sigma / 4 \Lambda_e \right]^2
\] (4)

\[
E[\omega^I_U] = \alpha \lambda_e \sigma ^2 / 2 \Lambda_e (2 + \delta). \Lambda_e ^2 \equiv \lambda_e ^2 + (2 + \delta) \sigma ^2 / 2
\] (5)

Finally, taking expectation of \(\pi^S_{a,U}\), \(\pi^N_{b,U}\) and \(\omega^{SN}_U\) (when firm \(a\) shares information and firm \(b\) does not) obtains

\[
E[\pi^S_{a,U}] = \left[\frac{\alpha}{2(2 + \delta)}\right]^2 = \left[E[q^S_{a,U}]\right]^2
\] (6)

\[
E[\pi^N_{b,U}] = \left[E[q^N_{b,U}]\right]^2 + \left[\alpha \lambda_e \sigma / (\lambda_e ^2 + \sigma ^2)\right]^2
\] (7)

\[
E[\omega^{SN}_U] = \alpha ^2 (4 \lambda_e ^2 + \sigma ^2) / 8 (\lambda_e ^2 + \sigma ^2)(2 + \delta)
\] (8)

and symmetrically when firm \(b\) shares and firm \(a\) does not.

With equations (2-8), the effect on profits and producer surplus can be calculated. First, turning to profits, from (2), (4), (6) and (7) it follows that

**Proposition 3:** If the firms compete in quantities, then it is a dominant strategy to not share productivity information. Expected downstream profits are lower with information sharing. The supplier’s expected profit increases with information sharing.

The results that downstream firms prefer to not share information runs counter to previous work that examines the sharing of cost information, both without a supplier (Shapiro 1986) and with a supplier as Li (2002). Both Shapiro (1986) and Li (2002) find that it is a dominant strategy to share cost information and that profits increase with information sharing. However, the new result here is not
driven primarily by the sharing of productivity rather than cost information, as it can be shown (not derived) that without a supplier it is a dominant strategy to share productivity information and profits increase with information sharing. Instead, price discrimination is likely to be the driving force as Creane (2005) shows it can be a dominant strategy not to share cost information if, like here, the supplier can charge different wages to the firms. That is, with both productivity and cost information it is a dominant strategy not to share information when the supplier sets the wage. Yet, it is still possible that the type of information – productivity instead of cost – may augment or ameliorate this effect. This hypothesis will be explored by considering next the effect information sharing has on producer surplus and in section 5 by considering the effect of the input supplier’s bargaining power on the results. That is, if cost and productivity uncertainty are equivalent, then the effect information sharing has on producer surplus here should be identical to that found in Creane (2005). Likewise, the effect of bargaining power should not depend on the type of uncertainty.

The next question is the effect a firm’s decision to share information has on its rival’s profits. As noted earlier, though sharing information raises the rival’s expected wage, the rival’s output weakly increases. A comparison of (2), (4), (6) and (7) shows that rival benefits from the firm’s decision to share information.

**Corollary:** If the firms compete in quantities, then sharing productivity information weakly increases the rival’s expected profits.

It has been shown that the firms are harmed by sharing information, and so they would not unilaterally share information. However, the supplier benefits and if the gain to the supplier is greater than the loss to firms, then the supplier would be willing to compensate the firms sufficiently to induce them to share their information. That is, if expected producer surplus (downstream and supplier profits) increases with information sharing, then with side payments information sharing
could arise. Moreover, since both Li (2002) and Creane (2005) find that expected producer surplus can increase with the sharing of cost information to both the supplier and the rival, it might be anticipated that will occur here. On the other hand, sharing productivity information affects the expected wage and output (propositions 1 and 2), while sharing cost information does not, and so the effect on producer surplus might change. Using (2-8) the effect on producer surplus can be derived:

**Proposition 4**: *If the firms compete in quantities, then producer surplus (firms’ and supplier’s profits) decreases with any sharing of productivity information. Expected producer surplus is greater with incomplete information than complete information.*

As Creane (2005) finds that with sufficiently homogenous goods and price discrimination, producer surplus increases with the sharing of cost information, proposition 4 suggests that productivity uncertainty has effects that cost uncertainty does not have.

