Turning Alphas into Betas: Arbitrage and Endogenous Risk

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Abstract

When arbitrage happens with limited capital, it can determine the cross-section of risks of the arbitraged assets. This happens as arbitrage turns "alphas" into "betas": an asset that is initially more mispriced attracts more arbitrage capital and attains a correspondingly large endogenous beta with respect to arbitrage capital shocks. This mechanism helps explain the cross-section of *betas* of 40 equity "anomalies" in multi-factor and intermediary-based asset pricing models. The presence of this alphas-into-betas effect means that a naive cross-sectional asset pricing regression suffers from endogeneity.

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1 Introduction

In a financial market with frictionless arbitrage, the role of arbitrageurs is limited to eliminating mispricings. When arbitrage happens with limited capital, however, arbitrageurs can play an additional role of determining the cross-section of risks of the arbitraged assets. They do this by turning "alphas" into "betas": an initially more mispriced asset attracts more arbitrage capital and attains a correspondingly large beta with respect to arbitrage capital shocks.[1] The goal of this paper is to document this risk-allocating role of arbitrage both theoretically in a model of capital-constrained arbitrageurs and empirically in the cross-section of equity "anomalies."

To fix ideas, consider assets "A" and "B" that are claims to some deterministic payoff of \$10 in present value. Suppose also that absent arbitrage capital, behavioral investors for whatever reason drive their prices down to $P_A = \$5$ and $P_B = \$8$, creating "pre-arbitrage" alphas of 100% (A) and 25% (B). Now, if arbitrageurs enter with unlimited capital, they would immediately drive up both P_A and P_B to $\$10$, and no further price movement would happen. Hence, frictionless arbitrage turns these assets with different pre-arbitrage alphas into effectively identical riskless assets.

If arbitrage capital faces shocks, however, the act of arbitrage itself makes A endogenously *riskier* than B. To see this, suppose that in the middle of the arbitrage, the arbitrage capital faces a negative shock and drops all the way to zero. In response, P_A and P_B would drop 50% (\$10 to \$5) and 20% (\$10 to \$8) respectively, assuming that the force that caused the initial underpricing stays. Hence, A suffers a worse return than B precisely when extra return is most valuable to the arbitrageurs, making A riskier.[2] That is, in equilibrium, the initially-more-mispriced assets rely more heavily on the arbitrageurs' price-correcting role, making them endogenously more sensitive to the variation in arbitrage capital and hence riskier. In this way, arbitrage turns alphas into betas.

This intuition carries over to a simple model of capital-constrained arbitrageurs. In my three-period model, a representative arbitrageur trades a continuum of assets but faces exogenous shocks to her wealth and funding constraint, which generates variation in the capital that she can deploy.

[1] Andrew Lo first used the phrase "alpha is becoming beta" to mean a situation where, as in Shleifer and Vishny (1997), quantitative trading strategies designed to have no systematic exposures become endogenously exposed to the risk of "unwinding" by the quantitative hedge funds (e.g., Khandani and Lo, 2011). Instead, my "alphas into betas" refers to a cross-sectional phenomenon.

[2] And this endogenous risk means that P_A and P_B would actually be lower than \$10 even with sufficiently large arbitrage capital, except in the period immediately before the deterministic payoff.

The assets are known to realize their fundamental value eventually (at time 3) but may be underpriced in the early (time 1) and intermediate (time 2) periods of arbitrage due to behavioral investors. Importantly, the degree of underpricing in the absence of arbitrage capital—the "pre-arbitrage" alpha—differs across the assets.

In this model, assets with high pre-arbitrage αs become assets with high endogenous "post-arbitrage" βs with respect to arbitrage capital shocks (Proposition 1). A large pre-arbitrage α means that a larger fraction of the asset is held by the arbitrageur since she plays a larger price-correcting role in that asset in equilibrium. This, however, also means that this asset responds more to the variation in the arbitrage capital at time 2, making the asset endogenously riskier for the arbitrageur. What follows from this explanation is that the cross-section of average arbitrage positions in the assets should also explain their post-arbitrage βs (Proposition 2).

Furthermore, this endogenous post-arbitrage β arises only when the arbitrageur is capital-constrained (Proposition 3); in times when the arbitrageur has a "deep pocket," the arbitrageur does not generate endogenous βs in the anomalies. A naive asset pricing test that does not account for this endogenous generation of βs can lead to a false discovery of a pricing factor (Proposition 4).

In light of these predictions, I study to what extent 40 equity anomaly portfolios' beta exposures to Fama-French (2015) (FF) five factors and Adrian, Etula, and Muir (2014) funding-liquidity factor are an outcome of arbitrage. The two papers find that anomalies have significant beta exposures to their respective factors. However, since the anomalies have been actively traded by arbitrageurs like hedge funds since the early 1990s, it seems possible that their betas partly come from the arbitrage capital having systematic exposures to those factors. My main approach is to study the anomalies' βs before and after 1993, the approximate year when arbitrage capital began entering into anomalies according to short interest data, and I supplement this with the cross-sectional differences in the anomalies' publication sample years.[3]

My tests suggest that the 40 anomalies' different betas with respect to FF five factors and the funding-liquidity factor are explained by the extent of arbitrage on their alphas. Among the FF five

[3] Chordia, Roll, and Subrahmanyam (2011) and Chordia, Subrahmanyam, and Tong (2014) are the first to suggest that anomaly trading grew rapidly around 1993. McLean and Pontiff (2016) are the first to exploit the cross-sectional differences in the publication sample years of anomalies.

factors, a simple long-short arbitrage involves a natural exposure to RMW and CMA ("profitability" and "investment" factors) but no significant exposure to MKT, SMB, and HML ("market," "size," and "value" factors). Consistent with this observation, the anomalies with high pre-1993 FF five-factor alphas attain higher post-1993 betas with respect to RMW and CMA but not with respect to SMB and HML (Proposition 1). Somewhat puzzling is that the anomalies with high pre-1993 alphas attain *lower* MKT betas in the post-1993 period, which may be because a set of dedicated "short" arbitrageurs have a negative exposure to MKT and transmit these exposures to short anomalies. Consistent with this view, the negative αs-into-βs effect for MKT betas only exists among short-side anomalies.

Although this αs-into-βs observation around 1993 supports the arbitrage-based explanation for βs, it is also consistent with betas of high-alpha anomalies in the pre-1993 period measured with errors that subsequently disappear in the post-1993 period. This problem is aggravated if anomaly "discovery" happens when researchers find strategies whose in-sample alpha is high because its in-sample beta is erroneously low, in which case a high in-sample alpha should predict an increase in the beta out-of-sample. However, two observations are at odds with this alternative explanation. First is the *absence* of the positive αs-into-βs effect in MKT, SMB, and HML, which suggests that measurement error is not a serious issue for the FF factors. Second, the RMW and CMA betas do *not* change predictably from in to out-of sample, indicating that there was no factor mining based on RMW and CMA betas.

More evidence suggests that part of the post-1993 RMW and CMA betas explained by the pre-1993 alpha are endogenous betas arising from arbitrage. Anomalies whose RMW and CMA betas increase more are the ones with larger arbitrage positions in the post-1993 period, consistent with the intuition that an anomaly in which arbitrageurs play a larger price-correcting role has a larger endogenous price sensitivity to the variation in arbitrage capital (Proposition 2). The part of the post-1993 betas predictable using the pre-1993 alpha comes from the subperiod of post 1993 in which the VIX is high, consistent with the risk-allocating role of arbitrage arising only when arbitrageurs are constrained (Proposition 3). Anomalies with high costs of arbitrage show a weaker αs-into-βs relation, consistent with the relation driven by arbitrage.

The intermediary-based asset pricing model of Adrian, Etula, and Muir (2014) offers another interesting opportunity to apply the αs-into-βs prediction. The funding-liquidity factor used in their model measures shocks to broker-dealers that also serve as the prime brokers of hedge funds, suggesting that the anomalies' exposures to this factor may also be endogenous.

My tests again point to the arbitrage-based explanation for the funding-liquidity betas: (i) the anomalies have no significant exposures to funding liquidity in the pre-1993 period but subsequently attain a large exposures in the post-1993 period that line up with their pre-1993 FF five-factor alphas; (ii) the cross-section of different funding-liquidity exposures are explained by the differences in their average arbitrage position in the post-1993 period; (iii) the funding-liquidity exposures arise exclusively in the "constrained" part of the post-1993 period.

This endogenous channel for the cross-section of betas is interesting in its own right, but it also has an important implication for conducting cross-sectional asset pricing tests: ignoring this potential endogeneity in the betas can bias the price of risk estimate upward, leading to a false discovery of a pricing factor (Proposition 4). To take an extreme example, consider systematic shocks to arbitrage capital that for whatever reason are not "priced" by the arbitrageurs. If anomalies with high pre-arbitrage returns due to alphas attain high beta exposures to these shocks through arbitrage, those betas can *appear* to explain the cross-section of returns in a sample that includes the pre-arbitrage period, when in fact causality goes in the opposite direction. The endogeneity issue can be especially severe for intermediary-based models of asset pricing (e.g., Adrian, Etula, and Muir, 2014; He, Kelly, and Manela, 2017; Avdjiev, Du, Koch, and Shin, 2017), as discussed in section 4.3. It is thus important to understand whether or not the suggested αs-into-βs phenomenon happens empirically and to what extent it affects the existing asset pricing test results. I provide evidence that this does occur, but it is not definitive evidence, and more future work should follow.

My theoretical point is related to the literature on asset pricing in the presence of arbitrageurs and financial intermediaries that trade against mispricings. My model is close to that of Kozak, Nagel, and Santos (2017) in which assets mispriced by behavioral investors become assets that can be "priced" using their betas with respect to the arbitrageur's stochastic discount factor (SDF). However, my model gains additional traction by introducing multiple assets that differ in their pre-arbitrage alphas, which generates additional predictions about a market in which factor betas arise endogenously through the act of arbitrage.

My prediction that alphas turn into betas is a cross-sectional extension of the insight that arbitrage with limited capital makes the arbitraged asset endogenously risky for the arbitrageur

(Shleifer and Vishny, 1997). My paper is thus related to the vast literature on limits of arbitrage (e.g., De Long, Shleifer, Summers, and Waldmann, 1990; Kyle and Xiong, 2001; Gromb and Vayanos 2002; Liu and Longstaff, 2004; Stein, 2009). My model is also related to recent models of arbitrage with multiple assets (Brunnermeier and Pedersen, 2009; Gromb and Vayanos, 2017; Kondor and Vayanos, 2018), but it differs in that the key cross-sectional difference in the assets in my model is their pre-arbitrage alpha.

I am not the first to find empirical evidence that arbitrage on anomalies change the nature of their risks. Brunnermeier, Nagel, and Pedersen (2009) find that more profitable carry trades are subject to higher currency crashes, consistent with the act of arbitrage generating crash risks. Lou and Polk (2013) find that arbitrage trading of momentum can lead to momentum crashes. Drechsler and Drechsler (2016) find that short-side anomalies are exposed to risks specific to shorting a stock. Liu, Lu, Sun, and Yan (2015) find that anomalies become correlated with hedge-fund wealth volatility after their discovery. McLean and Pontiff (2016) find that anomalies upon publication become more correlated with other published anomalies.

2 A Model of Arbitrage Determining the Cross-section of Risks

2.1 Model setup

Time horizon, assets, and investors. Consider a three-period ($t = 1, 2, 3$) economy with two types of securities: a risk-free bond and a continuum of anomaly assets $i \in [0, 1]$. The risk-free bond is supplied elastically at the zero interest rate. An anomaly asset ("asset") is a claim to a stream of cash flows $\{\delta_{i,2},\ \delta_{i,3} + p_{i,3}\}$ over $t = 2, 3$ and has a zero net supply. Both $\{\delta_{i,t}\}$ and $\{p_{i,3}\}$ are i.i.d. across assets and time (for dividends) with $E_t\left[\delta_{i,t+1}\right] = 0$ and $E_2\left[p_{i,3}\right] = v$, which makes the assets effectively risk-free for a diversified investor.

There are two types of investors: behavioral investors and a representative arbitrageur. The behavioral investors cause the assets to be mispriced and are modeled as a set of demand curves. An arbitrageur with mass μ trades against these mispricings but may be capital-constrained. I analyze the three-period equilibrium under two different assumptions about μ: the trivial "pre-arbitrage" equilibrium with $\mu = 0$ and the more interesting "post-arbitrage" equilibrium with $\mu = 1/2$. These two equilibria capture the period before the growth of arbitrage on the assets and after, respectively.

Behavioral investor demand. Behavioral investors cause the assets to be *underpriced*, with the severity of underpricing *increasing* in i.[4] This is modeled using the following aggregate behavioral investor demand for each asset i:

$$B_{i,t} = \frac{E_t\left[r^e_{i,t+1}\right]}{r_{max}} - i \quad \forall t, \tag{1}$$

where $B_{i,t}$ is the aggregate behavioral investor demand for anomaly i (in units of wealth), $E_t\left[r^e_{i,t+1}\right]$ is the *objective* conditional expected (excess) return, and $r_{max} > 0$ is a constant representing the abnormal return on the most underpriced when arbitrageurs have a zero mass ($r_{max} = E_t\left[r^e_{1,t+1} \middle| \mu = 0\right]$).

As a usual, this demand curve describes a negative relationship between price $p_{i,t}$ and demand $B_{i,t}$ (to see this, simply express $E_t\left[r^e_{i,t+1}\right]$ as $p^{-1}_{i,t} E_t\left[p_{i,t+1} + \delta_{i,t+1}\right] - 1$). Furthermore, as i increases, the demand curve shifts downward, implying that high-i assets have lower prices than low-i assets for any given level of behavioral investor demand. Since the demand curves are fixed over time, this cross-sectional ordering of the force behind the underpricing is time-invariant, a key assumption I maintain throughout the analysis.[5]

Specifying the demand curves rather than preferences of the behavioral investors allows me to abstract away from the underlying reason for the apparent mispricing, which could be behavioral, rational, or statistical. For instance, the assets may generate an "alpha" because they covary with consumption shocks that risk-averse households want to avoid but the arbitrageur does not care about. In this case, the alpha reflects a rational compensation for risk but can still turn into "beta" through the act of arbitrage. By construction, all assets have an equal "size" in the sense that for all assets, a given change in the arbitrage position leads to an equal change in the expected return on the asset: $\partial E_t\left[r^e_{i,t+1}\right] / \partial x_{i,t} = -\mu\left(\partial E_t\left[r^e_{i,t+1}\right] / \partial B_{i,t}\right) = -\mu\, r_{max}\ \forall i$, where $x_{i,t}$ is the arbitrageur's position on anomaly i at t with the first equality holding by market clearing.

[4] This direction of mispricing is chosen for convenience and does not affect my theoretical predictions.

[5] This can happen in practice despite the growth of institutional capital in the stock market if mutual fund managers exhibit behavioral patterns similar to retail investors, which seems to be true (e.g., Frazzini, 2006; Frazzini and Lamont, 2008).

Arbitrageur. A representative, risk-neutral arbitrageur trades assets to maximize the expected terminal wealth at time 3 but may be capital-constrained. Specifically, the arbitrageur faces a zero-mean wealth shock $\tilde{w}_t$ and can borrow only up to a funding constraint $f_t \in [0, \infty)$ that may also be time-varying. Making the arbitrageur wealth and funding shocks systematic but keeping the dividends i.i.d. and behavioral investor demand constant is a conscious choice that elucidates how assets with only idiosyncratic risks can attain systematic risks through the act of arbitrage.

The arbitrageur can take long or short positions in the assets but faces the margin rate of 1, which reflects that actual arbitrageurs like hedge funds are not short-sale constrained but face a non-zero margin requirement (e.g., Brunnermeier and Pedersen, 2009; Ang, Gorovyy, and van Inwegen, 2010; Gârleanu and Pedersen, 2011; Gromb and Vayanos, 2017).[6] If the arbitrageur wealth turns negative in any period, the arbitrageur is forced to exit the market immediately and pay an interest cost $c \geq r_{max}$ on the negative wealth in all future periods.[7]

Hence, the arbitrageur's objective at time t is as follows:

$$
\begin{aligned}
& \max\nolimits_{\{\boldsymbol{x}_i\}} E_t\left[w_3\right] \\
\text{s.t.} \qquad & w_{t+1} = \begin{cases} w_t + \int_0^1 r_{i,t+1} x_{i,t} di + \tilde{w}_{t+1} & \text{if } w_t > 0 \\ (1+c)\, w_t & \text{if } w_t \leq 0 \end{cases} \\
& \int_0^1 \left|x_{i,t}\right| di \leq \mathbf{1}\left(w_t > 0\right) k_t \\
& k_t = w_t + \mathbf{1}\left(w_t > 0\right) f_t,
\end{aligned}
\tag{2}
$$

where $\boldsymbol{x}_i$ is the unit arbitrageur's sequence of dollar positions on anomaly i at t and all future trading periods, w_t is the arbitrageur wealth (equity), $r_{i,t}$ is the asset return, $\mathbf{1}(\cdot)$ is an indicator function, and k_t is the arbitrageur's deployable capital ("arbitrage capital").

Equilibrium conditions. I look for a competitive equilibrium in which

1. The aggregate behavioral investor demand $\{B_{i,1}\}$ and $\{B_{i,2}\}$ satisfy equation (1) given prices $\{p_{i,1}\}$ and $\{p_{i,2}\}$;

[6] Keeping the margin constant rather than allowing it to depend on the volatility of the asset, I emphasize that different endogenous betas can arise without differences in the anomalies' idiosyncratic volatilities, the key feature in Brunnermeier and Pedersen (2009) and Gromb and Vayanos (2017).

[7] This allows me to obtain the marginal value of wealth and hence define the stochastic discount factor in the negative-wealth region.

2. The arbitrageur's chosen positions $\{x_{i,1}\}$ and $\{x_{i,2}\}$ solve problem (2) given prices $\{p_{i,1}\}$ and $\{p_{i,2}\}$; and

3. All markets for the assets clear: $x_{i,t} + B_{i,t} = 0 \; \forall i, t$.

2.2 The pre-arbitrage equilibrium

Given this setup, how do asset prices behave in the "pre-arbitrage economy" in which the mass of arbitrageurs is negligible ($\mu = 0$)? In this case, the assets generate different alphas but have no systematic risks (all proofs are in Appendix B):

Lemma 1. ***(Asset returns in the pre-arbitrage economy).*** *If $\mu = 0$, the return on asset i follows*

$$r_{i,t} = \alpha_i^{pre} + \epsilon_{i,t}, \tag{3}$$

where the "pre-arbitrage alpha,"

$$\alpha_i^{pre} = r_{max} i, \tag{4}$$

increases monotonically from asset 0 to asset 1 (from 0 to r_{max}) and $\epsilon_{i,t}$ is i.i.d. with mean zero.

This model of asset return will contrast sharply with that in the post-arbitrage economy in which the assets have different systematic risks that correspond to the strength of the arbitrage on the asset. Since the post-arbitrage economy will feature the same behavioral investors that generated α_i^{pre} in this pre-arbitrage economy, α_i^{pre} will continue to measure the degree of latent mispricing that pushes the asset price away from the correct level and as the key cross-sectional variable in my comparative statistics.

2.3 The post-arbitrage equilibrium

A. The nature of equilibrium

The equilibrium in this "post-arbitrage economy" with a non-negligible mass ($\mu = 1/2$) of arbitrageurs is solved backward, starting from time 2. Time 2 represents the period immediately before mispricing disappears and asset prices converge to their fundamental value. Hence, an arbitrageur at time 2 devotes all available capital into mispriced assets without worrying about asset returns covarying with the level of arbitrage capital in the future. Time 1 represents the earlier periods of arbitrage in which an arbitrageur does worry about asset returns covarying endogenously with the level of arbitrage capital before the assets realize their fundamental value. The asset prices at time 1 therefore take into account these endogenous risks.

The equilibrium price at time 2 is given by the following lemma:

Lemma 2. ***(Time-2 equilibrium prices).*** *The equilibrium price of asset i at time 2 is*

$$p_{i,2} = m_{i,3} v \tag{5}$$

s.t. *(i) $m_{i,3} = 1/\left(1 + r_{max} i_2^*\right)$ for the "exploited" assets $i \in (i_2^*, 1]$.*
(ii) $m_{i,3} = 1/\left(1 + r_{max} i\right)$ for the "unexploited" assets $i \in [0, i_2^*]$.
(iii) i_2^* is the marginal asset s.t. $i_2^* = 1$, $1 - \sqrt{k_2}$, and 0 for $k_2 \in (-\infty, 0]$, $(0, 1)$, and $[1, \infty)$, respectively.

This result is intuitive. If $k_2 \geq 1$ and thus $i_2^* = 0$, the arbitrageur has enough capital to restore all asset prices to the correct level v. If $k_2 < 1$, the arbitrageur faces a capital constraint. In this case, the assets that the arbitrageur exploits have price $v/\left(1 + r_{max} i_2^*\right)$ and generate a positive expected return of $r_{max} i_2^*$, which is also the arbitrageur's shadow cost of capital.[8] The lower-i assets $[0, i_2^*]$ remain unexploited since their expected return is lower than $r_{max} i_2^*$ even before arbitrage capital enters into those assets.

These time-2 prices offer a glimpse into why high-i assets become endogenously riskier in this post-arbitrage equilibrium. This is because the prices of high-i assets respond more to the variation in k_2: as k_2 ranges from 0 to 1, the price of asset i rises from $v/\left(1 + r_{max} i\right)$ to v, implying a $r_{max} i$ percent increase in its price. The intuition is that the an initially more-mispriced asset relies more heavily on the price-correcting role of arbitrage capital, which makes its price more sensitive to the variation in the level of arbitrage capital.

From Lemma 2 follows the arbitrageur's marginal value of wealth at time 2:

Lemma 3. ***(Time-2 marginal value of wealth).*** *The arbitrageur's value function at time 2 is*

$$V_2 = \Lambda_2 w_2 \tag{6}$$

where the marginal value of wealth of wealth in the non-default state ($w_2 > 0$) is $\Lambda_2 = 1 + r_{max} i_2^*$ and that in the default state ($w_2 \leq 0$) is $\Lambda_2 = 1 + c$.

[8] Recall that the risk-free rate is 0 so that an expected return is also the expected excess return.

Lemma 3 implies that a low-k_2 state is a "bad" state in which the arbitrageur's marginal value of wealth is high: Λ_2 rises from 1 to $1+r_{max}$ and to $1+c$ as k_2 decreases from ∞ to 0^+ and to $-\infty$. This inverse relationship between Λ_2 and k_2 is a natural result that is expected to occur regardless of the preference for risk or intertemporal substitution, similarly to how the decreasing marginal utility of consumption does not rely on the curvature of the utility function. With risk-neutrality in particular, this happens because arbitrage capital k_2 falls precisely when the investment opportunity $r_{max}i_2^*$ improves.

Given this, the equilibrium price at time 1 is as follows:

Lemma 4. ***(Time-1 equilibrium prices).*** *The equilibrium price of asset i at time 1 is*

$$p_{i,1} = E_1\left[m_{i,2}\left(p_{i,2} + \delta_{i,2}\right)\right] \tag{7}$$

s.t. (i) $m_{i,2} = m_2^A \equiv \Lambda_2/\Lambda_1$ for the exploited assets $i \in \boldsymbol{I}_1^*$ where $\boldsymbol{I}_1^*$ is the set of exploited assets.
(ii) $m_{i,2} = m_{i,2}^B \equiv 1/\left(1 + r_{max}i\right)$ for the unexploited assets $i \in \boldsymbol{I}_1^*$.
(iii) Λ_1 is the time-1 marginal value of wealth s.t. $\Lambda_1 = E_1\left[\Lambda_2\right] + \psi_1$ where $\psi_1 > 0$ if the arbitrageur is constrained and $\psi_1 = 0$ if the arbitrageur is unconstrained.
(iv) The arbitrageur is unconstrained if k_1 is above a threshold $k_1^* \leq 1$.

This result is also intuitive. The exploited assets $\boldsymbol{I}_1^*$ are priced by the arbitrageur's stochastic discount factor (SDF) m_2^A, whereas the unexploited assets $[0,1] \setminus \boldsymbol{I}_1^*$ are priced by the behavioral investors' discount factor $m_{i,2}^B$. The law of one price is violated if the non-zero margin requirement makes the arbitrageur's capital constraint bind, similarly to Gârleanu and Pedersen (2011), Geanakoplos and Zame (2014), and Gromb and Vayanos (2017). The capital constraint is slack when k_1 is above the threshold k_1^*, and this value may be lower than the threshold value at time 2 because the arbitrageur at time 1 may find it optimal not to fully correct the asset prices if these prices covary endogenously with the arbitrage capital at time 2 (Shleifer and Vishny, 1997).

B. The equilibrium with unconstrained arbitrageurs

Based on the derivations above, I first study the equilibrium in the benchmark case where the arbitrageur is always unconstrained with sufficient capital ($k_1, k_2 \geq 1$). In this case, the arbitrageur completely eliminates the mispricings, and no endogenous risk arises:

Lemma 5. ***(Asset returns with unconstrained arbitrageurs).*** *Suppose $\mu = 1/2$ and $k_1, k_2 \geq 1$ with certainty so that the arbitrageur is always unconstrained. Then, the return on asset i follows*

$$r_{i,t} = \epsilon_t \tag{8}$$

where ϵ_t is the idiosyncratic return with mean zero.

With $k_2 \geq 1$, all assets get fully arbitraged and are priced v since the arbitrageur is unconstrained and holding a large number of assets diversifies away the idiosyncratic risks of the assets. Knowing that $p_{i,2} = v$ with certainty, the arbitrageur at time 1 also fully arbitrages all assets and drives up their prices to v. Hence, with frictionless arbitrage, the assets that were differently mispriced in the pre-arbitrage economy become indistinguishable in the post-arbitrage economy.

C. The equilibrium with constrained arbitrageurs

In contrast, if arbitrage capital is limited in the sense that it can face large shocks, these assets become endogenously risky since their prices in the interim period (time 2) comove endogenously with the arbitrageurs' marginal value of wealth. This implies that an asset's expected return at time 2 is no longer zero and can be explained by its beta with respect to the arbitrageur's stochastic discount factor:

Lemma 6. ***(Asset returns with constrained arbitrageurs).*** *Suppose $\mu = 1/2$ and k_2 has a full support over $[0, 1]$ so that the arbitrageur may be constrained. Then, the expected return on asset i at time 2 follows*

$$E_1 r_{i,2} = \alpha_{i,0} + \lambda_m \beta_{i,m} \tag{9}$$

s.t. (i) $\beta_{i,m}$ is the negative of the beta with respect to m_2^A, the arbitrageur's SDF (see Lemma 4).
(ii) $\alpha_{i,0} = 1/E_1\left[m_2^A\right] - 1$ if $i \in \boldsymbol{I}_1^*$.
(iii) $\alpha_{i,0} \leq 1/E_1\left[m_2^A\right] - 1$ if $i \notin \boldsymbol{I}_1^*$.

Furthermore, $\beta_{i,m} > 0 \; \forall i \in (0, 1]$ and $\lambda_m > 0$ so that the "limits of arbitrage" arise even when the arbitrageur is unconstrained at time 1 ($\psi_1 = 0$ and$\alpha_{i,0} = 0$).

The ability of the arbitrageur's SDF to "price" the cross-section of expected returns is not surprising—absent constraints, any rational investor's SDF should price the assets. Since the arbitrageur in this model may be capital-constrained, the arbitrageur's SDF only prices the exploited assets, whereas the unexploited assets should have a zero-beta rate $\lambda_{i,0}$ lower than that of the exploited assets' (λ_0^A).

Equation (9) nonetheless has the appeal that it expresses the limits of arbitrage mechanism as a beta-pricing model. Intuitively, arbitrage trading of single mispriced asset is endogenously risky since the return on the asset and the level of arbitrage capital comove endogenously (Shleifer and Vishny, 1997). In a multi-asset economy, this insight implies that the act of arbitrage makes the arbitraged assets endogenously risky ($\beta_{i,m} > 0$), which limits a complete arbitrage even in unconstrained times as the arbitrageur demands positive risk premium ($\lambda_m \beta_{i,m} > 0 \Longrightarrow E_1 [r_{i,2}] > 0$).

Although $\beta_{i,m}$ is the correct measure of risk, it relies on being able to observe the arbitrageur's stochastic discount factor m_2^A. It is therefore useful to obtain an approximate factor model based on the observable determinants of m_2^A. I do this with the level of arbitrage capital k_2, the key determinant of the arbitrageur's marginal value of wealth at time 2, and the subcomponents of k_2. The resulting approximate factor model is as follows:

Lemma 7. ***(A factor model of asset returns).*** *Suppose $\mu = 1/2$ and k_2 has a full support over $[0, 1]$ so that the arbitrageur may be constrained. Then, the expected return on asset i at time 2 approximately follows a factor model with respect to arbitrage capital k_2:*

$$E_1 r_{i,2} \approx \alpha_{i,0} + \lambda_k \beta_{i,k} \tag{10}$$

where $\alpha_{i,0}$ is the asset-specific zero-beta rate defined in Lemma 6 and $\beta_{i,k}$ is the arbitrage capital beta with $\beta_{i,k} > 0 \ \forall i \in (0, 1]$. Furthermore, this factor model can be expanded using the components of k_2:

$$E_1 r_{i,2} \approx \alpha_{i,0} + \lambda_r \beta_{i,r} + \lambda_w \beta_{i,w} + \lambda_f \beta_{i,f} \tag{11}$$

where $\beta_{i,z} = Cov\left(r_{i,2}, z_2\right)/Var\left(z_2\right)$ is the beta with respect to a factor $z_2 \in \left\{r_2^A, \tilde{w}_2, f_2\right\}$ and

$$r_t^A \equiv \frac{\int_0^1 r_{i,t} x_{i,t-1} di}{\int_0^1 \left|x_{i,t-1}\right| di} \tag{12}$$

is the arbitrageur portfolio return. By Stein's lemma, these factor models hold exactly if k_2 and

$r_{i,2}$ *are jointly normally distributed.*

In an arbitrageur-based asset pricing model like the one here, the level of arbitrage capital k_2 is a key determinant of the arbitrageur's marginal value of wealth. Hence, an asset that covaries strongly with the level of arbitrage capital is likely to command a high risk premium since such an asset is also likely to have a strong negative covariance with the arbitrageur's marginal value of wealth. This means that the underlying sources of systematic shocks to arbitrage capital can also serve as the approximate asset pricing factors: shocks to assets held by the arbitrageurs r_2^A, other shocks to the arbitrageur's wealth $\tilde{w}_2$ (e.g., investor flow shocks), and funding-liquidity shocks f_2.