The main intuition behind this result rests on the effect information sharing has expected wages and, hence, outputs, which affects the “double-marginalization” inefficiency. Straightforward calculation shows that the downstream outputs with complete information are less than the outputs that maximize producer surplus – the “double-marginalization” inefficiency. By not sharing information, the expected wage is lower and as a result, expected outputs increase (Propositions 1 and 2), an effect that does not occur when cost information is shared. Moreover, as $\sigma$ increases, the incomplete information output increases (or, alternatively, as $\sigma \to 0$, the incomplete information output converges to the complete information output). In fact, with sufficient uncertainty the incomplete information output can be greater than the producer surplus maximizing level. Despite this, in quantity competition producer surplus is always greater with incomplete information than with complete information. That is, the reduction in producer surplus from the effect on expected output overwhelms the usual benefit (Shapiro 1986 and Li 2002) from information sharing.
These output effects also explain why with cost uncertainty (Creane 2005), producer surplus increases with information sharing. With cost uncertainty, the expected wage and outputs are unchanged, and hence this output effect with productivity uncertainty is not present with cost uncertainty. This also suggests that in price competition, if the goods are sufficiently undifferentiated, then the output effect could be positive, i.e., the reduction in expected output could raises producer surplus. This is because with price competition, unlike with quantity competition, as the goods become sufficiently undifferentiated, the double marginalization inefficiency goes to zero. In this case, the wages with complete information could be closer to the producer maximizing level than the incomplete information wages, and so sharing information in price competition could increase producer surplus even though in the previous literature it always reduces producer surplus.

4. Price competition/strategic complements

The outline for this section follows the previous. For analytical symmetry, let demand be \( q_i = A - p_i + d p_j \) with \( d \in [0,1) \). Otherwise, the model remains the same. Firm \( i \)'s profits are \( (p_i - \lambda_i w_i) \cdot (A - p_i + d \cdot p_j) \). Using the first order conditions, \( \text{firm a and firm b}'s profit maximizing price given its rival’s price is

\[
\begin{align*}
    p_a &= (A - \lambda_a w_a + d \cdot p_b)/2, \\
    p_b &= (A - \lambda_b w_b + d \cdot p_a)/2
\end{align*}
\]

As the analysis is analogous to that with quantity competition and, except for proposition 4, the propositions are the same, the derivations are only outlined here with the explicit derivation left for the appendix.

The framework follows the previous section. First, the fourth stage equilibrium prices (and hence quantities, profits, etc.) for each information structure are derived: when both share information \( p_i^c(w_a, w_b) \), when neither shares information \( p_i^f(w_a, w_b) \), and when only one shares information \( \{ p_i^s(w_a, w_b), p_j^N(w_a, w_b) \} \). Moving back to the third stage, the supplier given its
information sets the wages, denoted respectively $w_i^C$, $w_i^I$, $w_i^S$ and $w_j^N$. From the calculations in the appendix it can be seen that though the firms compete in prices instead of quantities, sharing has a similar effect on the expected wage. Sharing information increases the firm’s expected wage and more importantly has the “raising rival’s costs” aspect of raising the rival’s expected wage, just as with quantity competition. Hence, proposition one can be stated more generally:

**Proposition 1**: If the firms compete in prices or quantities, then a firm by sharing productivity information increases both its expected wage and the rival’s expected wage. The expected wage with complete information is greater than the expected wage with incomplete information.

Turning to the effect sharing has on expected output, calculation shows that sharing again has the exact same effect in price competition as in quantity competition, resulting in a more general version of proposition 2 as well.

**Proposition 2**: If the firms compete in prices or quantities, then sharing productivity information decreases the firm’s expected output and weakly increases the rival’s expected output. The expected output with complete information is less than the expected output with incomplete information.

With the stage four equilibrium prices and quantities the expected profits for each information structure can now be derived. Moving to the first-stage choice, taking the expectation of these fourth stage profits allows for the determination of whether the firms will choose to share information in stage one. Interestingly, the outcome with price competition is again the same as with quantity competition, and so proposition 3 and its corollary also can be stated more generally:

**Proposition 3**: If the firms compete in prices or quantities, then it is a dominant strategy to not share productivity information. Expected profits are lower with information sharing. The
supplier’s expected profit increases with information sharing.

**Corollary:** If the firms compete in prices or quantities, then sharing productivity information weakly increases the rival’s expected profits.

So far the type of competition has had no effect on the outcome. This might lead to the extrapolation that this will hold true in examining producer surplus. That is, that in price competition sharing productivity information reduces producer surplus as it does in quantity competition. Moreover, previous results in the literature also support this hypothesis: without a supplier, producer surplus decreases with the sharing of cost information (Gal-Or 1986) and with a supplier, producer surplus decreases with the sharing of cost information (Creane 2005). Finally, it can be shown that without a supplier, the dominant strategy is to not share productivity information and producer surplus is lower with information sharing. Despite this, with sufficiently undifferentiated goods, producer surplus increases:

**Proposition 5:** If the firms compete in prices, then expected producer surplus is greater with complete information than incomplete information if and only if \( d > \frac{9}{2} \left( \frac{\lambda_c}{\sigma^2} + \frac{\lambda_c^2}{9\sigma^2} \right) \).  

As the term \( 9(\lambda_c^2 + \sigma^2) - (\lambda_c^2 + \sigma^2)^{1/2}(\lambda_c^2 + 9\sigma^2)^{1/2} / 2(3\lambda_c^2 + 2\sigma^2) \) is a rather unintuitive expression, two simple sufficient conditions can help clarify the condition.

**Corollary:** Expected producer surplus is greater with complete information than incomplete information if \( d > 2/3 \). Expected producer surplus is greater with incomplete information than complete information if \( d < 1/2 \).

To summarize, expected producer surplus increases with the sharing of productivity information if
the goods are not too differentiated and so the supplier would be willing to compensate the firms sufficiently to persuade them to share information.

The intuition for this result follows on the final discussion after proposition 4. Information sharing raises the expected wages to a level above the producer surplus maximizing wages because of the double marginalization inefficiency. However, as the goods become less differentiated, the downstream competition becomes more intense thereby reducing the second price mark-up and as a result, this inefficiency. Furthermore, this reduction in the inefficiency can be greater with price competition than with quantity competition: in the limit with price competition as the downstream products become perfect substitutes, the downstream price converges to the firm’s marginal cost. Thus, the complete information wages approach the producer-surplus maximizing wages. Since expected wages are lower when the firms do not share information, then with sufficiently undifferentiated goods, the incomplete information wages lower expected producer surplus. On the other hand, information sharing by itself normally reduces producer surplus when the firms compete in prices (e.g., Gal-Or 1986). Thus, only if the goods are sufficiently undifferentiated can information sharing raise welfare.\(^{11}\) In contrast, with cost uncertainty expected wages and outputs are constant even with a monopoly supplier, that is, information sharing does not affect expected wages (Creane 2005). Hence, this gain with productivity uncertainty cannot exist with cost uncertainty and producer surplus always decreases (Creane 2005).

5. Extensions

In this section the importance of some of the main assumptions are discussed. The first subsection explores the effect of assuming that the supplier sets the wage, in effect, has all of the bargaining power. The second subsection explores the assumption that a firm that shares information

\(^{11}\) Likewise, since wages with incomplete information are decreasing in \(\sigma\), as \(\sigma\) increases, the critical \(d\) – the level of product homogeneity needed for producer surplus to increase from information sharing – decreases.
with its supplier simultaneously shares information with its rival. Though both assumptions are used in previous work, their examination is useful in better understanding the effects here.

A. The Role of Bargaining Power

In many environments downstream firms have some bargaining power when negotiating with the input supplier. For example, as there are more input suppliers one would expect the bargaining power to shift to the downstream firms. Intuitively, then, the result here that firms would not share information in quantity competition could change if the firms have sufficient bargaining power in determining the wage. To capture in a simple way the effect bargaining power can have in these types of setting, Creane (2005) examines the following variation to the standard structure of information sharing models. After firms choose whether to share the information there is a new stage in which nature determines whether the firms or the input supplier sets the wage. Let $B$ stand for the probability that the input supplier sets the wage. As $B$ increases, the effect that the supplier has on the outcome increases; if $B=0$, then the supplier has no effect and the model is the classic model of horizontal information sharing. Let $B^*$ be the bargaining power needed for the firms such that they are just indifferent between sharing and not sharing. Such a $B^*$ will exist when, e.g., the supplier using the information makes the firms worse off from sharing (as here), while without the supplier the firms would be better off, and captures in a simple way the relative importance of the two effects.