Next, I show that the cross-section of post-arbitrage risks of the assets measured by their arbitrage capital betas $\beta_{i,k}$ line up with their pre-arbitrage alphas α_i^{pre}:[9]

Proposition 1. ***(Pre-arbitrage alpha determines the cross-section of post-arbitrage betas).*** *Suppose* $\mu = 1/2$ *and* k_2 *has a full support over* $[0,1]$ *so that the arbitrageur may be constrained.*
(i) Then, the cross-section of arbitrage capital betas increase in their pre-arbitrage alphas:

$$\frac{\partial \beta_{i,k}}{\partial \alpha_i^{pre}} > 0. \tag{13}$$

That is, "alphas turn into betas."
(ii) Similarly, the cross-section of betas with respect to the factors of k *also line up with* α_i^{pre}*:*

$$\frac{\partial \beta_{i,r}}{\partial \alpha_i^{pre}} > 0,\ \frac{\partial \beta_{i,w}}{\partial \alpha_i^{pre}} > 0,\ \frac{\partial \beta_{i,f}}{\partial \alpha_i^{pre}} > 0. \tag{14}$$

A large pre-arbitrage means that a larger fraction of the asset is held by the arbitrageur since she plays a larger price-correcting role in the asset in equilibrium. This, however, also means that this asset responds more to the variation in the arbitrage capital at time 2, making the asset endogenously more exposed to each of the different systematic shocks to arbitrage capital. What follows from this explanation is that the cross-section of average arbitrage positions in the assets should also explain the cross-section of post-arbitrage βs:

[9] I show this using $\beta_{i,k}$ instead of the exact measure of risk $\beta_{i,m}$ for a tighter link to the empirical section. Appendix B provides the corresponding proof for $\beta_{i,m}$.

Proposition 2. ***(The cross-section of average arbitrage positions explains the cross-section of post-arbitrage betas).*** *The expected arbitrage position in an asset* $\mu E_1[x_{i,2}]$ *explains the cross-section of betas* $\beta_{i,k}$ *in the post-arbitrage equilibrium. This is also true with respect to* $\beta_{i,r}$, $\beta_{i,w}$, *and* $\beta_{i,f}$.

One restriction on the the alphas-into-betas relation in Proposition 1 that can be useful in an empirical application is that this cross-sectional beta "predictability" comes from the constrained states of time 2. Put differently, in times when the arbitrageur has a "deep pocket," the arbitrageur does not generate endogenous βs in the assets:

Proposition 3. ***(The endogenous post-arbitrage beta arises when the arbitrageur is constrained).*** *The endogenous post-arbitrage betas arise only when the arbitrageur is constrained. That is,*

$$\begin{aligned} \beta_{i,k}|\,(k_2 \geq 1) = 0 \\ \beta_{i,k}|\,(k_2 < 1) > 0 \end{aligned} \tag{15}$$

for all $i \in (0,1]$. *For this reason, if* k_t *follows a process* k_t^* *such that* $k_1^*, k_2^* > 1$ *almost surely, then neither beta nor abnormal return arises:*

$$\beta_i = 0 \textit{ and } E_1[r_{i,2}] = 0 \textit{ for all } i \in [0,1] \tag{16}$$

Finally, asset pricing should be handled with care if the econometrician cannot distinguish between pre- and post-arbitrage economies. The presence of the alphas-into-betas relation means that part of the ability of beta to explain the cross-section of returns can come erroneously from the fact that assets with high pre-arbitrage returns due to α_i^{pre} has turned into beta in the post-arbitrage equilibrium:

Proposition 4. ***(An upward-biased price of risk in a naive asset pricing test).*** *Suppose that the arbitrageur enters time 1 to face a pre-arbitrage economy (*$\mu = 0$*) with probability* ϕ *and a post-arbitrage economy (*$\mu = 1/2$*) with probability* $1 - \phi$. *If this arbitrageur's capital* k_t *has a positive support on* $[0,1]$, *an econometrician who does not condition on* μ *observes an upward-biased price* *of risk associated with $\beta_{i,k}$:*

$$E\left[r_{i,2}\right] \approx (1-\phi)\,\alpha_{i,0} + \left(\lambda_k + \underbrace{\phi b^{-1}}_{\textit{bias}}\right)\beta_{i,k} \tag{17}$$

where b is the coefficient on the linearized alphas-into-betas effect, $\beta_{i,k} = b\,\alpha_i^{pre}$.

3 Application to Multifactor Models

Does the act of arbitrage help determine the cross-section of risks by turning alphas into betas in practice? If so, what does this imply about asset pricing using these risks?

3.1 The anomalies

To answer these questions, I study the cross-section of betas of equity anomalies. Equity anomalies are well-suited for this purpose for two reasons. First, they have been actively traded by arbitrageurs such as the quantitative equity hedge funds since the early 1990s (e.g., Hanson and Sunderam, 2014), suggesting that some of these anomalies' risks may be an endogenous outcome of the arbitrage itself. Second, they offer a reasonably rich cross-section as noted in Green, Hand, and Zhang (2016), allowing me to test the cross-sectional predictions of the model.

In particular, I use a set of 40 equity anomalies which are the "long" and "short" portfolios (top and deciles) of 20 anomaly characteristics (see the list in Table 1).[10] The return on an anomaly is the monthly value-weighted average return of all domestic, common stocks from the three major exchanges (NYSE, AMEX, and NASDAQ) that belonged to the extreme decile portfolios in the previous month.[11] I use data from 1974m1 to 2016m12.

[10]This list of 20 characteristics represents a standard set of low-turnover anomaly characteristics. One can arrive at this set by taking the 32 characteristics surveyed by Novy-Marx and Velikov (2016) and excluding the 5 redundant (e.g., "high-frequency combo") and 7 highest-turnover (e.g., short-term reversal) characteristics. I thank Mihail Velikov for several correspondences that helped ensure my replication of their anomalies. I exclude the high turnover anomalies since endogenous beta should *not* arise in anomalies with a short mispricing horizon (i.e., if the asset price achieves the fundamental value v at time 2, in the context of my model) (for a formal treatment of this point, see Gromb and Vayanos, 2017).

[11]See the online data appendix to this paper as well as Novy-Marx and Velikov (2016) for more information on the

3.2 The Fama-French (2015) five factors

I study the anomalies' cross-section of beta with respect to the five factors in the asset pricing model of Fama and French (2015). The model suggests that the market, size, value, profitability, and investment factors (labeled "MKT," "SMB," "HML," "RMW," and "CMA") summarize the systematic risks of a stock. Consistent with this claim, the five-factor model have an ability to explain high returns to beta arbitrage, net issuance, and volatility anomalies through their positive exposures to RMW and CMA in addition to the the size, value, investment and profitability anomalies that the model is intended to explain (Fama and French, 2015, 2016). Table 2 summarizes the 40 anomalies' beta exposures to the five factors in two different sample periods based on the 1993 cutoff explained next.[12]

3.3 The 1993 cutoff

I use 1993 as the approximate year in which the shift from the "pre-arbitrage" equilibrium to the "post-arbitrage" equilibrium occurred in equity anomalies. There are several reasons. First, the amount of hedge fund capital grew rapidly in the 1990s, with the total assets under management of hedge funds undergoing a dramatic change from \$39 billion in 1990 to \$1.73 trillion in 2008 (Stein, 2009). Second, some of the most influential papers in equity anomalies were published around 1993, including Fama and French (1993) and Jegadeesh and Titman (1993), igniting the practitioner interest. Third, the stock market liquidity improved significantly around 1993 (Chordia, Roll, and Subrahmanyam, 2011), lowering the cost of arbitrage on the anomalies (Chordia, Subrahmanyam, and Tong, 2014).

I back up this anecdotal evidence using actual arbitrage activity inferred from short interests. Since endogenous risk in equity anomalies, if exists, is likely to arise due to the activity of long-short arbitrageurs such as the equity quantitative hedge funds, abnormal shorting of a stock can serve as a noisy proxy for arbitrage activity by these arbitrageurs.[13] I therefore measure arbitrage

anomaly construction.

[12]I thank Kenneth French for providing the factor data through his data library.

[13]Ben-David, Frazoni, and Moussawi (2012) and Boehmer, Jones, and Zhang (2013) both argue that hedge funds are responsible for most of the short interest. Other papers that measure arbitrage activity using short interest include Hanson and Sunderam (2014) and Hwang, Liu, and Xu (2018).

activity as the negative ($\times -1$) of the abnormal short interest, which I define to be the value-weighted average of short interest ratio (shares shorted / shares outstanding) minus the cross-sectional average short interest ratio of stocks that belong to the size NYSE size decile, where the average is taken over all stocks that belong to the anomaly portfolio.

Arbitrage activity inferred from short interests on the anomalies tell a consistent story about the 1993 cutoff (Table 3). The post-1993 dummy has the single largest effect on the growth of arbitrage activity on the anomalies, controlling for both the post-publication and post-sample effects (read the coefficients on variables interacted with the long anomaly dummy or pre-1993 alpha).[14] The publication effect is significant only in the post-1993 period, when hedge funds had sufficient capital to generate observable effect on short interests in response to an academic publication. Multifactor alphas seem to be a better measure of the anomalies' latent mispricing assessed by the arbitrageurs (the R^2 is lower when CAPM α is used), but the exact horizon in which the alpha is measured during the pre-1993 period matters less.

3.4 The evidence

Is there evidence of the alphas-into-betas effect arising in practice, and if so, to what extent are the anomalies' five-factor betas endogenous? (Note: Throughout this paper, "endogenous beta" refers to the part of the beta that arises from arbitrage on past returns, which makes the beta an endogenous variable in an asset pricing regression with realized past returns on the left-hand side).

The first task is to infer which of the five factors represent systematic shocks for a long-short arbitrageur. Table 4 shows that in the pre-1993 period, both long and short anomalies are positively exposed to MKT and SMB and negatively exposed to HML, allowing a long-short strategy to neutralize these exposures. In contrast, the long and short anomalies respectively have positive and negative exposures to RMW and CMA, making them "slope" factors in the language of Clarke (2016). Consistent with this evidence, Table A3 shows that a 90/10 mix of equity market-neutral

[14] My finding on the 1993 cutoff is at odds with the finding that no return decay is observed in the anomalies following 1993 (McLean and Pontiff, 2016). The main reason for this difference is that short interest measures the arbitrage activity by a group of sophisticated arbitrageurs, whereas return decay reflects investment by all types of investors as well as other determinants of return changes such as the change in risk premium. Another contributing factor is that I use the year in which the anomaly was first published, not when it was first well-publicized (e.g., the academic publication of value anomaly is Rosenberg, Reid, and Lanstein 1985 in my data but Fama and French 1992 in McLean and Pontiff.

and equity short-bias hedge funds neutralizes the exposures to MKT, SMB, and HML but has positive exposures to RMW and CMA. This suggests that a long-short arbitrage on the anomalies has a natural exposure to both RMW and CMA (and more so to RMW), implying that the anomalies' post-1993 exposures to these two factors may be endogenous.

I therefore test whether the anomalies' pre-1993 five-factor alphas predict their post-1993 betas with respect to RMW and CMA but *not* those with respect to MKT, SMB, and HML (Proposition 1). Table 5 shows that a high pre-1993 alpha predicts high post-1993 RMW and CMA betas, controlling for the effect of pre-1993 beta. Comparing the R^2s suggests that as much as 38% (RMW) and 21% (CMA) of the cross-sectional variation in the post-1993 betas may be endogenous. The coefficient on pre-1993 beta is around .6 for both RMW and CMA (and for SMB and HML as well), suggesting that these factor betas shrink by a factor of 0.6 around the cross-sectional mean of zero between the two sample periods, similarly to how the market beta shrinks around one over time (e.g., Vasicek, 1973).

The αs-into-βs effect is stronger for RMW betas than for CMA betas, which may be because a long-short strategy is slightly more exposed to the RMW factor (Table 4). The effect exists in both long and short anomalies for RMW betas, but it only exists in short anomalies for CMA betas, which is interesting since only the long anomalies had significant exposure to CMA in the pre-1993 period (Table 4). This can mean that CMA shocks from the long positions have been transmitted to short positions, but the weak effect in long anomalies remains a puzzle.

The effect does not exist for SMB and HML, consistent with the earlier observation that a long-short strategy neutralizes the exposure to these factors. With MKT, a high pre-1993 alpha predicts a *lower* post-1993 beta, which is consistent with a simple long-short strategy in Table 4 having a sightly negative exposure to MKT but inconsistent with equity market-neutral hedge funds having a slightly positive market exposure. One explanation is that dedicated "short" arbitrageurs whose capital is negatively exposed to market movements have significant positions in the short anomalies and cause the prices of short anomalies to escalate quickly when these group of arbitrageurs unwind their positions quickly in a sudden bullish movement in the market. And when this happens, the lower-alpha anomalies with more arbitrage short interests would experience a larger price increase, which explains their higher post-1993 market betas. This explanation is consistent with the negative αs-into-βs effect for MKT existing only in the short-side anomalies.

A. The measurement-error explanation

The results above are consistent with the arbitrage-based explanation for αs-into-βs, but what other explanations exist? To answer this, I take a step back and consider the αs-into-βs regression. The estimated equation is

$$\hat{\beta}_i^{post} = b_0 + b_1 \hat{\alpha}_i^{pre} + b_2 \hat{\beta}_i^{pre} + u_i, \tag{18}$$

where $\hat{\alpha}$ and $\hat{\beta}$ are the estimated alphas and betas. Assuming for simplicity a single-factor model, this can be rewritten as

$$\beta_i^{post} + e_i^{post} = b_0 + b_1 \left(\alpha_i^{pre} + \overline{\epsilon}_i^{pre} - \overline{g}_i e_i^{pre}\right) + b_2 \left(\beta_i^{pre} + e_i^{pre}\right) + u_i, \tag{19}$$

which expresses the estimated alpha as the actual alpha α plus the realized mean error of the return process $\overline{\epsilon}$ minus the measurement error in the risk premium (realized factor mean $\overline{g}$ times the measurement error in the beta e_i). Hence, even with $b_1 = 0$, the estimated $\hat{b}_1$ is *expected* to be positive when b_2 is positive: a positive measurement error e^{pre} means that the actual beta is lower. The problem is aggravated if anomaly "discovery" happens when a researcher finds a strategy whose realized in-sample alpha is higher because its in-sample beta is erroneously lower—i.e., if "factor mining" has focused on anomalies with erroneously low (for long anomalies) or high (for short anomalies) realized in-sample risk premia.