Following Creane (2005) the critical $B^*$ with productivity uncertainty can be derived:\textsuperscript{12}

$$B = \frac{4\sigma^2(8-\delta^2)(\lambda^2+\sigma^2)}{3r^2\delta^2(8-\delta^2)(\lambda^2+\sigma^2)+\alpha(2+\delta)(2-\delta)^2[2\delta\lambda+\alpha(6+\delta)]}$$

The analogous $B^*$ in Creane (2005), which has cost uncertainty, is the simpler $\delta^2(8-\delta^2)/12$, but with productivity uncertainty the critical bargaining power also depends on the demand intercept, the

\textsuperscript{12} The explicit derivations are available on request.
distributional properties of $\lambda$ as well as the marginal cost of input production $r$.\textsuperscript{13} Comparative statics show that this $B$ is increasing in the variance ($\sigma^2$) of $\lambda$.\textsuperscript{14} Further, this critical $B$ is decreasing in the intercept ($\alpha$, which reduces the importance of $\sigma^2$). Using conditions for an interior solution as restrictions for the upper limit on $\sigma$ and the lower limit on $\alpha$ it can be shown that an upper bound (not necessarily the lowest) on $B$ such that the firms would unilaterally share information is $\delta^2(8-\delta^2)/12$ (the condition with cost uncertainty). That is, except for the limit case, the firms need more bargaining power to be willing to share information with productivity uncertainty than with cost uncertainty. A similar analysis can be done to show that an upper bound on the $B$ such that firms benefit from jointly sharing information is $\delta^2(12-\delta^2)/3(4+\delta^2)$, which again is the condition with cost uncertainty. That is, the firms need more bargaining power (lower $B$) with productivity uncertainty than with cost uncertainty to benefit from information sharing. Thus, the productivity and cost uncertainties do not have identical effects on a firm’s incentive to share. These results reinforce the hypothesis put forward after Proposition 3: that the result here was not purely a result of the supplier’s ability to price discriminate.

B. Information sharing with the supplier

This paper was motivated by the observation that firms often share information (perhaps

\textsuperscript{13} When the downstream firms have all of the bargaining power, zero input costs ($r=0$) would make the problem trivial and more importantly firms would not share if the input supplier has any bargaining power (i.e., for all $B > 0$). That is, the result here would be even stronger: productivity uncertainty never results in information sharing unless the firms have all of the bargaining power, instead of just for some range as shown here. As this stronger result is driven by the triviality, a positive input cost ($r>0$) is assumed in this section.

\textsuperscript{14} That $B$ is increasing in $\sigma$ may seem surprising since, when the supplier sets the wage, the size of the wage increase caused by information sharing is increasing in $\sigma$, i.e., a greater harm to firms from information sharing. All else equal, this would imply that the firms would require a higher probability that they are setting the wage (a lower $B$) for them to still be willing to share information. However there is also the classic information sharing effect: holding wages constant, information sharing increases the firms’ profits and this benefit is increasing in $\sigma$. This effect ameliorates the loss to the firms from the higher wages when the supplier sets the wage and increases the benefit to the firms when they set the wage (i.e., all else equal, this effect would imply that firms would be willing to share with a higher $B$). That this classic effect can dominate is not surprising since the firms’ profits are four times greater when they set the wage (e.g., if an increase in $\sigma$ has the same percentage change in profits, then the increase in profit from when the firms set the wage is greater than the loss when the supplier sets the wage, i.e., $B^*$ increases).
inadvertently) with their suppliers and yet there has been little theoretical examination of the effect the supplier’s learning can have. In modeling this, it was assumed (as previous work did) that the rival also learns when a firm shares information with the supplier. This assumption was motivated by noting that the intentional leakage of information by the supplier to the firm’s rival is well known in several literatures and furthermore, that the environment here (in which the supplier and the rival negotiated over the wage) was particularly well suited for the inadvertent leakage of information. In this subsection, this motivation is given a more detailed discussion, that is, why, for the environment studied here, a firm that shares with its supplier should include the effect of the rival learning as well in its calculus. The attention is then turned to what occurs if the firm was able to prevent the supplier from explicitly sharing the information, that is, only the supplier directly learns. In such a case, if the supplier uses this information in the wages it sets, the rival should be able to infer the information.