However, two observations are at odds with this measurement-error explanation. First is the *absence* of a positive αs-into-βs effect in MKT, SMB, and HML. All five factors have positive realized mean in the pre-1993 period, implying that the measurement error issue, if exists, should apply to all five factors. Instead, the absence of a positive αs-into-βs effect in those three factors suggests that betas are measured with a sufficient precision over 240 months in the pre-1993 period. Second, I test if RMW and CMA betas of the anomalies change predictably from in to out-of sample, finding that they do not once the post-1993 effect has been controlled for (Table A1). This is in contrast to MKT betas of long (short) anomalies dropping (increasing) predictably out-of-sample, suggesting that "beta mining" did occur with respect to MKT (Table A2).[15] This helps alleviate further the concern that anomalies are the ones whose in-sample RMW and CMA betas

[15] This null finding on RMW and CMA betas is also consistent with RMW and CMA not being the original Fama and French (1992, 1993) factors, although beta mining could have occurred indirectly through RMW and CMA being closely related to HML (Fama and French, 2015).

are measured with errors due to factor mining.

B. Further evidence for the arbitrage channel

On the other hand, two additional pieces of evidence are consistent with arbitrage-based explanation for the αs-into-βs effect in RMW and CMA. First, using the time-series average of the arbitrage activity measure in section 3.3 to measure average arbitrage activity in an anomaly in the post-1993 period, I test whether endogenous beta is larger in an anomaly that has a larger average arbitrage activity (Proposition 2).

The results in Table 6 are strongly in favor of arbitrage activity explaining the cross-section of betas in the post-1993 period: anomalies that attain high post-1993 RMW and CMA betas are the ones with larger average arbitrage positions in the same period, consistent with the intuition that an anomaly in which arbitrageurs play a larger price-correcting role endogenously has a larger price sensitivity to the variation in arbitrage capital. The effect is not symmetric for CMA betas, however, which may be because my measure of arbitrage activity based on short interests is a poorer proxy for arbitrageur price-correcting role in long anomalies.

It is useful to consider this result jointly with the αs-into-βs result in Table 5. In both regressions, I use α^{pre} or arbitrage activity as the determinant of endogenous beta, but the conceptual motivation is different. The advantage of the arbitrage activity measure is that it directly measures the underlying cause of the endogenous betas, but inferring arbitrage activity from short interests is a noisy exercise that introduces large measurement errors. When this measurement error is large, the pre-1993 alpha acts as an instrument for the actual arbitrage activity in the post-1993 period and can provide a cleaner estimate of the αs-into-βs effect if arbitrage capital is allocated to anomalies based on past alphas.[16]

The predictability of post-1993 betas using pre-1993 alphas comes from the constrained times of the post-1993 period, consistent with the notion that in times when the arbitrageur has a "deep pocket," the arbitrageur does not generate endogenous βs in the anomalies (Proposition 3). To test this, I proxy the constrained times within the post-1993 as the months in which the 3-month moving average of VIX was above the median, which roughly gives 1997m1-2003m8 and 2007m7-

[16] I thank Yao Zeng for this point.

2012m6 as the times in which arbitrageurs' capital constraint was likely to be binding (Figure 1). The first period begins in the aftermath of the Peso crisis and includes the LTCM, 9/11, and the Iraq War, whereas the second period coincides with the beginning and the aftermath of the Great Recession. I then ask if the αs-into-βs result earlier is coming from these constrained times of the post-1993 period in which shocks to arbitrage capital should translate into a larger price response in the anomalies with the arbitrageur finding it harder to replenish its capital.

Comparing the R^2s in Table 7 shows that the ability of pre-1993 alphas to explain the post-1993 betas is stronger during constrained times, consistent with the risk-allocating role of arbitrage arising only when arbitrageurs are constrained. Some predictive power of pre-1993 alphas remains in my proxy for unconstrained times, suggesting that the constrained vs. unconstrained classification is imperfect. The larger alphas and lower arbitrage activities both indicate that the high-VIX times are indeed when arbitrageurs operate with a reduced level of capital.

An alternative explanation for this result is that pre-1993 alpha represents compensation for conditional risk with respect to these two factors. However, I find that the differential αs-into-βs result does not arise in MKT, SMB, and HML, implying that the pre-1993 alpha is not a compensation for conditional risks with respect to the five factors (Table A4). Another useful exercise would be to find a measure comparable to VIX in the pre-1993 period (VIX begins in 1990) and repeat the exercise to see if the conditional beta also arises in the pre-1993 period as well, which I have not done.

C. Costs of arbitrage

Costs of arbitrage such as idiosyncratic volatility and liquidity may matter in arbitraging anomalies, especially for anomalies that involve short selling (Knez and Ready, 1996; Mitchell and Pulvino, 2001; Korajczyk and Sadka, 2004; Lesmond, Schill, and Zhou, 2004; Pontiff, 2006; Duan, Hu, and McLean, 2010). Hence, if the αs-into-βs effect arises through arbitrage, the strength of the effect may depend on the costs of arbitraging each anomaly. I therefore include arbitrage cost and its interaction with pre-1993 alpha in my main αs-into-βs regression, using size, idiosyncratic volatility, illiquidity (Amihud, 2002), and bid-ask spread (Corwin and Schultz, 2012) to measure the cost.

Arbitrage costs affect the αs-into-βs relation in the direction one would expect if the relation were driven by arbitrage (Table 8; each cost variable is signed so that a high value means high arbitrage cost). Anomalies whose underlying stocks are small, volatile, and have a large bid-ask spread see their alphas turn into smaller betas, although the the effect—albeit weak—has an opposite direction for the Amihud measure of illiquidity. Among different costs of arbitrage, idiosyncratic volatility and bid-ask spread have the largest magnitudes of the effect, and their effect is statistically significant for CMA but not for RMW. The finding that idiosyncratic volatility plays an important role in determining the level of arbitrage activity inferred from the αs-into-βs relation is consistent with the finding of McLean and Pontiff (2016) based on anomaly return decay after academic publication.

3.5 Implications for asset pricing tests

So far I have presented theoretical and empirical evidence that the act of arbitrage may be contributing to the determination of the cross-section of risks. Understanding the role of arbitrage in determining the cross-section of risks is interesting in its own right, but it also important for interpreting asset pricing test results correctly.

This is because the αs-into-βs effect represents a source of endogeneity in the β in an asset pricing test (Proposition 4). Consider a cross-sectional asset pricing regression with a single factor, assuming for simplicity that all quantities are perfectly measured:

$$E\left[r_i^e\right] = \lambda_0 + \lambda_1 \beta_i + \upsilon_i, \tag{20}$$

where λ_1 is the price of risk associated with the factor. Also, rewrite the αs-into-βs in terms of expected return to obtain

$$\beta_i^{post} = b_0 + b_1 \left(\underbrace{E\left[r_i^{e,pre}\right] - \lambda_1 \beta_i^{pre}}_{=\alpha_i^{pre}} \right) + b_2 \beta_i^{pre} + u_i, \tag{21}$$

again assuming that no measurement issue arises.

This should remind the reader of the classic simultaneity problem in econometrics. If the regression in eq. (7) is run in a sample in which the fraction ϕ of the time periods is the pre-arbitrage equilibrium, the asset pricing regression becomes

$$\phi E\left[r_i^{e,pre}\right] + (1-\phi) E\left[r_i^{e,pre}\right] \approx \lambda_0 + \underbrace{\phi\lambda_1\beta_i^{pre} + (1-\phi)\lambda_1\beta_i^{post}}_{\approx\lambda_1\beta_i} + \upsilon_i. \tag{22}$$

But eq. (21) implies that β_i^{post} is partly determined by $E\left[r_i^{e,pre}\right]$, which biases the estimated λ_1 upward if $b_1 > 0$. That is, a naive asset pricing test attributes part of the realized return due to (realized) alpha to risk premium associated with beta, when in fact the positive relationship between the two in the cross-section of test assets is due to high past alphas turning into high future betas through arbitrage.

If the empirical evidence presented in this paper is correct, this "endogenous beta" problem will arise in using the FF five-factor model to price the cross-section of expected returns, especially when the test assets have large exposures to RMW and CMA. Addressing this issue more formally will be an interesting future work. Some immediate remedies include (i) comparing the price of risk estimates from a cross-sectional regression with the realized factor means and (ii) estimating the price of risk in samples with decreasing exposure to the possible pre-arbitrage sample to see if the estimated price of risk falls.

3.6 Implications for discount-rate vs. cash-flow channels

My model and empirical evidence does not help distinguish whether the systematic factor in question is a discount-rate shock or a cash-flow shock, an interesting question that matters especially for long-term investors.[17] However, it does suggest that the arbitrage-driven change in the anomalies' betas with respect to a factor represents a discount-rate exposure of the anomaly to that factor. Hence, this component of the beta represents risk that dissipates if the anomaly is held for a sufficiently long period. This is good news for long-term investors trading anomalies with a long horizon (e.g., value) but matters less for investment in short-horizon anomalies (e.g., momentum).

[17] See Cohen, Polk, and Vuolteenaho (2009) for an analysis of size and value anomalies from this perspective.

3.7 Other multifactor models

There are other factor models of equity anomalies suggested in the literature. These include, but are not limited to, the models motivated by the arbitrage pricing theory (APT) (e.g., Fama and French, 1992, 1993; Carhart, 1997), intertemporal CAPM (e.g., Campbell and Vuolteenaho, 2002; Campbell, Giglio, Polk, and Turley, 2017), q-theory (e.g., Hou, Xue, and Zhang, 2015), consumption and macroeconomic risks (e.g., Parker and Julliard, 2005; Yogo, 2006; Lettau, Ludvigson, and Ma, 2017), and behavioral factors (e.g., Stambaugh and Yuan, 2016; Daniel, Hirschleifer, and Sun, 2017). My findings about the FF 5-factor model and their asset pricing implication should apply similarly to these models: asset pricing tests should be handled with care if their factors represent systematic shocks that the long-short arbitrageurs of anomalies must bear so that the act of arbitrage changes the cross-section of betas. This makes the "level, slope, and curve" factor analysis of Clarke (2016) useful since a long-short arbitrage is likely to be exposed only to "slope" factors, making those factors likely suspects for the alphas-into-betas effect and hence the endogenous beta problem in an asset pricing test.

4 Application to Intermediary-based Asset Pricing

Intermediary-based asset pricing models offer another interesting opportunity to test the alphas-into-betas prediction. These models have enjoyed some empirical success in pricing the cross-section of assets by measuring risks from the perspective of financial intermediaries (e.g., Adrian, Etula, and Muir, 2014; He, Kelly, and Manela, 2017). These models, however, seem incomplete without an adequate explanation on where the cross-section of different intermediary βs come from:

> ". . . ultimately we want to understand not only the pricing of assets' covariances with risk factors, but also the determinants of these covariances themselves. In the context of [Adrian, Etula, and Muir (2014)]'s empirical analysis specifically, a full account of margin risk should explain why some assets covary more strongly than others with the leverage of security broker-dealers" (p.484 of Campbell, 2017).

Why do some assets covary more strongly with systematic shocks to financial intermediaries than others? To what extent are the intermediary asset pricing betas an endogenous outcome of arbitrage?

4.1 Funding-liquidity factor of Adrian, Etula, and Muir (2014)

I answer these questions using the funding-liquidity factor of Adrian, Etula, and Muir (2014) (AEM). Motivated by funding-liquidity model of Brunnermeier and Pedersen (2009), AEM focuses on aggregate funding-liquidity shocks to financial intermediaries, which they proxy using shocks to the leverage of security broker-dealers (plotted in Figure 2). Using these shocks, they find that high returns to value and momentum anomalies are explained by their positive exposures to aggregate funding-liquidity shocks.

However, these funding-liquidity exposures of anomalies may be endogenous given that long-short arbitrageurs like hedge funds are strongly exposed to these shocks (Brunnermeier and Pedersen, 2009; Aragon and Strahan, 2012; Mitchell and Pulvino, 2012). This seems especially plausible in the context of the AEM factor given that the majority of security broker-dealers provide funding to hedge funds as prime brokers.

4.2 The evidence

I therefore test if the funding-liquidity betas of 40 anomalies are consistent with the arbitrage-based alphas-into-betas explanation (Table 9). In the pre-1993 period, the anomalies did not have strong exposures to this measure of funding liquidity. It is only in the post-1993 period that the anomalies attain strong beta exposures, consistent with anomalies on average having no inherent exposure to funding liquidity. These exposures are concentrated in short anomalies, which may be due to short positions being exposed to margin shocks.

Additional evidence also point to the arbitrage-based explanation for the betas. The cross-section of betas are explained by the cross-section of average arbitrage positions in the post-1993 period, consistent with anomalies with more arbitrageur involvement being endogenously more exposed to arbitrage capital shocks. The endogenous part of the beta comes from parts of post-1993 period in which arbitrageurs are likely to be capital-constrained, proxied by quarters in which the VIX is above the post-1993 median. This is again consistent the alphas-into-betas effect arising when arbitrageurs are constrained but not when they are unconstrained.

4.3 Implications for intermediary-based asset pricing

The endogenous beta problem discussed in section 3.5 is is especially relevant for intermediary-based models. The premise of these models is to identify important shocks to intermediary capital or risk aversion, and these shocks can easily be transmitted to assets traded by these intermediaries. This transmission of shocks would be especially strong for assets that were initially more mispriced in the absence of intermediary capital from the intermediaries' perspective. In this way, the "pre-intermediary" alpha can turn into "post-intermediary" beta, generating the endogenous beta problem discussed earlier. This issue is missing from the current intermediary-based asset pricing literature but needs to be addressed as the literature matures.

5 Conclusion

In this paper, I show both theoretically and empirically that the act of arbitrage plays an important role of determining the cross-section of risks of the arbitraged assets. When risks arise in this manner, a cross-sectional asset pricing test can lead to a false discovery of a pricing factor. Useful future work is to provide additional empirical evidence for or against this risk-allocating role of arbitrage in the equity and other markets and to examine the severity of the endogeneity problem caused by the alphas-into-betas effect in popular asset pricing models.

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Table 1: **List of Anomaly Characteristics**

No	Anomaly	Label	Academic Publication			Market Cap Share	
			Authors	Year	Sample	Short	Long
1	Beta arbitrage	beta	Fama and MacBeth	1973	1926-1968	0.09	0.09
2	Return on market equity	rome	Basu	1977	1956-1971	0.05	0.03
3	Ohlson's O-score	ohlson	Ohlson	1980	1970-1976	0.29	0.01
4	Size	size	Banz	1981	1926-1975	0.02	0.58
5	Long-run reversals	rev60m	DeBondt and Thaler	1985	1926-1982	0.03	0.13
6	Value	value	Rosenberg, Reid, and Lanstein	1985	1980-1990	0.04	0.2
7	Momentum	mom12m	Jegadeesh	1990	1964-1987	0.1	0.04
8	Net issuance	netissue	Ikenberry, Lakonishok, and Vermaelen	1995	1980-1990	0.11	0.09
9	Net issuance monthly	netissue_m	Ikenberry, Lakonishok, and Vermaelen	1995	1980-1990	0.1	0.09
10	Accruals	acc	Sloan	1996	1962-1991	0.06	0.05
11	Return on assets	roa	Haugen and Baker	1996	1979-1993	0.17	0.03
12	Return on book equity	roe	Haugen and Baker	1996	1979-1993	0.14	0.04
13	Failure probability	failprob	Dichev	1998	1981-1996	0.16	0.02
14	Piotroski's f-score	piotroski	Piotroski	2000	1976-1997	0.21	0.09
15	Investment	invest	Titman, Wei, and Xie	2004	1973-1996	0.03	0.07
16	Idiosyncratic volatility	idiovol	Ang et al.	2006	1986-2000	0.25	0.04
17	Asset growth	atgrowth	Cooper, Gulen, and Schill	2008	1968-2003	0.03	0.1
18	Asset turnover	ato	Soliman	2008	1984-2002	0.05	0.09
19	Gross margins	gm	Soliman	2008	1984-2002	0.2	0.04
20	Gross profitability	profit	Balakrishnan, Bartov, and Faurel	2010	1976-2005	0.1	0.07

Table 2: **Beta Exposures to Fama-French (2015) Five Factors**

For failure probability, the pre-arbitrage period is computed from 1981m1 to account for its sensitivity to sample period emphasized in Dichev (1998). Significance at the 5% level based on heteroskedasticity-robust OLS standard errors is expressed in boldface. Returns are in annualized percentages.