At an intuitive level, there are several reasons for why information shared with the supplier will be transmitted to the rival as well. First, there already exists the means for the supplier to pass on the information as it must interact with the rival when setting its wage. Second, there are several different reasons why the supplier would want to pass on the information: (a) the information is valuable and so it could sell it to the rival (see examples cited in the introduction); (b) the supplier may try to pass information only when it is strategically advantageous (i.e., to induce greater input demand) however, this could unintentionally lead to a complete revelation equilibrium (Okuno-Fujiwara et al. 1990); (c) a policy of revealing (for free) the information may increase the supplier’s expected profits (see, e.g., Milgrom and Weber 1982 and Ottaviani and Prat 2001, as well as the discussion below) and (d) the supplier may be acting as a conduit for the firm that wants to circumvent antitrust rules against information sharing with rivals. Thus, unless the firm can detect and punish the supplier for passing on the information, it should be expected that the supplier will pass on the information. Third, the information could be passed between employees as a matter of course, much like the intuition often given for how R&D spillovers occur in specific localities (e.g.,
Silicon Valley). Finally, if the input supplier uses the information in setting its wages for the firms, then the rival should be able to infer the information. This idea is examined in more detail next.

If the firm shares the information only with its supplier and the supplier uses this information in the wage it sets, it seems intuitive that the wages will retransmit this information. The simplest way to see this is to note that if the wages the supplier sets are the same wages it would have set when the rival firm also learn (i.e., the wages in sections 3 and 4), then the rival can infer the information from the wage. For example, if the supplier learns both firms’ productivities and sets the complete information wages ($w^C$), then the firms would be able to infer each other’s productivities.\(^{15}\) Moreover, it can be shown that there are plausible beliefs that would support such an equilibrium. Likewise, in a separating equilibrium the wages would transmit the information as well. In such a case it is well known from the signaling literature that a privately informed party – the supplier – may be worse off when its private information is indirectly inferred by the uninformed parties at a separating equilibrium. Hence, in this case if the supplier can avoid the separating equilibrium by passing on the information directly to the rival firm, it will increase its expected profits. That is, the supplier can increase its expected profit if it has a policy of making any information shared public, which echoes the results in Milgrom and Weber (1982), and Ottaviani and Prat (2001). This also implies that the supplier can increase its expected profit if it can commit to the wages it would set when the information is public.

6. Conclusion

It is well known that firms not only share information with their horizontal rivals but with their vertical rivals as well. Despite this, until recently there has been little theoretic examination of how the supplier’s learning affects a firm’s decision to share. Li (2002) and Zhang (2002) were the

\(^{15}\) Note that the issue here is inferring the rival’s productivity in contrast to the secret price discount literature in which firms know their rivals type but not the contract offered. Thus, the question here is not what wage is offered to the rival, but how the supplier determined that wage.
first to explore this by placing the classic model of information sharing into an environment where firms share information with both their supplier and their rivals. This paper considers instead a variable more commonly studied in work examining vertical relationships – productivity information. Despite productivity uncertainty seeming to be equivalent to cost uncertainty at first glance, there are new results. First, a firm by agreeing to share productivity information increases its expected wage, which does not occur in the same setting if it is sharing cost information (Creane 2005). Likewise, there is a new “raising rival’s costs” type effect: a firm by agreeing to share information can increase the rival’s expected wage. This increase in expected wages also increases expected prices. Not surprisingly then, expected outputs decrease with information sharing, while in the previously cited work, information sharing does not affect expected outputs. This leads to perhaps the most interesting result here: if firms compete in prices, then producer surplus (supplier and downstream profits) increases in expectation when the products are sufficiently undifferentiated. Thus, the supplier would be willing to compensate the downstream firms so as to induce them to share their productivity information. In contrast, in the previous literature if firms compete in prices then they would not share cost information (e.g., Gal-Or 1986), even if there was a supplier with price setting power (Creane 2005).

These new results arise because productivity uncertainty has an effect on the supplier that cost uncertainty does not have. Specifically, while the firms’ productivity uncertainty affects the supplier’s profits by affecting the equilibrium outputs as does cost uncertainty, it also affects the input demanded for a given level of output unlike cost uncertainty. This new effect affects the supplier’s optimal wages, which leads to an increase in the expected wage.
References


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Appendix A: Proof to proposition 1.

\[ \gamma \equiv (2 - \delta), \Delta \equiv 4 - \delta^2, \Lambda_e^2 \equiv \lambda_e^2 + (2 + \delta)\sigma^2/2 \]

**Proposition 1**: If the firms compete in quantities, then a firm by sharing productivity information increases both its expected wage and the rival’s expected wage. The expected wage with complete information is greater than the expected wage with incomplete information.

Proof: Consider first the comparison of the expected wage with complete information to that with incomplete information. Taking the expectation of \( w_a^c - w_a^i \), \( \frac{\alpha}{2} \int_\lambda^\Lambda_e \frac{\Lambda_e - 2\lambda_e \lambda_a}{\lambda_a \Lambda_e} f(\lambda_a) d\lambda_a \), the numerator is positive in expectation, decreasing in \( \lambda_a \), and, hence, at \( \lambda_a \) positive. Since the denominator is increasing in \( \lambda_a \), the integral is positive. Taking the expectation of the wage differences when a firm shares holding its rival’s decision constant (both share and not share) and using an analogous analysis on the numerator and denominator of the expectation shows that the own expected wage increases and taking the expectation of the rival’s wage differences for the same variations shows that the rival’s expected wage increases. //

Appendix B: derivation of the price case

As before, the derivations work from the last stage backwards.

A2.a The fourth stage

In the fourth stage each firm simultaneously sets price. As with quantity competition, the equilibrium prices depend on the firm’s and its rival’s information: whether both firms share information, neither does or if one shares but the other does not. When both firms share information, the game is a standard Bertrand type game. From (9) the equilibrium prices are:

\[ p_a^c(w_a, w_b) = [\alpha(2 + \delta) + 2\lambda_a w_a + \delta\lambda_a w_b]/\Delta, \]

\[ p_b^c(w_a, w_b) = [\alpha(2 + \delta) + 2\lambda_b w_b + \delta\lambda_b w_a]/\Delta \]
where the C superscript represents complete information and \( \Delta = 4 - \delta^2 \). Outputs, profits and producer surplus (downstream and supplier profits) in the fourth stage are likewise evaluated at 
\[ \{ p_a^C(w_a, w_b), p_b^C(w_a, w_b) \} \], e.g., \( q_a^C(w_a, w_b) = [A - p_a^C(w_a, w_b) + d p_b^C(w_a, w_b)] \), etc.

When neither firm shares information, it is a game of incomplete information with respect to the other’s productivity. The standard derivation yields the equilibrium prices:
\[
p_a^I(w_a, w_b) = \frac{\alpha(2 + \delta) + 2\lambda_a w_a + \delta\lambda_a w_b}{\Delta} + \delta^2(\lambda_a - \lambda_b) / 2 \Delta,
\]
\[
p_b^I(w_a, w_b) = \frac{\alpha(2 + \delta) + 2\lambda_b w_b + \delta\lambda_a w_a}{\Delta} + \delta^2(\lambda_a - \lambda_b) w_b / 2 \Delta,
\]
with the outputs \( q_a^I(w_a, w_b) \), etc. being derived analogously.

Finally, consider when only one firm shares information. Assume for concreteness that firm \( a \) shares information (firm \( b \) learns \( a \)'s cost) while firm \( b \) does not. This is a game of asymmetric information. The Bayesian-Nash equilibrium prices are
\[
p_a^S(w_a, w_b) = \frac{\alpha(2 + \delta) + 2\lambda_a w_a + \delta\lambda_a w_b}{\Delta},
\]
\[
p_b^N(w_a, w_b) = \frac{\alpha(2 + \delta) + 2\lambda_b w_b + \delta\lambda_a w_a}{\Delta} + \delta^2(\lambda_a - \lambda_b) / 2 \Delta,
\]
where \( S \) indicates share and \( N \) not share. Outputs are realized after prices are set and equal \( q_{a,S}(w_a, w_b) = [A - p_a^S(w_a, w_b) + d p_b^N(w_a, w_b)] \) and \( q_{b,N}(w_a, w_b) = [A - p_b^N(w_a, w_b) + d p_a^S(w_a, w_b)] \).

\( A2.b \) The third stage

If information was share by both firms in the second stage, then the supplier not only knows the values of \( \lambda_a \) and \( \lambda_b \), but it also knows what the stage-four equilibrium prices and hence outputs will be. Thus, it chooses \( w_a \) and \( w_b \) to maximize its profits
\[
\omega = \lambda_a w_a q_a^C(w_a, w_b) + \lambda_b w_b q_b^C(w_a, w_b).
\]

From the first order conditions, the optimal wages are
\[ w_a^c = \alpha/2(1-\delta)\lambda_a \quad \quad \quad w_b^c = \alpha/2(1-\delta)\lambda_b. \]