		Pre-1993 Period (1974m1-1993m12)							Post-1993 Period (1994m1-2016m12)						
				β							β				
No	Anomaly	$\overline{r^e}$	α	MKT	SMB	HML	RMW	CMA	$\overline{r^e}$	α	MKT	SMB	HML	RMW	CMA
1	beta(L)	**7.92**	2.58	**0.67**	0.06	**0.40**	**-0.28**	-0.19	**7.75**	0.8	**0.54**	-0.01	0.09	**0.28**	**0.43**
2	rome(L)	**16.39**	**5.43**	**1.10**	**0.30**	**0.58**	0	**-0.22**	**13.74**	2.93	**1.06**	**0.20**	**0.30**	**0.30**	0.08
3	ohlson(L)	5.96	0.7	**0.97**	**-0.22**	**-0.21**	**0.17**	**0.14**	**8.45**	**2.14**	**0.97**	**-0.15**	**-0.28**	0.03	-0.01
4	size(L)	**9.48**	-2.21	**0.89**	**1.19**	0.01	**-0.20**	0.15	**9.73**	2.53	**0.81**	**0.97**	0.07	**-0.40**	0.12
5	rev60m(L)	**11.23**	-4.63	**1.18**	**0.89**	**0.36**	-0.08	0.4	**12.61**	2.12	**1.16**	**0.59**	**0.33**	**-0.44**	**0.39**
6	value(L)	**13.38**	-2.43	**1.15**	**0.49**	**0.82**	0.08	0.17	**8.93**	0	**0.93**	**0.15**	**0.65**	-0.12	0.06
7	mom12m(L)	**14.06**	**6.29**	**1.11**	**0.38**	**-0.48**	-0.03	**0.32**	**8.74**	1.62	**1.02**	**0.36**	**-0.42**	0.01	-0.07
8	netissue(L)	**10.92**	1.05	**1.02**	**0.08**	**0.16**	**0.24**	**0.23**	**12.48**	1.85	**1.02**	**0.11**	**0.12**	**0.41**	**0.20**
9	netissue_m(L)	**11.04**	1.18	**1.05**	**0.10**	**0.27**	**0.33**	-0.02	**10.72**	0.29	**1.04**	-0.04	0.12	**0.33**	**0.29**
10	acc(L)	7.88	-0.37	**1.02**	**0.21**	**-0.18**	0.15	**0.24**	7.65	0.69	**1.16**	0.04	-0.18	**-0.27**	-0.09
11	roa(L)	7.05	**2.68**	**0.98**	-0.07	**-0.46**	**0.20**	0.08	**9.80**	**2.74**	**0.98**	-0.02	**-0.42**	**0.30**	-0.1
12	roe(L)	8.72	**3.58**	**1.04**	0.03	**-0.45**	**0.22**	0.04	**9.76**	0.97	**1.01**	-0.04	**-0.25**	**0.40**	0.1
13	failprob(L)	7.75	2.48	**0.93**	0	**-0.41**	0.08	0.23	**10.02**	3.03	**0.87**	0.16	**-0.31**	0.14	0.11
14	piotroski(L)	7.16	1.69	**0.96**	-0.03	**-0.10**	**0.14**	-0.1	**7.70**	-0.31	**1.02**	**0.06**	-0.1	**0.19**	-0.09
15	invest(L)	**12.03**	0.58	**1.14**	**0.37**	-0.06	0	**0.53**	**9.84**	0.51	**1.02**	**0.24**	-0.11	-0.09	**0.53**
16	idiovol(L)	**6.31**	1.09	**0.83**	**-0.30**	**0.20**	0.03	-0.02	**8.04**	0.33	**0.79**	**-0.17**	0.06	**0.24**	**0.27**
17	atgrowth(L)	**10.70**	-1.82	**1.14**	**0.62**	**-0.23**	-0.15	**0.80**	**10.17**	0.4	**1.08**	**0.11**	-0.06	**-0.15**	**0.63**
18	ato(L)	**10.56**	0.82	**1.03**	**0.37**	-0.04	**0.37**	0.07	**8.84**	0.34	**0.95**	**0.16**	-0.06	**0.44**	-0.2
19	gm(L)	4.7	0.45	**0.93**	**-0.11**	**-0.37**	0.11	0.1	**9.27**	**3.71**	**0.97**	**-0.14**	**-0.35**	-0.06	-0.06
20	profit(L)	7.05	1.56	**0.94**	-0.02	**-0.31**	**0.28**	0.07	**10.48**	2.17	**0.94**	0.08	**-0.27**	**0.29**	0.17
21	beta(S)	4.29	-3.71	**1.31**	**0.55**	0.02	**-0.26**	**-0.51**	5.73	-4.07	**1.53**	0.16	**0.52**	**-0.54**	**-0.42**
22	rome(S)	-1.13	**-11.53**	**1.16**	**0.30**	-0.1	-0.2	**0.52**	4.31	-3.59	**1.38**	**0.34**	0.25	**-0.89**	-0.1
23	ohlson(S)	3.08	**-6.49**	**1.10**	**0.81**	-0.13	**-0.45**	0.13	5.07	-2.14	**1.18**	**0.66**	0.1	**-0.57**	**-0.33**
24	size(S)	5.06	-0.73	**0.98**	**-0.29**	-0.01	**0.15**	**0.07**	**7.10**	0.19	**0.99**	**-0.26**	0	0.02	-0.02
25	rev60m(S)	4.86	-0.72	**1.10**	0.1	**-0.52**	**0.20**	0.08	**9.13**	2.12	**1.14**	0.03	**-0.18**	**0.13**	**-0.50**
26	value(S)	3	-0.34	**0.99**	-0.02	**-0.55**	**0.12**	-0.03	**7.45**	1.18	**1.03**	**-0.12**	**-0.35**	**0.07**	**-0.16**
27	mom12m(S)	-4.5	**-11.32**	**1.07**	**0.51**	0.23	**-0.48**	**-0.51**	2.72	-5.35	**1.44**	0.19	**0.62**	**-0.64**	**-0.70**
28	netissue(S)	3.16	-0.6	**0.97**	**0.19**	-0.04	**-0.41**	**-0.42**	3.44	**-3.78**	**1.10**	0.04	**0.24**	**-0.14**	**-0.39**
29	netissue_m(S)	5.31	0.48	**0.99**	**0.22**	-0.01	**-0.34**	**-0.34**	3.92	**-3.09**	**1.12**	**0.12**	**0.26**	**-0.28**	**-0.41**
30	acc(S)	3.81	**-2.73**	**1.09**	**0.42**	**-0.27**	0.09	**-0.27**	5.58	-1.83	**1.08**	**0.44**	**-0.18**	0.09	**-0.47**
31	roa(S)	-0.29	**-11.89**	**1.09**	**0.66**	0.05	**-0.33**	**0.41**	3.27	-1.83	**1.22**	**0.37**	**-0.20**	**-0.88**	**-0.21**
32	roe(S)	0.51	**-11.45**	**1.11**	**0.63**	0.18	**-0.30**	0.3	1.57	**-4.33**	**1.29**	**0.32**	-0.11	**-0.91**	-0.14
33	failprob(S)	-2.39	**-11.46**	**1.29**	**1.09**	**0.31**	**-0.47**	-0.11	0.67	-8.94	**1.69**	0.32	0.46	**-0.78**	-0.59
34	piotroski(S)	3.36	**-3.51**	**1.04**	0	-0.04	**-0.33**	**0.26**	7.09	-1.07	**1.02**	**0.14**	-0.04	**-0.21**	**0.34**
35	invest(S)	3.77	-2.2	**1.12**	**0.25**	**-0.18**	0.11	**-0.38**	5.2	-3.1	**1.06**	**0.30**	0.11	**0.22**	**-0.48**
36	idiovol(S)	-3.6	**-14.52**	**1.09**	**1.16**	-0.02	**-0.37**	-0.1	3.67	-2.23	**1.37**	**0.64**	-0.19	**-0.99**	**-0.36**
37	atgrowth(S)	4.19	-1	**1.09**	**0.31**	**-0.29**	0.02	**-0.36**	6.07	-0.33	**1.10**	**0.19**	**-0.24**	0.08	**-0.58**
38	ato(S)	5.53	**3.15**	**0.77**	**-0.19**	**0.33**	**-0.60**	**-0.39**	3.69	-2.53	**1.02**	**-0.12**	0	**-0.27**	-0.04
39	gm(S)	7.46	-1.58	**1.08**	**0.27**	-0.02	**-0.20**	**0.28**	6.43	-0.83	**0.94**	**0.39**	-0.02	**-0.20**	0.05
40	profit(S)	4.88	1.8	**0.90**	-0.01	**0.29**	**-0.92**	**-0.36**	6.12	1.27	**0.84**	**-0.10**	-0.06	**-0.33**	0.07

Table 3: **Arbitrage Activity on Anomalies Inferred from Short Interests**

$$\text{Baseline:}\quad ArbActivity_{i,t} = b_0 + (\alpha_i^{pre} \times \boldsymbol{X}_{i,t})'\,\boldsymbol{b}_1 + \boldsymbol{X}'_{i,t}\boldsymbol{b}_2 + u_i + \epsilon_{i,t}$$

The table studies the determinants of arbitrage activity on an anomaly using panel data (40 anomalies $\times$ 1974m1-2016m12) of short interests. The dependent variable measures arbitrage activity on anomaly i in month t using the negative of (-1×10^4) the "abnormal" short interest defined as the value-weighted average of short interest ratio (shares shorted / shares outstanding) minus the cross-sectional average short interest ratio of stocks that belong to the size NYSE size decile, where the average is taken over all stocks that belong to the anomaly portfolio. I use short interests reported in mid month and shares outstanding on the same day (if available) or the previous trading day. The post-1993 dummy is 0 for the pre-1993 (1974m1-1993m12) and 1 for the post-1993 (1994m1-2016m12) periods. For columns (6)-(12), an anomaly's "pre-arbitrage" alpha, denoted α^{pre}, is measured by its alpha with respect to the multifactor model specified in the column head in the pre-1993 period. Columns (11) and (12) use α^{pre} computed in the last 15 (1979m1-1993m12) and 10 years (1984m1-1993m12) leading to 1993m12, respectively. For failure probability, α^{pre} is computed from 1981m1 onward. Post-Publication, Post-Sample, Post-1993, Post-1993$\times$Post-Pub, and constant terms are included in the regression (whenever appropriate) but not reported in the table. In the parentheses are standard errors adjusted for cross-anomaly covariances. ***, **, and * indicate 1%, 5%, and 10% significance levels.

	Long vs. Short					Model: FF5		CAPM	FF3	Carhart	FF5('79-)	FF5('84-)
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
Long $\times$ Post-Publication	66.09***			5.42	-15.26							
	(18.74)			(14.19)	(18.77)							
Long $\times$ Post-Sample		71.70***		11.20	16.59							
		(18.53)		(18.69)	(16.76)							
Long $\times$ Post-1993			79.08***	69.12***	55.31***							
			(17.11)	(18.90)	(14.61)							
Long $\times$ Post-1993 $\times$ Post-Pub					29.04							
					(24.14)							
α^{pre} $\times$ Post-Pub						3.40**	-0.25	-0.49	-1.48	-0.58	-1.22	-1.58
						(1.50)	(1.57)	(1.44)	(1.33)	(1.99)	(1.54)	(1.61)
α^{pre} $\times$ Post-Sample						0.77	2.11	3.36**	1.80	0.66	2.24*	2.51
						(1.72)	(1.35)	(1.54)	(1.18)	(1.39)	(1.25)	(1.63)
α^{pre} $\times$ Post-1993						8.86***	6.14***	5.71***	5.21***	7.62***	6.50***	6.77***
						(1.43)	(1.06)	(1.19)	(1.02)	(0.91)	(1.08)	(1.40)
α^{pre} $\times$ Post-1993 $\times$ Post-Pub							5.04***	2.69	5.41***	7.35***	6.32***	6.24***
							(1.64)	(1.76)	(1.44)	(2.26)	(1.36)	(1.71)
Anomaly FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Observations	20640	20640	20640	20640	20640	20640	20640	20640	20640	20640	20640	20640
Adjusted R^2	0.11	0.11	0.16	0.17	0.18	0.28	0.28	0.23	0.29	0.28	0.30	0.28

Table 4: **Pre-1993 Beta Exposures of a Simple Long-Short Strategy on Anomalies**

$$\text{Baseline:} \quad \beta_i^{pre} = b_0 + b_1 \left(\mathbf{1} \left(i \in \boldsymbol{I}_{Long} \right) - \mathbf{1} \left(i \in \boldsymbol{I}_{Short} \right) \right) / 2 + u_i$$

The table regresses the pre-1993 factor betas of 40 anomalies on the "long - short" dummy variable $\left(2^{-1} \left(\mathbf{1} \left(i \in \boldsymbol{I}_{Long} \right) - \mathbf{1} \left(i \in \boldsymbol{I}_{Short} \right) \right)\right)$ to show that a simple long-short strategy on the anomalies has a natural positive exposure to RMW and CMA but no significant exposure to MKT, SMB, and HML. I use the pre-1993 period to infer exposures uncontaminated by the act of arbitrage. In the parantheses are standard errors adjusted for cross-anomaly covariances in measurement errors. ***, **, and * indicate 1%, 5%, and 10% significance levels.

	MKT β^{pre}		SMB β^{pre}		HML β^{pre}		RMW β^{pre}		CMA β^{pre}	
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
Long - Short	-0.063		-0.128		0.015		0.331***		0.238**	
	(0.043)		(0.125)		(0.108)		(0.090)		(0.110)	
Long		1.004***		0.218***		-0.025		0.082*		0.151**
		(0.029)		(0.084)		(0.086)		(0.048)		(0.061)
Short		1.068***		0.346***		-0.039		-0.249***		-0.086
		(0.031)		(0.091)		(0.065)		(0.075)		(0.091)
Constant	1.036***		0.282***		-0.032		-0.083*		0.032	
	(0.021)		(0.062)		(0.054)		(0.044)		(0.055)	
Observations	40	40	40	40	40	40	40	40	40	40
Adjusted R^2	0.04	0.99	0.00	0.35	-0.03	-0.04	0.32	0.36	0.13	0.12

Table 5: **"Turning Alphas into Betas" in Beta Exposures to Fama-French Five Factors**

$$\text{Baseline: } \beta_i^{post} = b_0 + b_1 \alpha_i^{pre} + b_2 \beta_i^{pre} + u_i$$

The table shows the ability of pre-1993 alpha to explain the cross-section of post-1993 betas. I do this separately for factors that are likely to be systematic shocks to arbitrage capital (RMW and CMA) and for factors that are not (MKT, SMB, and HML), in light of the result in Table 4. Both alphas and betas are with respect to the Fama-French (2015) five-factor model. In the parentheses are standard errors adjusted for cross-anomaly covariances in measurement errors. ***, **, and * indicate 1%, 5%, and 10% significance levels.