With the wages determined, the resulting outputs from the prices set in the fourth stage can be computed:

\[ q_{a,\text{u}}^c = \alpha/2(2-\delta) \quad \quad \quad q_{b,\text{u}}^c = \alpha/2(2-\delta) \]

where the subscripts \{i,\text{u}\} indicate which firm and that the upstream supplier set the wage. Substituting these prices into the definitions for profit (\( \pi_i^c \)) to obtain equilibrium profits

\[ \pi_{i,\text{u}}^c = [q_{i,\text{u}}^c]^2 \]

\[ \omega_{\text{u}}^c = \alpha^2/(2-\delta)(1-\delta) \]

with which producer surplus can be calculated, \( \text{PS}_i^c = \alpha^2(3-2\delta)/(2-\delta)(1-\delta) \)

If neither firm shares information, then the supplier’s profit in the fourth stage is

\[ \omega = q_a^f(w_a, w_b) \lambda_a w_a + q_b^f(w_a, w_b) \lambda_b w_b. \]

In the third stage the supplier chooses \( w_a \) and \( w_b \) to maximize the expectation of the fourth-stage profits

\[ E[\omega] = E[q_a^f(w_a, w_b)] \lambda_a w_a + E[q_b^f(w_a, w_b)] \lambda_a w_b - (w_a^2 + w_b^2)\sigma^2/2 \]

which obtains

\[ w_a^f = \alpha\lambda_a/[2(1-\delta)\lambda_c^2 + (2-\delta)\sigma^2] \quad \quad \quad w_b^f = \alpha\lambda_a/[2(1-\delta)\lambda_c^2 + (2-\delta)\sigma^2], \]

noting that if \( \sigma = 0 \), then \( w_a^f = \alpha/2(1-\delta)\lambda_c \): if there were no uncertainty (\( \sigma = 0 \)), then the wage is the complete information wage evaluated at the expected productivity level.

From the resulting prices, fourth stage outputs are

\[ q_{a,\text{u}}^f = q_{a,\text{u}}^c + \alpha[(1-\delta)\lambda_c^2 + \lambda_a(\lambda_a - \delta\lambda_b) + \sigma^2]/[2(1-\delta)\lambda_c^2 + (2-\delta)\sigma^2], \]

\[ q_{b,\text{u}}^f = q_{b,\text{u}}^c + \alpha[(1-\delta)\lambda_c^2 + \lambda_b(\lambda_b - \delta\lambda_a) + \sigma^2]/[2(1-\delta)\lambda_c^2 + (2-\delta)\sigma^2]. \]
As the rival’s productivity enters linearly, it is convenient to use the profit that a firm expects given its \( \lambda \):

\[
E_A[\pi_{i,t}^L] = [E_A[q_{i,t}^L]]^2
\]

\[
\omega_i^L = \alpha^2\lambda_e^2/(2-\delta)[2(1-\delta)\lambda_e^2 + (2-\delta)\sigma^2].
\]

Finally, if firm \( a \) shares its cost observation but firm \( b \) does not, then the supplier’s profit is

\[
\omega = q_s^A(w_a, w_b) \lambda_a w_a + q_n^N(w_a, w_b) \lambda_b w_b.
\]

The supplier knows that firm \( a \) will set output \( q^s_s \) as the supplier has the same information that firm \( a \) does. However, the supplier does not know the value of \( \lambda_b \) so taking the expectation yields

\[
E_{\lambda_a}[\omega] = E_{\lambda_a}[q_s^A(w_a, w_b) \lambda_a w_a + E_{\lambda_a}[q_n^N(w_a, w_b) \lambda_b w_b] - w_b \sigma^2/2.
\]

From the maximization the optimal wages are

\[
w_a^{SN} = \alpha[2(1+\delta)\lambda_e^2 + (2+\delta)\sigma^2]/2\lambda_a[2(1-\delta)\lambda_e^2 + (2-\delta)\sigma^2]
\]

\[
w_b^{SN} = \alpha \lambda_a (1+\delta)/[2(1-\delta^2)\lambda_e^2 + (2-\delta^2)\sigma^2].
\]

The from the prices, fourth stage outputs then are

\[
q_{s,a,U} = q_{a,U}^C + \alpha(1+\delta)\delta \lambda_e (\lambda_e - \lambda_b)/2[2(1-\delta^2)\lambda_e^2 + (2-\delta^2)\sigma^2]
\]

\[
q_{b,U} = q_{b,U}^C + \alpha(1+\delta)(\lambda_e^2 + \sigma^2 - \lambda_b \lambda_e)/2[2(1-\delta^2)\lambda_e^2 + (2-\delta^2)\sigma^2].
\]

with the corresponding profits

\[
\pi_{a,U}^S = [q_{a,U}^S]^2 = [q_{a,U}^C]^2,
\]

\[
\pi_{b,U}^N = [q_{b,U}^N]^2,
\]

\[
\omega_{U}^{SN} = \alpha^2(4(1+\delta)\lambda_e^2 + \gamma\sigma^2)/4[2(1-\delta^2)\lambda_e^2 + (2-\delta^2)\sigma^2](2-\delta).
\]

The case when firm \( b \) shares but firm \( a \) does not follows symmetrically.