	Factors That Proxy Arbitrage Capital Shocks								Other Fama-French Factors						
	RMW β^{post}				CMA β^{post}				MKT β^{post}			SMB β^{post}		HML β^{post}	
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)
Long - Short	0.444***				0.409***										
	(0.129)				(0.110)										
α^{pre}		0.054***				0.030**			-0.018***			0.005		-0.011	
		(0.011)				(0.014)			(0.006)			(0.009)		(0.012)	
β^{pre}		0.579***	0.981***			0.676***	0.627***		0.904***	1.265***		0.628***	0.582***	0.625***	0.658***
		(0.115)	(0.243)			(0.139)	(0.157)		(0.244)	(0.270)		(0.099)	(0.100)	(0.083)	(0.103)
$\alpha^{pre} \times$ Long				0.061***				-0.013			0.001				
				(0.022)				(0.026)			(0.008)				
$\alpha^{pre} \times$ Short				0.056***				0.026			-0.018**				
				(0.012)				(0.019)			(0.008)				
$\beta^{pre} \times$ Long				0.703**				0.368			0.936***				
				(0.337)				(0.307)			(0.202)				
$\beta^{pre} \times$ Short				0.616***				0.605***			0.834*				
				(0.131)				(0.216)			(0.454)				
Long				-0.083				0.197			-0.178				
				(0.069)				(0.128)			(0.505)				
Constant	-0.130**	0.012	-0.048	0.055	-0.066	-0.036	-0.086	-0.101	0.104	-0.239	0.204	-0.000	0.004	-0.002	0.018
	(0.062)	(0.034)	(0.045)	(0.051)	(0.053)	(0.048)	(0.056)	(0.095)	(0.251)	(0.269)	(0.471)	(0.025)	(0.024)	(0.034)	(0.047)
Observations	40	40	40	40	40	40	40	40	40	40	40	40	40	40	40
Adjusted R^2	0.29	0.86	0.48	0.85	0.40	0.55	0.34	0.59	0.72	0.57	0.74	0.75	0.75	0.58	0.55

Table 6: **Arbitrage Activity Explains the Endogenous Post-1993 Betas**

Baseline: $\beta_i^{post} = b_0 + b_1 ArbActivity_i^{post} + b_2\beta_i^{pre} + u_i$

The table relates the cross-section of post-1993 betas to the cross-section of post-1993 arbitrage activity using a cross-sectional regression with 40 anomalies. Arbitrage activity is measured as the negative ($\times -1$) of the "abnormal" short interest defined as the value-weighted average of short interest ratio (shares shorted / shares outstanding) minus the cross-sectional average short interest ratio of stocks that belong to the size NYSE size decile, where the average is taken over all stocks that belong to the anomaly portfolio and then over the post-1993 period. I use short interests reported in mid month and shares outstanding on the same day (if available) or the previous trading day. In the parentheses are standard errors adjusted for cross-anomaly covariances in measurement errors. ***, **, and * indicate 1%, 5%, and 10% significance levels.

	RMW β^{post}			CMA β^{post}		
	(1)	(2)	(3)	(4)	(5)	(6)
Post-1993 Arbitrage Activity	0.306***			0.216***		
	(0.073)			(0.083)		
Pre to Post-1993 Change in Arbitrage Activity		0.392***			0.259**	
		(0.088)			(0.101)	
Pre-1993 Arbitrage Activity × Long			0.310***			0.142
			(0.110)			(0.142)
Pre-1993 Arbitrage Activity × Short			0.333***			0.184*
			(0.090)			(0.109)
β^{pre}	0.574***	0.554***		0.592***	0.614***	
	(0.118)	(0.118)		(0.120)	(0.122)	
β^{pre} × Long			0.748***			0.552**
			(0.267)			(0.241)
β^{pre} × Short			0.584***			0.498***
			(0.156)			(0.159)
Long			-0.085			0.106
			(0.098)			(0.110)
Constant	0.057	0.053	0.106	0.013	0.004	-0.055
	(0.038)	(0.039)	(0.087)	(0.046)	(0.046)	(0.093)
Observations	40	40	40	40	40	40
Adjusted R^2	0.78	0.80	0.77	0.64	0.63	0.63

Table 7: **The Part of Post-1993 Beta Predicted by Pre-1993 Alpha Arises in Constrained Post-1993 Period**

$$\text{Baseline: } \beta_i^{post,\,constrained} = b_0 + b_1\alpha_i^{pre} + b_2\beta_i^{pre} + u_i$$

The table shows that the ability of pre-1993 alpha to explain the cross-section of post-1993 betas for RMW and CMA comes from its ability to predict betas in the constrained times of the post-1993 period. The constrained vs. unconstrained post-1993 periods are proxied by months in which the 3-month moving average of VIX is above vs. below the median (see Figure 1). Both alphas and betas are with respect to the Fama-French (2015) five-factor model. In the parentheses are standard errors adjusted for cross-anomaly covariances in measurement errors. ***, **, and * indicate 1%, 5%, and 10% significance levels.

	Unconstrained Post-1993 Period (Low-VIX)						Constrained Post-1993 Period (High-VIX)					
	α	ArbActivity	RMW β		CMA β		α	ArbActivity	RMW β		CMA β	
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
Long - Short	2.483*	0.011***	0.125		0.184		3.478	0.009***	0.194		0.285**	
	(1.485)	(0.002)	(0.124)		(0.126)		(2.301)	(0.002)	(0.124)		(0.135)	
α^{pre}				0.036**		0.008				0.058***		0.031
				(0.017)		(0.016)				(0.013)		(0.020)
β^{pre}			0.584**	0.439**	0.446**	0.578***			0.819***	0.580***	0.437***	0.673***
			(0.270)	(0.183)	(0.206)	(0.220)			(0.270)	(0.126)	(0.166)	(0.167)
Constant	-0.294	-0.006***	-0.075	-0.024	-0.012	-0.002	0.284	-0.003***	-0.080	0.001	-0.098	-0.051
	(0.794)	(0.001)	(0.057)	(0.046)	(0.071)	(0.069)	(1.073)	(0.001)	(0.055)	(0.038)	(0.063)	(0.054)
Observations	40	40	40	40	40	40	40	40	40	40	40	40
Adjusted R^2	0.18	0.38	0.38	0.62	0.27	0.22	0.29	0.28	0.50	0.88	0.49	0.56

Table 8: **The Role of Arbitrage Costs in the Alphas-into-Betas Relation**

Baseline: $\beta_i^{post} = b_0 + b_1\alpha_i^{pre} \times Cost_i + b_2\alpha_i^{pre} + b_3 Cost_i + b_4\beta_i^{pre} + u_i$

The table shows how the anomaly's size (expressed in $\times - 1$), idiosyncratic volatility ("Idivol"), illiquidity ("Amihud"), and bid-ask spread affects the level of arbitrage involvement inferred from the regression of post-1993 betas on pre-1993 alphas. The dependent variable is the post-1993 beta with respect to RMW or CMA, a proxy for systematic shocks to arbitrage capital. Both alphas and betas are with respect to the Fama-French (2015) five-factor model. The cost of arbitrage of an anomaly is calculated as value-weighted decile ranks (based on NYSE stocks) of the underlying stocks' costs of arbitrage as measured by size, illiquidity (Amihud, 2002), idiosyncratic volatility, or bid-ask spread (Corwin and Schultz, 2012). This measure is averaged over time within the pre-1993 period with little arbitrage disturbance and then cross-sectionally standardized for the ease of interpretation. In the parantheses are standard errors adjusted for cross-anomaly covariances in measurement errors. ***, **, and * indicate 1%, 5%, and 10% significance levels.

	RMW β^{post}				CMA β^{post}			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
α^{pre} × Cost	-0.001	-0.005	0.001	-0.005	-0.002	-0.012**	0.000	-0.016**
	(0.005)	(0.004)	(0.005)	(0.005)	(0.007)	(0.006)	(0.008)	(0.007)
α^{pre}	0.052***	0.049***	0.051***	0.048***	0.033***	0.028**	0.033***	0.031**
	(0.009)	(0.010)	(0.009)	(0.010)	(0.012)	(0.013)	(0.012)	(0.014)
Cost	-0.024	-0.070*	-0.020	-0.073**	0.022	-0.104*	0.036	-0.100*
	(0.030)	(0.038)	(0.029)	(0.034)	(0.042)	(0.063)	(0.041)	(0.060)
β^{pre}	0.559***	0.575***	0.563***	0.595***	0.675***	0.723***	0.665***	0.747***
	(0.094)	(0.092)	(0.092)	(0.091)	(0.119)	(0.121)	(0.120)	(0.125)
Constant	0.006	-0.014	0.009	-0.014	-0.035	-0.084*	-0.030	-0.092**
	(0.035)	(0.034)	(0.034)	(0.033)	(0.050)	(0.048)	(0.048)	(0.045)
Cost of Arbitrage	Size	Idivol	Amihud	Spread	Size	Idivol	Amihud	Spread
Observations	40	40	40	40	40	40	40	40
Adjusted R^2	0.85	0.86	0.85	0.87	0.53	0.59	0.54	0.61

Table 9: **Explaining the Cross-section of Funding-liquidity Betas**

The table applies the key cross-sectional tests to understand the cross-section of betas with respect to the funding-liquidity factor (also called “leverage factor”) of Adrian, Etula, and Muir (2014), controlling for the Fama-French (2015) (FF) five factors. The alphas are with respect to FF five factors. The anomaly data used in this analysis are quarterly (1974Q1-2016Q4) to match the quarterly frequency of the funding-liquidity factor. The constrained vs. unconstrained post-1993 periods are proxied by quarters in which the VIX is above vs. below the median. In the parentheses are standard errors adjusted for cross-anomaly covariances in measurement errors. ***, **, and * indicate 1%, 5%, and 10% significance levels.

	β_f^{pre}		β_f^{post}						Unconstrained β_f^{post}		Constrained β_f^{post}	
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
Long - Short	0.24		9.03						-5.03		16.80	
	(3.42)		(6.86)						(4.88)		(10.56)	
α^{pre}		-0.03		0.54*	0.56**	0.15				-0.01		0.93**
		(0.16)		(0.28)	(0.28)	(0.26)				(0.23)		(0.42)
β_f^{pre}					0.70	0.71	0.35	0.44	0.32	0.30	0.70	1.07
					(0.49)	(0.45)	(0.44)	(0.45)	(0.62)	(0.62)	(1.01)	(0.88)
α^{pre} × Short						0.61**						
						(0.30)						
β_f^{pre} × Short						0.17						
						(0.66)						
Post-1993 Arbitrage Activity							11.89*					
							(6.14)					
Pre to Post-1993 Change in Arbitrage Activity								14.51*				
								(7.48)				
Short						3.07						
						(3.38)						
Constant	0.99	0.79	-4.51	-0.97	-1.53	-1.13	0.54	0.10	-0.07	-0.13	-8.08	-2.36
	(1.64)	(1.13)	(3.70)	(2.45)	(2.67)	(2.29)	(2.12)	(2.17)	(3.01)	(2.19)	(6.95)	(4.73)
Observations	40	40	40	40	40	40	40	40	40	40	40	40
Adjusted R^2	-0.03	-0.02	0.10	0.53	0.60	0.69	0.58	0.57	0.11	-0.01	0.15	0.60

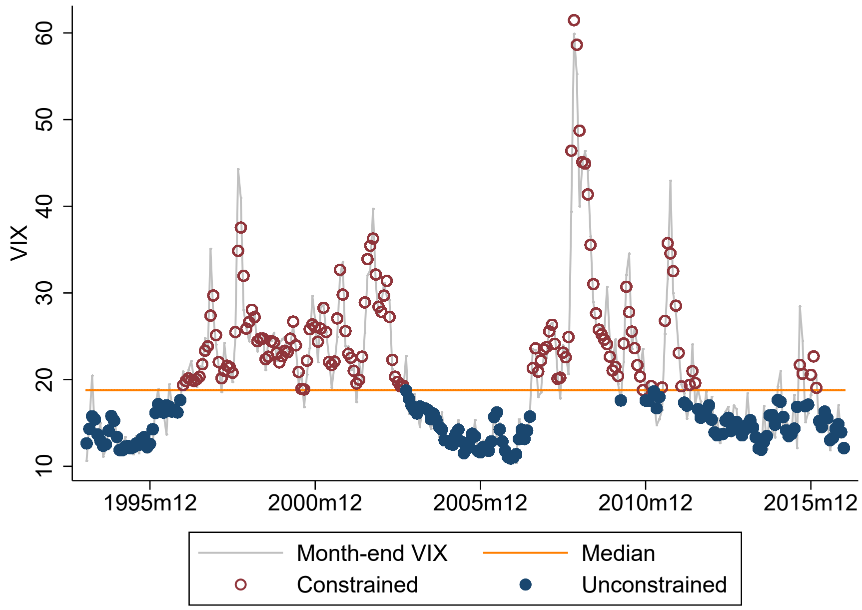

Figure 1: **Constrained vs. Unconstrained Post-1993 Periods Inferred from the VIX**

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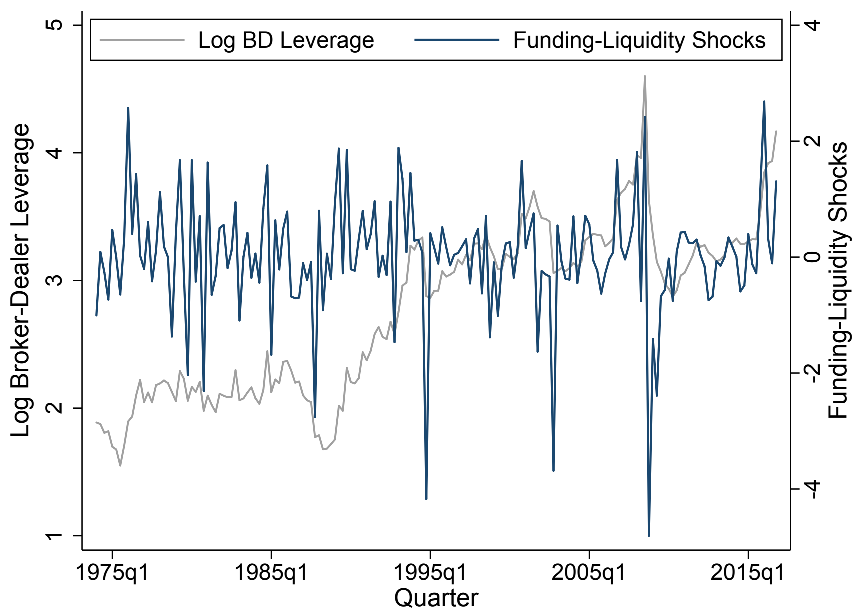

Figure 2: **Funding-Liquidity Shocks Measured by Broker-Dealer Leverage Shocks**

A Additional Tables

Table A1: **In-Sample to Out-of-Sample Change in the Estimated RMW and CMA Betas**

$$\text{Baseline: } \hat{\beta}_{i,t} = b_0 + \boldsymbol{b}_1 \left(\alpha_i^{pre} \times \mathbf{1}\left(t \notin sample\right)\right) + \boldsymbol{b}_2 \mathbf{1}\left(t \notin sample\right) + u_i + \epsilon_{i,t}$$

The dependent variable is the beta estimated over one year using daily returns on the anomaly and the factor. In the parentheses are standard errors not adjusted for cross-anomaly covariances in betas but adjusted for serial correlations in the betas within an anomaly. ***, **, and * indicate 1%, 5%, and 10% significance levels.

	RMW β^{post}				CMA β^{post}			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Long × Out-of-Sample	-0.006	-0.069			0.057	0.018		
	(0.048)	(0.051)			(0.067)	(0.066)		
Long × Post-1993		0.104*				0.065		
		(0.059)				(0.070)		
α^{pre} × Out-of-Sample			0.012***	-0.001			0.007	-0.001
			(0.003)	(0.003)			(0.007)	(0.006)
α^{pre} × Post-1993				0.022***				0.013**
				(0.005)				(0.006)
Out-of-Sample	-0.016	0.065*	0.002	0.030	-0.078*	-0.013	-0.038	-0.005
	(0.032)	(0.034)	(0.023)	(0.026)	(0.039)	(0.036)	(0.040)	(0.039)
Post-1993		-0.133**		-0.043		-0.108**		-0.053
		(0.050)		(0.028)		(0.047)		(0.040)
Constant	-0.128***	-0.117***	-0.126***	-0.118***	-0.012	-0.003	-0.012	-0.003
	(0.016)	(0.017)	(0.014)	(0.015)	(0.022)	(0.023)	(0.022)	(0.023)
Anomaly FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Observations	1,720	1,720	1,720	1,720	1,720	1,720	1,720	1,720
Adjusted R^2	-0.00	0.03	0.01	0.06	0.00	0.01	0.00	0.02

Table A2: **In-Sample to Out-of-Sample Change in the Estimated MKT, SMB, and HML Betas**

Baseline: $\hat{\beta}_{i,t} = b_0 + \boldsymbol{b}_1 \left(\alpha_i^{pre} \times \mathbf{1}\left(t \notin sample\right)\right) + \boldsymbol{b}_2 \mathbf{1}\left(t \notin sample\right) + u_i + \epsilon_{i,t}$

The dependent variable is the beta estimated over one year using daily returns on the anomaly and the factor. In the parentheses are standard errors not adjusted for cross-anomaly covariances in betas but adjusted for serial correlations in the betas within an anomaly. ***, **, and * indicate 1%, 5%, and 10% significance levels.

	MKT β^{post}		SMB β^{post}		HML β^{post}	
	(1)	(2)	(3)	(4)	(5)	(6)
Long $\times$ Out-of-Sample	-0.002		0.098**		0.005	
	(0.027)		(0.047)		(0.054)	
Long $\times$ Post-1993	0.009		-0.092		-0.005	
	(0.035)		(0.056)		(0.056)	
α^{pre} $\times$ Out-of-Sample		-0.006**		-0.001		-0.004
		(0.002)		(0.004)		(0.004)
α^{pre} $\times$ Post-1993		0.010***		-0.003		-0.002
		(0.003)		(0.005)		(0.007)
Out-of-Sample	0.009	-0.001	-0.025	0.023	0.079*	0.076***
	(0.021)	(0.012)	(0.033)	(0.025)	(0.043)	(0.027)
Post-1993	-0.059**	-0.037**	0.025	-0.027	-0.023	-0.028
	(0.026)	(0.016)	(0.036)	(0.030)	(0.047)	(0.026)
Constant	1.052***	1.051***	0.155***	0.155***	-0.088***	-0.089***
	(0.006)	(0.006)	(0.012)	(0.013)	(0.018)	(0.018)
Anomaly FE	Yes	Yes	Yes	Yes	Yes	Yes
Observations	1,720	1,720	1,720	1,720	1,720	1,720
AdjustedR^2	0.04	0.07	0.01	0.00	0.01	0.01

Table A3: **Post-1993 Beta Exposures of Equity Long-Short Hedge Funds**

Baseline: $r^e_{i,t} = \beta_0 + \beta_{MKT} r_{MKT,t} + \beta_{SMB} r_{SMB,t} + \beta_{HML} r_{HML,t} + \beta_{RMW} r_{RMW,t} + \beta_{CMA} r_{CMA,t} + \epsilon_{i,t}$

The table computes the post-1993 beta exposures of equity long-short hedge funds proxied by a 90/10 mix of equity market neutral hedge fund return and equity short-bias hedge fund indices from Hedge Fund Research (HFR). The standard errors are heteroskedasticity-robust OLS standard errors. ***, **, and * indicate 1%, 5%, and 10% significance levels.

	Equity Long-Short	Alternative Mix of Equity Market Neutral + Short Bias Hedge Funds					
	Hedge Funds (90/10)	100/0	80/20	60/40	40/60	20/80	0/100
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
MKT	0.007	0.085***	-0.070***	-0.225***	-0.380***	-0.534***	-0.689***
	(0.015)	(0.016)	(0.016)	(0.021)	(0.029)	(0.038)	(0.047)
SMB	0.005	0.045**	-0.035*	-0.114***	-0.194***	-0.273***	-0.352***
	(0.018)	(0.019)	(0.018)	(0.025)	(0.034)	(0.046)	(0.057)
HML	0.008	-0.020	0.036	0.091***	0.147***	0.202***	0.258***
	(0.023)	(0.024)	(0.025)	(0.032)	(0.042)	(0.053)	(0.065)
RMW	0.087***	0.068**	0.105***	0.142***	0.178***	0.215***	0.252***
	(0.023)	(0.028)	(0.022)	(0.031)	(0.046)	(0.063)	(0.081)
CMA	0.076**	0.058	0.093***	0.127***	0.162***	0.196***	0.231**
	(0.032)	(0.036)	(0.031)	(0.038)	(0.054)	(0.072)	(0.092)
Constant	0.136***	0.146***	0.126***	0.107*	0.087	0.067	0.048
	(0.047)	(0.050)	(0.048)	(0.064)	(0.089)	(0.117)	(0.146)
Observations	276	276	276	276	276	276	276
Adjusted R^2	0.16	0.15	0.48	0.72	0.77	0.79	0.80

Table A4: **Constrained vs. Unconstrained Post-1993 Period Betas: MKT, SMB, and HML**

$$\text{Baseline: } \beta_i^{post,\,constrained} = b_0 + b_1\alpha_i^{pre} + b_2\beta_i^{pre} + u_i$$

The table studies the ability of pre-1993 alpha to explain the cross-section of constrained vs. unconstrained post-1993 period betas for MKT, SMB, and HML. The constrained vs. unconstrained post-1993 periods are indicated in Figure 1. Both alphas and betas are with respect to the Fama-French (2015) five-factor model. In the parentheses are standard errors adjusted for cross-anomaly covariances in measurement errors. ***, **, and * indicate 1%, 5%, and 10% significance levels.

	Unconstrained Post-1993 Period (Low-VIX)						Constrained Post-1993 Period (High-VIX)					
	MKT β		SMB β		HML β		MKT β		SMB β		HML β	
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
Long - Short	-0.057		-0.135*		-0.083		-0.157***		0.046		-0.111	
	(0.046)		(0.070)		(0.090)		(0.060)		(0.077)		(0.123)	
β^{pre}	1.049***	0.940***	0.634***	0.589***	0.585***	0.575***	1.139***	0.914***	0.573***	0.643***	0.688***	0.640***
	(0.219)	(0.208)	(0.098)	(0.109)	(0.148)	(0.153)	(0.265)	(0.281)	(0.123)	(0.119)	(0.152)	(0.118)
α^{pre}		-0.009		-0.009		-0.002		-0.020**		0.009		-0.015
		(0.006)		(0.011)		(0.012)		(0.008)		(0.012)		(0.018)
Constant	-0.031	0.067	0.060**	0.058*	-0.047	-0.051	-0.111	0.088	-0.022	-0.026	0.043	0.016
	(0.214)	(0.205)	(0.029)	(0.030)	(0.048)	(0.045)	(0.264)	(0.290)	(0.029)	(0.029)	(0.070)	(0.049)
Observations	40	40	40	40	40	40	40	40	40	40	40	40
Adjusted R^2	0.70	0.72	0.78	0.74	0.47	0.45	0.64	0.68	0.65	0.66	0.50	0.52

B Theory Appendix

B.1 Solving the pre-arbitrage equilibrium

Proof of Lemma 1 (Asset returns in the pre-arbitrage economy). Since the behavioral investors alone clear the market, equation (1) implies that the price at time t is simply the price at time $t+1$ discounted by the behavioral investors' asset-specific discount factor $1 + r_{max} i$:

$$p_{i,t} = (1 + r_{max} i)^{-1} \, p_{i,t+1},$$

where the expected dividend term drops out by the zero-mean assumption $E_t[\delta_{i,t+1}] = 0$. It follows that the return on asset i at time $t+1$ is

$$r_{i,t+1} = \underbrace{r_{max} i}_{\equiv \alpha_i^{pre}} + \underbrace{(1 + r_{max} i)^{3-t} \frac{\delta_{i,t+1}}{v}}_{\equiv \epsilon_{i,t+1}}.$$

with $\delta_{i,3} = 0$ and $\partial \alpha_i^{pre} / \partial i = r_{max} > 0$.

B.2 Solving the post-arbitrage equilibrium

The arbitrageur's objective function in (2) implies the following value function at $t \in \{1, 2\}$:

$$\begin{gathered} V_t(w_t, f_t) = \max_{\{x_{i,t}\}} \; E_t[V_{t+1}(w_{t+1}, f_{t+1})] \\ \text{s.t. } \int_0^1 |x_{i,t}| \, di \leq (w_t + f_t) \\ w_{t+1} = w_t + \int_0^1 \left(\frac{p_{i,t+1} + \delta_{i,t+1}}{p_{i,t}} - 1 \right) x_{i,t} di + \tilde{w}_{t+1} \\ V_3 = w_3 \end{gathered}$$

in the non-default state ($w_t > 0$), and

$$V_t = (1 + c)^{3-t} \, w_t \tag{23}$$

in the default state ($w_t \leq 0$). Then, it is straightforward to solve the time-2 equilibrium prices:

Proof of Lemma 2 (Time-2 equilibrium prices). The arbitrageur's value function at time 2 in the non-default state ($w_2 > 0$) is

$$V_2 = w_2 + \max_{\{x_{i,2}\}} \left\{ \int_0^1 E_2 \left[r_{i,3}\right] x_{i,2} di + \psi_2 \left[w_2 + f_2 - \int_0^1 |x_{i,2}| \, di \right] \right\} \tag{24}$$

where $\psi_2 = 0$ if the arbitrageur is unconstrained. Since the arbitrageur does not take a negative position on the assets in equilibrium (doing so would generate a negative expected return due to the behavioral investor demand), the first order condition with respect to $x_{i,2}$ within the value function at time 2 implies

$$E_2 \left[r_{i,3}\right] \leq \psi_2$$

Now, suppose that there exists a marginal asset i_2^* such that the condition binds for $i \geq i_2^*$ and is slack for $i < i_2^*$. The unexploited assets ($i < i_2^*$) are priced by the behavioral investors using the demand curve in (1) so that $m_{i,3} = 1/\left(1 + r_{max} i\right)$. The exploited assets share the same discount factor $1/\left(1 + r_{max} i_2^*\right)$, which coincides with the behavioral investors' discount factor for the marginal asset. Finally, since the behavioral investors' demand in (1) and market clearing imply $\mu x_{i,1} = 2^{-1} x_{i,1} = i - i_2^*$, the marginal asset is given by

$$k_2 = \int_{i_2^*}^1 x_{i,1} di = \left(1 - i_2^*\right)^2 \Longleftrightarrow i_2^* = 1 - \sqrt{k_2}$$

in the constrained case. Given these discount factors, $E_2 \left[r_{i,3}\right] = r_{max} i_2^*$ for any $i \geq i_2^*$ and $E_2 \left[r_{i,3}\right] = r_{max} i < r_{max} i_2^*$ for $i < i_2^*$ so that i_2^* is indeed the marginal asset. If $k_2 \geq 1$, $1 - \sqrt{k_2} \leq 0$ and all assets are exploited so that $i_2^* = 0$. If $k_2 \leq 0$, no asset is exploited so that $i_2^* = 1$.

For completeness, one can obtain the equilibrium arbitrage position on asset i at time 2 from this proof:

$$x_{i,2} = \begin{cases} i - i_2^* & \text{if} \quad i \geq i_2^* \\ 0 & \text{if} \quad i < i_2^* \end{cases} .$$

Next is the marginal value of wealth and the value function at time 2:

Proof of Lemma 3 (Time-2 marginal value of wealth). First, consider $w_2 > 0$. The derivative of the value function (24) with respect to w_2 gives $\Lambda_2 = 1 + \psi_2$. For ψ_2 , the derivative with respect to any exploited asset's $x_{i,2}$ within the bracket implies $\psi_2 = E_2[r_{i,3}] = r_{max} i_2^*$, where the second equality follows from equation (5). Next, Λ_2 for $w_2 \le 0$ follows from equation (23). Finally, $V_2 = \Lambda_2 w_2$ since Lemma 2 implies that the marginal value of wealth $\Lambda_2 = 1 + \psi_2 = 1 + r_{max} i_2^*$ is also the average return on wealth in the non-default state and $w_3 = (1 + c) w_2$ in the non-default case.

Next, I derive the equilibrium prices at time 1:

Proof of Lemma 4 (Time-1 equilibrium prices). Since again the arbitrageur does not take negative positions in the assets in equilibrium, in the unconstrained case where $\psi_1 = 0$, taking the first order condition with respect to $x_{i,1}$ within the maximization bracket implies the fundamental theorem of asset pricing with $\Lambda_1 = E_1[\Lambda_2]$. In the constrained case, taking the first order condition with respect to $x_{i,1}$ within the maximization bracket implies $(E_1[\Lambda_2] + \psi_1) p_{i,1} \ge E_1[\Lambda_2 (p_{i,2} + \delta_{i,2})]$, which holds with equality if $x_{i,1} > 0$. Then, $\Lambda_1 = dV_1/dw_1 = E_1[\Lambda_2] + \psi_1$ pins down the prices of the exploited assets. The unexploited assets are priced by the behavioral investors so that $p_{i,1} = E_1\left[(1 + r_{max} i)^{-1} (p_{i,2} + \delta_{i,2})\right]$. Given these prices, the equilibrium arbitrage positions are given by behavioral investor demand (1). Finally, to obtain k_1^*, assume that all assets are exploited and combine (1) and (7) to obtain

$$E_1\left[\frac{\Lambda_2}{E_1[\Lambda_2]} (p_{i,2} + \delta_{i,2})\right] = \frac{E_1[p_{i,2} + \delta_{i,2}]}{1 + r_{max}(i - \mu x_{i,1})},$$

which gives

$$x_{i,1} = 2\left(i - \frac{1}{r_{max}}\left[\left(1 + Cov_1\left(\frac{\Lambda_2}{E_1[\Lambda_2]}, \frac{p_{i,2} + \delta_{i,2}}{E_1[p_{i,2}]}\right)\right)^{-1} - 1\right]\right).$$

Rearranging and setting $k_1^* = \int_0^1 x_{i,1} di$ gives

$$k_1^* = 1 - \frac{2}{r_{max}} \int_0^1 \left\{\left(1 + Cov_1\left(\frac{\Lambda_2}{E_1[\Lambda_2]}, \frac{p_{i,2} + \delta_{i,2}}{E_1[p_{i,2}]}\right)\right)^{-1} - 1\right\},$$

which is less than or equal to 1 since $Cov(\Lambda_2, p_{i,2} + \delta_{i,2}) = Cov(1 + r_{max} i_2^*, p_{i,2} + \delta_2) \le 0$ $\forall i$ since $p_{i,2} = v/(1 + r_{max} i_1^*)$ or $p_{i,2} = v/(1 + r_{max} i)$ and $i_2^* = 1 - \sqrt{k_2}$ where $k_2 = w_1 + \int_0^1 (p_{i,2} + \delta_2) x_{i,1} di$.