Comparison of the expectation of \( w_a^C, w_a^L, w_a^{SN} \) and \( w_b^{SN} \) yields:
**Proposition 1**: If the firms compete in prices or quantities, then a firm by sharing productivity information increases both its expected wage and the rival’s expected wage. The expected wage with complete information is greater than the expected wage with incomplete information.

The proof follows analogously to the proof of proposition 1 in appendix A. Straightforward comparison of the expectation of $q^C_{b,U}, q^I_{b,U}, q^S_{a,U}$ and $q^N_{b,U}$ obtains:

**Proposition 2**: If the firms compete in prices or quantities, then sharing productivity information decreases the firm’s expected output and weakly increases the rival’s expected output. The expected output with complete information is less than the expected output with incomplete information.

**A2.c The first stage**

Calculating expected profits and producer surplus with complete information are straightforward from the profit functions derive in section A2.b: $\pi^C_{i,U}$ and $\omega^C_U$.

\[
E[\pi^C_{i,U}] = \left[\frac{\alpha}{2(2-\delta)}\right]^2 = \left[E[q^C_{a,U}]\right]^2
\]  \hspace{1cm} (A1)

\[
E[\omega^C_U] = \frac{\alpha^2}{2(2-\delta)(1-\delta)}
\]  \hspace{1cm} (A2)

Turning to the case of incomplete information taking the expectation of $\pi^I_{i,U}$ and $\omega^I_U$ yields

\[
E[\pi^I_{i,U}] = \left[E[q^I_{a,U}]\right]^2 + \left[\alpha\lambda_c\sigma/2[2(1-\delta)]\lambda_c^2 + (2-\delta)\sigma^2]\right]^2
\]  \hspace{1cm} (A3)

\[
E[\omega^I_U] = \frac{\alpha^2\lambda_c^2}{2(2-\delta)}\left[2(1-\delta)\lambda_c^2 + (2-\delta)\sigma^2]\right].
\]  \hspace{1cm} (A4)

Finally, taking expectation of $\pi^S_{a,U}$, $\pi^N_{b,U}$ and $\omega^SN_U$ when firm $a$ shares information and firm $b$ does not

\[
E[\pi^S_{a,U}] = \left[\alpha/(2-\delta)\right]^2 = \left[E[q^S_{a,U}]\right]^2,
\]  \hspace{1cm} (A5)

\[
E[\pi^N_{b,U}] = \left[E[q^N_{b,U}]\right]^2 + \left[\alpha\lambda_c\sigma(1+\delta)/[2(1-\delta)]\lambda_c^2 + (2-\delta^3)\sigma^2\right]^2
\]  \hspace{1cm} (A6)

\[
E[\omega^SN_U] = \frac{\alpha^2[4(1+\delta)]\lambda_c^2 + \gamma\sigma^2]}{4[2(1-\delta^3)]\lambda_c^2 + (2-\delta^3)\sigma^2]}(2-\delta).
\]  \hspace{1cm} (A7)
From (A1), (A3) and (A6) the profit comparison can be made to derive:

**Proposition 3**: If the firms compete in prices or quantities, then it is a dominant strategy to not share productivity information. Expected profits are lower with information sharing. The supplier’s expected profit increases with information sharing.

**Corollary**: If the firms compete in prices or quantities, then sharing productivity information weakly increases the rival’s expected profits.

Straightforward manipulation of (A1-A4) obtains

**Proposition 5**: If the firms compete in prices, then expected producer surplus is greater with complete information than incomplete information if and only if \( d > \frac{5(\lambda_c^2 + \sigma^2)}{4(3\lambda_c^2 + 2\sigma^2)} \). 

Though not presented in the main body, it can be shown that when \( d > 2/3 \) and one firm shares information, summation of the supplier’s profit and the other firm’s profit increases if the other firm shares information. Hence, both firms sharing information is a stable equilibrium if \( d > 2/3 \).