B.3 Proof of other lemmas

Proof of Lemma 5 (Equilibrium with unconstrained arbitrageurs). Since $k_2 \geq 1$ with certainty, $\Lambda_2 = 1$ and $i_2^* = 1$ with certainty. It follows from equation (5) that the prices of all assets at time 2 is $p_{i,2} = v$ and $r_{i,3} = \epsilon_{i,3} \equiv \delta_{i,3}/v$. It follows from this and the assumption $k_1 \geq k_1^* = 1$ (note $k_1^* = 1$ in this special case with no endogenous risk) that, by Lemma 4, $p_{i,1} = E_1 \left[p_{i,2} + \delta_{i,2}\right] = v$ so that $r_{i,2} = \epsilon_{i,2} \equiv \delta_{i,2}/v$.

Proof of Lemma 6 (Asset returns with constrained arbitrageurs). The expected return formula follows from an algebraic manipulation of Lemma 4. $\beta_{i,m} > 0$ is true because

$$Cov_1\left(r_{i,2}, m_2^A\right) = Cov_1\left(p_{i,2} + \delta_{i,2}, m_2^A\right) = Cov_1\left(\frac{v}{m_{i,2}}, m_2^A\right)$$

where $m_{i,2} = m_2^A$ when $i > i_2^*$ and $m_{i,2} = \left(1 + r_{max} i\right)^{-1}$ when $i \leq i_2^*$. Finally, $\lambda_m > 0$ by assumption since k_2 has a full support over $[0, 1]$.

Proof of Lemma 7 (A factor model of asset returns). Since $m_2^A = \left(1 + r_{max}\left(1 - \sqrt{k_2}\right)\right)/\Lambda_1$ at $k \in (0, 1)$, a first-order approximation around $\overline{k}_2 \equiv \left(1 - r_{max}^{-1}\left(E_1\left[\Lambda_2\right] - 1\right)\right)^2$ is $m_2^A \approx E_1\left[m_2^A\right] - r_{max}\left(2\Lambda_1\sqrt{\overline{k}_2}\right)^{-1}\left(k_2 - \overline{k}_2\right)$. Thus,

$$E\left[r_{i,2}\right] = \lambda_{i,0} + \lambda_m \beta_{i,m} \approx \lambda_{i,0} + \underbrace{\frac{r_{max} Var_1\left(k_2\right)}{2\Lambda_1 E_1\left[m_2^A\right]\sqrt{\overline{k}_2}}}_{\equiv \lambda_k} \underbrace{\frac{Cov_1\left(r_{i,2}, k_2\right)}{Var_1\left(k_2\right)}}_{\equiv \beta_{i,k}}.$$

Since $k_2 = w_2 + f_2 = w_1 + \left(\int_0^1 \left|x_{i,1}\right| di\right) r_2^A + \tilde{w}_2 + f_2$, this also means

$$E\left[r_{i,2}\right] \approx \lambda_{i,0} + \underbrace{\frac{r_{max}\left(\int_0^1 \left|x_{i,1}\right| di + a\right) Var_1\left(r_2^A\right)}{2\Lambda_1 E_1\left[m_2^A\right]\sqrt{\overline{k}_2}}}_{\equiv \lambda_r} \beta_{i,r} + \underbrace{\frac{r_{max} Var_1\left(\tilde{w}_2\right)}{2\Lambda_1 E_1\left[m_2^A\right]\sqrt{\overline{k}_2}}}_{\equiv \lambda_w} \beta_{i,w} + \underbrace{\frac{r_{max} Var_1\left(f_2\right)}{2\Lambda_1 E_1\left[m_2^A\right]\sqrt{\overline{k}_2}}}_{\equiv \lambda_f} \beta_{i,f}$$

To see $\beta_{i,k} > 0$ for $i > 0$, note

$$Cov_1 (r_{i,2}, k_2) = p_{i,1}^{-1} Cov_1 (p_{i,2} + \delta_{i,2}, k_2) = p_{i,1}^{-1} Cov_1 \left(\frac{v}{m_{i,2}}, k_2 \right)$$

where we know

$$\frac{\partial m_{i,2}}{\partial k_2} \leq 0$$

for $i > 0$. Also, for any random variable x, we know

$$Cov (x, f (x)) = E [(x - E [x]) (f (x) - E [f (x)])]$$

$$= \underbrace{E [(x - E [x]) (f (x) - f (E [x]))]}_{\geq 0}$$

$$+ \underbrace{E [(x - E [x]) (f (E [x]) - E [f (x)])]}_{=0}$$

$$\geq 0$$

if $f' (x) \geq 0$, which is the case when x is k_2 and $f (x)$ is $m_{i,2} (k_2)$.

B.4 Proof of the propositions

Proof of Proposition 1 (Pre-arbitrage alpha determines the cross-section of post-arbitrage betas). (i) The steps itself has two steps: first prove that the prices of high-i assets respond more strongly to the variation in arbitrage capital and then prove that this implies those assets have higher arbitrage capital betas. For the first step, since $Cov_1 (p_{i,2}, k_2) = E_1 [p_{i,2} k_2] - E_1 [p_{i,2}] E_1 [k_2]$,

$$Cov_1 (p_{i,2}, k_2) = v \int_{-\infty}^{k_2(i)} \frac{k_2}{1 + r_{max} i} dF (k_2) + v \int_{k_2(i)}^{\infty} \frac{k_2}{1 + r_{max} i_2^*} dF (k_2)$$

$$-v E_1 [k_2] \left(\int_{-\infty}^{k_2(i)} \frac{1}{1 + r_{max} i} dF (k_2) + \int_{k_2(i)}^{\infty} \frac{1}{1 + r_{max} i_2^*} dF (k_2) \right),$$

where $k_2 (i)$ denotes the value of k_2 that makes i the marginal asset, and F is the conditional conditional cumulative density function of k_2. The derivative of the covariance with respect to i gives

$$\frac{\partial Cov_1\left(p_{i,2},k_2\right)}{\partial i}=-v\int_{-\infty}^{k_2(i)}\frac{k_2}{\left(1+r_{max}i\right)^2}dF\left(k_2\right)+vE_1\left[k_2\right]\int_{-\infty}^{k_2(i)}\frac{1}{\left(1+r_{max}i\right)^2}dF\left(k_2\right),$$

where the Leibniz terms cancel out by the fact that $i_2^*\left(k_2\left(i\right)\right)=i$. Rearranging the terms gives

$$\frac{\partial Cov_1\left(p_{i,2},k_2\right)}{\partial i}=\frac{v}{\left(1+r_{max}i\right)^2}\left(E_1\left[k_2\right]-E_1\left[k_2\right|k_2\leq k_2\left(i\right)\right]\right)F\left(k_2\left(i\right)\right)$$

$$>0.$$

Next, to show how this monotonicity of the price covariance implies $\partial Cov_1\left(r_{i,2},k_2\right)/\partial i>0$, it suffices to show that the equilibrium time-1 prices are non-increasing in i:

$$\frac{\partial p_{i,1}}{\partial i}\leq 0.$$

To see this, suppose for a contradiction that $i<j$ but $p_{i,1}<p_{j,1}$. Suppose also that j is priced by the arbitrageur so that $p_{j,1}=E_0\left[\frac{\Lambda_2}{\Lambda_1}p_{j,2}\right]$. Since $p_{i,2}\geq p_{j,2}$ in all states of $t=2$, it must be that

$$p_{i,1}\geq E_1\left[\frac{\Lambda_1}{\Lambda_0}p_{i,2}\right]\geq E_1\left[\frac{\Lambda_1}{\Lambda_0}p_{j,2}\right]$$

which is a contradiction. Now suppose that j is priced by the behavioral investors so that $p_{j,1}=\frac{1}{1+r_{max}j}E_1\left[p_{j,2}\right]$. Again, since $p_{i,2}\geq p_{j,2}$ in all states of $t=2$, it must be that

$$p_{i,1}\geq\frac{1}{1+r_{max}i}E_1\left[p_{i,2}\right]\geq\frac{1}{1+r_{max}j}E_1\left[p_{j,2}\right]$$

which is also a contradiction. Hence, $p_{i,1}$ is non-increasing in i. Putting these together, we see that $Cov_1\left(r_{i,2},k_2\right)$ is non-decreasing in i:

$$\frac{\partial Cov_1\left(r_{i,2},k_2\right)}{\partial i}>0.$$

It follows that

$$\frac{\partial\beta_{i,k}}{\partial\alpha_i}=\frac{1}{r_{max}Var_1\left(k_2\right)}\times\frac{\partial Cov_1\left(r_{i,2},k_2\right)}{\partial i}>0.$$

(ii) This follows from a similar application of the method above, which I skip here.

(iii) Now I show that the proof holds similarly for $\beta_{i,m}$. This, I show formally. Note

$$\frac{\partial Cov_1\left(p_{i,2},\Lambda_2\right)}{\partial\left(\alpha_i\right)}=\frac{\partial Cov_1\left(p_{i,2},\Lambda_2\right)}{\partial\left(r_{max}i\right)}=\frac{\frac{\partial Cov_1\left(p_{i,2},\Lambda_2\right)}{\partial i}}{\frac{\partial\left(r_{max}i\right)}{\partial i}}=r_{max}^{-1}\times\frac{\partial Cov_1\left(p_{i,2},\Lambda_2\right)}{\partial i}$$

Now, since $Cov_1\left(p_{i,2},\Lambda_2\right)=E_1\left[\Lambda_2 p_{i,2}\right]-E_1\left[\Lambda_2\right]E_1\left[p_{i,2}\right]$,

$$Cov_1\left(p_{i,2},\Lambda_2\right)=v\int_{-\infty}^{0}\frac{1+c}{1+r_{max}i}dF\left(k_2\right)+v\int_{0}^{k_2(i)}\frac{1+r_{max}i_2^*}{1+r_{max}i}dF\left(k_2\right)+v\int_{k_2(i)}^{\infty}dF\left(k_2\right)$$

$$-vE_1\left[\Lambda_2\right](\int_{-\infty}^{0}\frac{1}{1+r_{max}i}dF\left(k_2\right)+v\int_{0}^{k_2(i)}\frac{1}{1+r_{max}i}dF\left(k_2\right)$$
$$+v\int_{k_1(i)}^{1}\frac{1}{1+r_{max}i_2^*}dF\left(k_2\right)+v\int_{1}^{\infty}dF\left(k_2\right)),$$

where $k_2\left(i\right)$ denotes the value of k_2 that makes i the marginal anomaly, and F is the conditional cumulative density function of k_1. Thus, the derivative of the covariance with respect to i gives

$$\frac{\partial Cov_1\left(p_{i,2},\Lambda_2\right)}{\partial i}=-v\left(\int_{-\infty}^{0}\frac{\left(1+c\right)r_{max}}{\left(1+r_{max}i\right)^2}dF\left(k_2\right)+\int_{0}^{k_2(i)}\frac{\left(1+r_{max}i_1^*\right)r_{max}}{\left(1+r_{max}i\right)^2}dF\left(k_2\right)\right)$$
$$+E_1\left[\Lambda_2\right]v\left(\int_{-\infty}^{0}\frac{r_{max}}{\left(1+r_{max}i\right)^2}dF\left(k_2\right)+\int_{0}^{k_2(i)}\frac{r_{max}}{\left(1+r_{max}i\right)^2}dF\left(k_2\right)\right),$$

where the Leibniz terms cancel out by the fact that $i_2^*\left(k_2\left(i\right)\right)=i$. Rearranging the terms gives

$$\frac{\partial Cov_1\left(p_{i,2},\Lambda_2\right)}{\partial i}=-\frac{v\,r_{max}}{\left(1+r_{max}i\right)^2}\left(\int_{-\infty}^{k_2(i)}\Lambda_2 dF\left(k_2\right)-E_1\left[\Lambda_2\right]\int_{-\infty}^{k_2(i)}dF\left(k_2\right)\right)$$
$$=-\frac{v\,r_{max}}{\left(1+r_{max}i\right)^2}\left(E_1\left[\Lambda_2\middle|\,i<i_2^*\right]-E_1\left[\Lambda_2\right]\right)F\left(k_2\left(i\right)\right)$$
$$<0$$

since $E_1\left[\Lambda_2\middle|\,i<i_2^*\right]>E_1\left[\Lambda_2\right]$. This monotonicity of price covariance implies the monotonicity of return covariance by the same logic as in (i).

Proof of Proposition 2 (The cross-section of average arbitrage positions explains the cross-section of post-arbitrage betas). Since $x_{i,2} = i - i_2^*$, the expectation of aggregate arbitrage position is $\mu E_1[x_{i,2}] = \mu(i - E_1[i_2^*])$. Thus, $\partial\beta_i/\partial(\mu E_1[x_{i,2}]) = (\partial\beta_i/\partial i) \times (\partial i/\partial(\mu E_1[x_{i,2}])) \propto \partial\beta_i/\partial i > 0$.

Proof of Proposition 3 (The endogenous post-arbitrage beta arises when the arbitrageur is constrained). Follows trivially from the analysis in Lemma 5.

Proof of Proposition 4 (An upward-biased price of risk in a naive asset pricing test). The data-generating process considered by the econometrician is

$$r_{i,2} = \begin{cases} r_{i,2}^{pre} = \alpha_i^{pre} + \epsilon_{i,2}^{pre} & \text{with probability } \phi \\ r_{i,2}^{post} \approx \alpha_{i,0} + \lambda_k \beta_{i,k} + \epsilon_{i,2}^{post} & \text{with probability } 1-\phi \end{cases}$$

where $\beta_{i,k} \approx b\,\alpha_i^{pre}$. Hence, $E[r_{i,2}] \approx \phi\alpha_i^{pre} + (1-\phi)(\alpha_{i,0} + \lambda_k\beta_{i,k}) \approx (1-\rho)\lambda_{i,0} + \left(\lambda_{k,0} + \underbrace{\phi b^{-1}}_{\text{bias}}\right)\beta_{i,k}$.